THE BACKGROUND RADIATION IN ISOTROPIC WORLD MODELS

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Summary

A general equation for the extragalactic background intensity is derived for the general case of an isotropic homogeneous universe. The extragalactic radio spectrum depends critically on the thermal history of the universe and numerical results have been obtained for different possible models. A brief discussion is given on the conclusions drawn from the comparison of these results with the observed background spectrum.

I. Introduction

Parameters such as the density and temperature of the intergalactic material are of fundamental importance to all cosmologies, and on the basis of proposed thermal histories of the universe several cosmological models have been suggested with widely differing properties. Layzer (1963, 1966) has advanced the concept of a constant temperature universe in which local non-uniformities in the distribution of matter give rise to a negative gravitational contribution to the internal energy density. The expansion of the metagalaxy tends to decrease the kinetic contribution to the internal energy, and to increase the gravitational contribution. Layzer suggests that when account is taken of energy losses by radiation, the temperature of the gas remains approximately constant with time. Ginzburg and Ozernoi (1966) have calculated theoretical thermal models from considerations of heating and cooling mechanisms in an adiabatically expanding universe. Special difficulties are raised in the interpretation of the microwave observations in relation to the thermal history of the universe, and this problem is examined in Section IV. There is also a whole series of conjectural thermal models based on simple evolutionary and steady-state cosmologies. It is the aim of the present paper to examine these models in terms of the expected background radiation with particular emphasis on the low frequency end of the spectrum.

Cosmological theories for the background radiation are, of course, restricted by the lack of information on the extragalactic radio spectrum. However, it is hoped that the present series of observations of galactic emission will yield estimates of the extragalactic background intensity. Much attention has been given in the past to the high frequency observations of sky brightness, but it is felt that a considerable amount of useful information can be obtained from measurements at low frequencies. There is at present no direct evidence for the existence of intergalactic ionized hydrogen, and an estimate (from low frequency results) of its density could resolve many of the difficulties now confronting current theories.

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The radio background is considered here to be the sum of contributions from two sources: (1) the emission by ordinary radio galaxies and (2) free–free emission from intergalactic ionized hydrogen. Account will also be taken of the free–free absorption that must occur at low frequencies if significant amounts of ionized hydrogen actually exist in the space between galaxies.

(a) *Emission from Radio Sources*

The background intensity due to radio sources depends fairly critically (at least at low frequencies) on the luminosity distribution, the proper density of sources at the present epoch, and the source average spectral index. It is well known that the number–flux relation for sources in an isotropic homogeneous universe can be written approximately as

$$N = \tfrac{1}{3} n P^{3/2} S^{-3/2}, \tag{1}$$

where N is the number of radio sources per steradian with flux densities exceeding S, P is the mean luminosity, and n is the number of sources per unit proper volume. If it is assumed that P has some arbitrary dispersion function and the number of sources in the infinitesimal range of P is of the form

$$\rho(P/P_0)\,\mathrm{d}(P/P_0),$$

where ρ is the cosmic density and P_0 the mean of the luminosity distribution, then we must write instead of (1)

$$N = \tfrac{1}{3} S^{-3/2} \int_{P_1}^{P_2} \rho(P/P_0)\, P^{3/2}\, \mathrm{d}(P/P_0) = \tfrac{1}{3} n_0 P_0^{3/2} S^{-3/2}, \tag{2}$$

where n_0 may now be regarded as the weighted mean density of radio sources.

The luminosity distribution has been derived by Longair and Scott (1965) using extensive optical data and they find the mean luminosity of radio sources at 178 MHz to be $P_0 = 8 \times 10^{25}$ W Hz^{-1} sr^{-1}. Gower (1966) has derived the complete $(\log N)/(\log S)$ curve using revised 3C data, together with the results from the 4C and North Polar survey (Ryle and Neville 1962). According to equation (2) the local value for the number of sources per unit proper volume can be estimated from the value of N at large flux densities where the effects of redshift are negligible and the statistical fluctuations are small. This method yields a value for the local source density of $n_0 = 4 \cdot 24 \times 10^{-75}$ m^{-3}. The mean spectral index for radio sources will be taken from the recently completed sections of the Parkes catalogue for the declination zones 0° to +20° and −20° to 0° (Day *et al.* 1966; Shimmins *et al.* 1966). These surveys have the important advantage that the same instrument has been used to observe the sources at three different frequencies, thus eliminating errors due to source confusion and selection. The average spectral index over the frequency range 85·5–2650 MHz is 0·88, and this value will be used throughout the present analysis. It will be assumed here (at least initially) that the spectral index and radio luminosity of a radio source do not change with advancing epoch, although it must be noted that if the universe is evolving then it is likely that the sources will also evolve.

(*b*) *Free–Free Emission from Ionized Hydrogen*

Various cosmological theories predict a mean density for the universe which, according to Oort (1958), is about two orders of magnitude greater than the mean density of matter in galaxies averaged over all space. Similar results have been found by other workers (Sandage 1961; Hoyle and Narlikar 1962; Sciama 1964) from analyses based on steady-state and evolutionary cosmologies. They find a mean value for the density of matter in the universe of approximately 2×10^{-29} g cm^{-3}, whereas the mean density of matter in the galaxies is about 5×10^{-31} g cm^{-3}. If it is assumed that there are no aggregations of matter in the universe other than the types of galaxies already known, then there is a problem of the specification of the intergalactic material. For the lack of any other evidence, the bulk of the matter density is attributed to uncondensed intergalactic hydrogen. There are several experimental results that support the argument for an intergalactic medium of ionized hydrogen, although it should be noted that there is no direct evidence for an ionized medium and the only information obtained so far concerns atomic hydrogen. Goldstein (1963) made measurements of the 21 cm line emission in the direction of the north celestial pole, and concluded that the intergalactic atomic hydrogen density is $n_{\mathrm{H}} < 2 \cdot 1 \times 10^{-25}$ cm^{-3}. According to the measurements by Davies (1964), on the absorption spectrum of Cygnus A, n_{H} is less than 9×10^{-8} cm^{-3} for an Einstein–de Sitter universe. Gunn and Peterson (1965) examined the red-shifted Lyman α-line of the quasi-stellar source 3C 9 for any evidence of photon scattering by intergalactic hydrogen. They found the spatial density of atomic hydrogen to be almost negligible for several cosmological models. On the basis of these results, it will be assumed in the following work that all the matter in the universe exists as ionized hydrogen.

The thermal history of the universal plasma is of fundamental importance to any analysis of the background radiation. Several authors have considered the question of the temperature of the intergalactic gas, and a brief account will be given summarizing these investigations. A useful upper limit to the temperature at the present epoch can be calculated from the requirement that the background intensity cannot exceed the observed flux in the X-ray region of the spectrum. This method was employed by Field and Henry (1964) to derive an upper limit of 4×10^{6} °K in the steady-state model and 3×10^{6} °K in evolutionary models. These results proved. the hot (10^{9} °K) universe proposed by Gold and Hoyle (1959) to be invalid. Kahn and Woltjer (1959) obtained a temperature of 5×10^{5} °K for the ionized hydrogen in the local group by assuming that the gravitational self-attraction was balanced by the kinetic pressure nkT. The temperature of the intergalactic gas was estimated by Field (1965) to be 5×10^{4} °K on the basis of theoretical investigations of thermal instabilities leading to the formation of galactic clusters. Sciama (1964) proposed that the temperature of the intergalactic gas would be about 10^{5} °K if an equilibrium state exists in which heating of the gas by cosmic ray ionization is balanced by cooling due to bremsstrahlung and recombination radiation.

Gould and Ramsay (1966) have examined the question of the temperature of the intergalactic gas in terms of a quasi-equilibrium state in which a thermal balance is attained in a time less than the characteristic time of expansion of the universe. They consider that heating of the gas is a result of ionization by a universal cosmic ray flux and the dissipation of hydrodynamic turbulence while cooling is due to inelastic electron collisions with H, He, and He^- and by free–free emission and recombination radiation. It should be pointed out here that the density flux of these universal cosmic rays, and indeed their very existence in intergalactic space, is at present the subject of considerable conjecture. However, Gould and Ramsay adopt a universal flux mainly because of the simplicity that this assumption introduces into the equations, and, by assuming the cosmic ray spectrum calculated by Pollack and Fazio (1963), they find a probable equilibrium temperature in the range 10^4–5×10^4 °K, depending on the density and a possible non-equilibrium temperature greater than 2×10^5 °K.

On the basis of these investigations, we shall take, as a very reasonable assumption, a temperature at the present epoch in the range 10^4–10^6 °K, regardless of the density of the intergalactic material. In Section IV, we examine in closer detail possible thermal histories of the universe and the expected background radiation.

II. Equation for Background Radiation

This analysis is confined to homogeneous isotropic world models that can be specified by the Robertson–Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{R^2}{c^2}\left(\frac{\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2)}{(1+\frac{1}{4}kr^2)^2}\right), \tag{3}$$

where $\mathrm{d}s$ is the time interval between events, t is the cosmic time, $R(t)$ represents the linear scale of the universe, (r,θ,ϕ) are dimensionless spatial coordinates relative to the Earth as origin, and k, which can take the values 1, 0, -1, is the curvature of space at the present time.

(a) *Evolutionary Models*

Equation (3) can be derived from the set of Einstein field equations

$$-\kappa c^2 T^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}(R^\rho_\rho - 2\Lambda), \tag{4}$$

where κc^2 is a constant, $T^{\mu\nu}$ is the energy tensor, $g^{\mu\nu}$ is the contravariant metrical tensor, Λ is the cosmical constant, and R is the Ricci tensor. For a uniform isotropic medium the energy tensor can be written

$$T^{\mu\nu} = (\rho + p/c^2)u^\mu u^\nu - g^{\mu\nu}p/c^2, \tag{5}$$

where $u^\mu = \mathrm{d}x^\mu/\mathrm{d}s$, and p and ρ are the cosmic pressure and density respectively. Equations (4) and (5) reduce to the well-known equations for the density and pressure distribution in uniform world models

$$8\pi G\rho = \frac{3}{R^2}\left\{kc^2 + \left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2\right\} - \Lambda, \tag{6}$$

and

$$\frac{8\pi G p}{c^2} = -\frac{2}{R}\frac{\mathrm{d}^2R}{\mathrm{d}t^2} - \frac{1}{R^2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{kc^2}{R^2} + \Lambda, \tag{7}$$

where $\kappa c^2 = 8\pi G$.

It follows from (6) and (7) that

$$\frac{\mathrm{d}(\rho R^3)}{\mathrm{d}t} + \frac{p}{c^2}\frac{\mathrm{d}R^3}{\mathrm{d}t} = 0, \tag{8}$$

an equation that is applicable to all evolving models of the universe. We define the Hubble "constant" by

$$H = (1/R_0)\,\mathrm{d}R_0/\mathrm{d}t_0 \tag{9}$$

and the acceleration parameter by

$$q_0 = -\frac{1}{R_0}\frac{\mathrm{d}^2R_0}{\mathrm{d}t_0^2}\left(\frac{R_0}{\mathrm{d}R_0/\mathrm{d}t_0}\right)^2. \tag{10}$$

At the present epoch t_0, we can write equations (6) and (7) with the aid of (9) and (10) as

$$8\pi G\rho_0 = 3(H^2 + kc^2/R_0^2) - \Lambda \tag{11a}$$

and

$$8\pi G p_0/c^2 = (2q_0 - 1)H^2 - kc^2/R_0^2 + \Lambda, \tag{11b}$$

and introducing the density parameter σ_0 and the pressure parameter ϵ_0 by

$$\sigma_0 = (4\pi G/3H^2)\rho_0 \qquad \text{and} \qquad \epsilon_0 = p_0 c^{-2}/\rho_0, \tag{12}$$

we obtain the equations

$$3H^2\{(1+3\epsilon_0)\sigma_0 - q_0\} = \Lambda \tag{13}$$

and

$$3(1+\epsilon_0)\sigma_0 - (q_0+1) = kc^2/H^2R_0^2. \tag{14}$$

Observations indicate that the pressure term $p_0 c^{-2}$ is small compared with the density ρ_0. Accordingly we take $p_0 c^{-2} = 0$ in the following analysis.

Integration of equation (8) therefore yields

$$\rho R^3 = \rho_0 R_0^3, \tag{15}$$

and with $\epsilon_0 = 0$ in equations (13) and (14) we have finally

$$3H^2(\sigma_0 - q_0) = \Lambda \tag{16}$$

and

$$H^2(3\sigma_0 - q_0 - 1) = kc^2/R_0^2. \tag{17}$$

We now have sufficient equations to derive the general equations for the background radiation in evolving models. Consider the 3-space specified by the coordinates (r,θ,ϕ). According to equation (3), these three mutually perpendicular coordinates define a volume element

$$\mathrm{d}v = \{R^3r^2/(1+\tfrac{1}{4}kr^2)^3\}\mathrm{d}r\,\mathrm{d}\Omega\,, \tag{18}$$

where $\mathrm{d}\Omega$ is the solid angle subtended by the element, as measured by the origin observer.

Further, let us suppose that the emission coefficient (watts per unit proper volume) is $j(\nu,t)$. The total flux received at the origin at time t_0 will be $\mathrm{d}S(\nu_0)\,\mathrm{d}\nu_0$ ($\mathrm{W\,m^{-2}}$) given by

$$\mathrm{d}S(\nu_0)\,\mathrm{d}\nu_0 = \{j(\nu,t)/4\pi D^2(t)\}\mathrm{d}v\,\mathrm{d}\nu\,, \tag{19}$$

where D is the luminosity distance defined by

$$D = R_0^2\,r/R(1+\tfrac{1}{4}kr^2) \tag{20}$$

and

$$\mathrm{d}\nu = \mathrm{d}\nu_0\,.\,R_0/R\,. \tag{21}$$

Combining equations (18) to (20), we obtain for the received flux density at frequency ν_0

$$\mathrm{d}S(\nu_0) = \{j(\nu,t)\,R^4/R_0^3(1+\tfrac{1}{4}kr^2)\}\mathrm{d}r\,\mathrm{d}\Omega/4\pi\,. \tag{22}$$

Now the equation for the null geodesic is

$$\mathrm{d}r/(1+\tfrac{1}{4}kr^2) = -(c/R)\mathrm{d}t\,, \tag{23}$$

and substituting in (22) we have

$$S(\nu_0) = -c\left(\int_{t_0}^{t} j(\nu,t)\,(R/R_0)^3\,\mathrm{d}t\right)\mathrm{d}\Omega/4\pi\,. \tag{24}$$

If the absorption coefficient of ionized hydrogen in intergalactic space is denoted by $K(\nu,t)$, then equation (24) must be modified appropriately so that we have finally for the background radiation in evolving world models

$$I(\nu_0) = \frac{c}{4\pi}\int_{\omega_0}^{1} j(\nu_0\,\omega^{-1},\omega)\,\omega^3\exp\left(-c\int_{\omega}^{1} K(\nu_0\,\omega'^{-1},\omega')\frac{\partial t}{\partial\omega'}\,\mathrm{d}\omega'\right)\frac{\partial t'}{\partial\omega}\,\mathrm{d}\omega\,, \tag{25}$$

where ω is the dimensionless ratio R/R_0 and the exponential is analogous to optical depth. Equation (25) is not directly integrable until expressions have been found for $\partial t/\partial\omega$ as a function of ω. The differential $\partial t/\partial\omega$ is calculated easily by substituting R from equation (6) into the identity $\partial t/\partial\omega = R_0(\mathrm{d}R/\mathrm{d}t)^{-1}$, and we find

$$\partial t/\partial\omega = R_0\{\tfrac{1}{3}(8\pi G\rho+\Lambda)R^2-kc^2\}^{-\frac{1}{2}}. \tag{26}$$

With the aid of equations (12), (15), (16), and (17), this yields for $k \neq 0$

$$\partial t/\partial\omega = (\omega^{\frac{1}{2}}/H)\{2\sigma_0+(q_0+1-3\sigma_0)\omega-3(q_0-\sigma_0)\omega^3\}^{-\frac{1}{2}}\,, \tag{27}$$

and for $k = 0$

$$\partial t/\partial\omega = (\omega^{\frac{1}{2}}/H)\{2\sigma_0-3(q_0-\sigma_0)\omega^3\}^{-\frac{1}{2}}\,. \tag{28}$$

If the parameters q_0 and σ_0 are specified, then a particular world model is uniquely defined, and together with an estimate of the function $j(\nu, \omega)$ we can calculate the expected background radiation by the numerical integration of equation (25).

The ordinary absorption coefficient in ionized hydrogen at the epoch defined by the parameter ω can be written

$$K(\nu, \omega) = Ag(\omega)\{1-\exp(-h\nu_0/kT\omega)\}\omega^{-3}\,T^{-\frac{1}{2}}\,N_0^2\,\nu_0^{-3}\,, \tag{29}$$

where the subscripts correspond to values at the present epoch, A is a constant, $N = N_0\,\omega^{-3}$ is the electron density, and $T \equiv T(\omega)$ is the kinetic temperature of the intergalactic medium. The type of functional dependence of T on epoch will largely determine the absorption characteristics of the intergalactic ionized hydrogen, and in Section IV we consider this problem in more detail. The Gaunt factor $g(\omega)$ is a slowly varying function of frequency and temperature, and in the following work we shall approximate this function by the use of numerical results obtained by Karzas and Latter (1961), which indicate that g is between $0\cdot8$ and $1\cdot0$ for a temperature range of 10^4 to 10^{10} °K.

(b) *Steady-state Models*

If a vector $C^\mu = 3H(0,0,0,1)$ is introduced into the field equations (4) and the cosmological constant is neglected, then we have

$$-\kappa c^2 T^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}\,R^\rho_\rho + C^{\mu\nu}\,, \tag{30}$$

where $C^{\mu\nu}$ is obtained by the covariant differentiation of C^ν. The field equations (30) now yield

$$\frac{2}{R}\frac{\mathrm{d}^2R}{\mathrm{d}t^2} + \frac{1}{R^2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{3H}{R}\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = 0 \qquad \text{and} \qquad \frac{3}{R^2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \kappa\rho c^2\,. \tag{31}$$

If $R = R_0$ at $t = t_0$, then it follows that

$$R = R_0\exp\{-H(t-t_0)\} \qquad \text{and} \qquad \rho = 3H^2/4\pi G\,.$$

This metric is identical to the de Sitter metric obtained from equations (16) and (17) by defining $q_0 = -1$ and $k = 0$, with the density parameter σ_0 consequently equal to zero. However, in the steady-state universe the density is a constant nonzero quantity and we can write the equation to the background radiation in a form analogous to equation (25) as

$$I(\nu_0) = \frac{c}{4\pi}\int_{\omega_0}^{1} j(\nu_0\,\omega^{-1})\,\omega^3\exp\left(-c\int_{\omega}^{1} K(\nu_0\,\omega'^{-1})\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)\frac{\partial t}{\partial\omega}\,\mathrm{d}\omega\,, \tag{32}$$

the emission and absorption coefficients being now functions of frequency only. Following equation (29), the absorption coefficient in the steady-state model will be

$$K(\nu) = Ag(\omega)\{1-\exp(-h\nu_0/kT_0\,\omega)\}\omega^3\,T_0^{-\frac{1}{2}}\,N_0^2\,\nu_0^{-3}. \tag{33}$$

The intensity of the radio background can now be derived for both evolutionary and steady-state models after the emission coefficient has been calculated.

The lower limit ω_0 of equations (25) and (32) depends on the ω corresponding to the observer's particle horizon. In evolutionary cosmologies with $\epsilon_0 = 0$ the limit is usually equal to zero and the geometry of the model assists the convergence of the integral. In oscillating models the limit is undefined, unless the universe is treated as existing for a finite time (the time for one oscillation). Some models with large negative values for q_0 collapse from an infinitely rarefied state to a state of finite density corresponding to an expansion parameter value of $\omega = \omega_{\min} > 0$. At this stage, the universe reverses its motion and expands monotonically into an empty de Sitter model. The limit of integration in this case will be $\omega_0 = \infty$ and (25) must be integrated through a singularity at $\omega_{\min}$ where $\mathrm{d}\omega/\mathrm{d}t = 0$. Unless a cutoff is introduced for the emission coefficient at a particular finite ω, the derived background will diverge to infinity. In the analyses of possible thermal models in Section IV it will be assumed that $\sigma_0 = q_0 = 0\cdot5$. This means that $\partial t/\partial\omega = \omega^{0\cdot5}$, and not only will the equations have a greater simplicity but the time taken for the numerical computation of equation (25) will be significantly reduced. The integral of equation (32) for the background radiation in steady-state cosmologies has a lower limit $\omega_0 = 0$ and the geometry of the metric aids the convergence of the integral, although not as strongly as in most of the evolutionary models.

III. Emission Coefficient

(a) *Radio Sources*

It is well known that many radio sources exhibit low frequency turnovers in their spectra due to a variety of possible causes including thermal absorption, synchrotron self-absorption in the source itself, and a cutoff in the electron energy distribution. We will approximate the low frequency spectral behaviour of sources by assuming that the power emitted is of the form

$$P(\nu) = C(\nu^{-\alpha}/\nu_c^2)[1-\exp\{-(\nu/\nu_c)^2\}], \tag{34}$$

where ν_c is the critical frequency corresponding to unit optical depth. We have further assumed here that P is not a function of epoch, the dispersion in the radio source luminosity is zero, and the spectrum is linear for frequencies $\nu \gg \nu_c$. The last assumption may, in some models, lead to an excess in the expected background radiation at high frequencies since a common feature of source spectra is a high frequency cutoff due to electron energy losses by synchrotron emission (Kellermann 1964, 1966). This effect will be negligible in those models for which the contribution to the background intensity by free–free emissions is relatively high.

Now in all models $\nu = \nu_0\,\omega^{-1}$, and in evolving zero-pressure models $n = n_0\,\omega^{-3}$, while for the steady-state models $n = n_0$, where n is the number of sources per unit proper volume and n_0 is the value of n at the present epoch. Hence we have by substitution into equation (34)

$$nP = n_0\frac{\bar{P}_{178}}{178\times10^6}\frac{\nu_0^{-\alpha+2}}{\nu_c^2}\,\omega^{\alpha-5}\left[1-\exp\left\{-\left(\frac{\nu_c\omega}{\nu_0}\right)^2\right\}\right] \tag{35}$$

for evolutionary universes with $p/c^2 = 0$, and

$$nP = n_0 \frac{\overline{P}_{178}}{178\times10^6} \frac{\nu_0^{-\alpha+2}}{\nu_c^2} \omega^{\alpha-2}\left[1-\exp\left\{-\left(\frac{\nu_c\,\omega}{\nu_0}\right)^2\right\}\right] \tag{36}$$

for steady-state cosmologies, where nP is the emission per unit volume for radio sources. From the values obtained for n_0 and $P_0 = \overline{P}_{178}/4\pi$ in Section I(a), the constant term $n_0\overline{P}_{178}/(178\times10^6)$ in (35) and (36) is equal to $2\cdot39\times10^{-56}$ MKS units.

(b) *Free–Free Emission*

According to Allen (1965), the free–free emission per unit volume and unit frequency range for ionized hydrogen of density N and temperature T is

$$\epsilon(\nu) = BgN^2\,T^{-\frac{1}{2}}\exp(-h\nu/kT)\,, \tag{37}$$

where $B = 6\cdot8\times10^{-39}$ MKS units and g is the Gaunt factor. Substituting $N = N_0\,\omega^{-3}$ we obtain for evolutionary universes

$$\epsilon(\nu,\omega) = Bg(\omega)\exp(-h\nu_0/kT\omega)\,\omega^{-6}\,N_0^2T^{-\frac{1}{2}}\,, \tag{38}$$

where, as before, $T \equiv T(\omega)$ is as yet an unknown function of the expansion factor ω.

We have, similarly, for the steady-state models

$$\epsilon(\nu) = Bg(\omega)\exp(-h\nu_0/kT_0\,\omega)\,N_0^2T_0^{-\frac{1}{2}}\,. \tag{39}$$

Equations (38) and (39) clearly show that, unless the temperature T is a steep function of ω, the background radiation due to free–free emission will be considerably greater in evolutionary cosmologies than in those of the steady-state theory. The total emission coefficient in either cosmology may now be written in the general form

$$j(\nu,\omega) = n(\omega)\,P(\nu,\omega) + \epsilon(\nu,\omega)\,, \tag{40}$$

where the function $j(\nu,\omega)$ will be appropriate to equations (25) and (32). With the aid of equations (35)–(39), we now have sufficient information to derive the background radiation on the basis of different possible thermal histories of the universe.

IV. Background Intensity

It is the purpose of this section to examine the predicted background spectrum as a function of possible thermal histories of the universe. As far as the author is aware, only one account has been given of the background radiation in the low frequency region of the radio spectrum. Kaufman (1965), who restricted her analysis to the Einstein–de Sitter universe, derived theoretical background spectra with the assumption that the ionized gas behaved isothermally back to infinite densities. Kaufman assumed that the observed background spectrum was due entirely to extragalactic sources, and by fitting theoretical curves to the experimental spectrum in the frequency range 5–100 MHz, and to the spectral point at 4080 MHz (Penzias and Wilson 1965), she was able to determine limits for the intergalactic ionized hydrogen density. However, on the basis of available evidence it is unlikely that the extragalactic component could contribute more than 10% to the observed background intensity, and until reliable experimental estimates can be made for the extragalactic intensity it will not be possible for any direct comparison between the theoretical and observed background spectra.

(*a*) *Steady-state Models*

In steady-state cosmologies, the density and temperature of the universal plasma must remain constant, independent of epoch. Assuming 100% ionization, and a value for H of 100 km sec^{-1} mpc^{-1}, the electron density in the steady-state universe (Section II(*b*)) originally proposed by Hoyle (1948) will be given by

$$N_0 = 3H^2/8\pi G m_{\mathrm{H}} = 1\cdot 1\times 10^{-5} \quad \mathrm{cm}^{-3}. \tag{41}$$

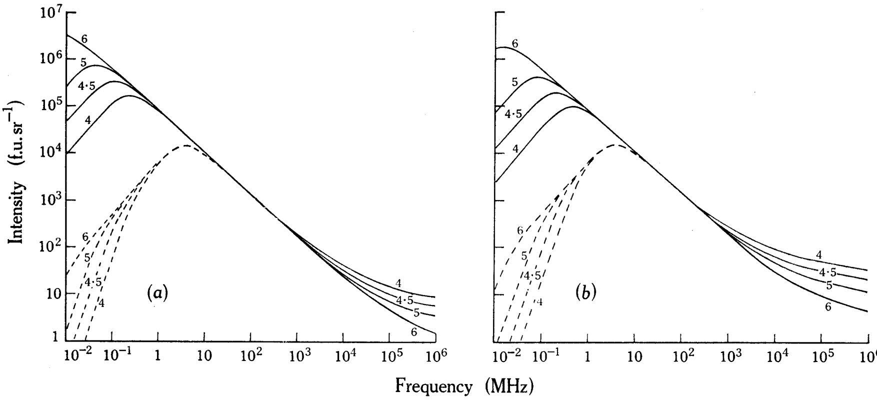

Fig. 1.—Expected extragalactic radio spectrum for a steady-state universe in which the electron density N_0 is (*a*) $1\cdot 1\times 10^{-5}$ and (*b*) $2\cdot 2\times 10^{-5}$ cm^{-3}. The dashed curves indicate the possible low frequency spectra if the source average spectrum has a 5 MHz cutoff. The numbers on the curves are the logarithms of the constant temperatures of the intergalactic gas. (Note that 1 f.u. = 10^{-26} W m^{-2} Hz^{-1}.)

In another version of the steady-state theory, Hoyle and Narlikar (1962) suggested that, as a consequence of the existence of the creation field C^μ, the observed homogeneity and isotropy of the universe is an asymptotic state for all initial conditions. It follows from the theory that a perturbation $C^i_i \neq 0$, introduced into the C field, will cause the expansion factor $\omega(t)$ to converge to an exponential form while the matter density ρ approaches $3H^2/4\pi G$. Hence we have an alternative electron density in a steady-state model which is twice the value predicted by ordinary theory. The free–free emission and optical depth for a particular ω will be correspondingly four times the values calculated on the basis of equation (41). The expected background radiation can now be derived for both cases with the aid of equations (32), (33), (36), and (39), and the numerical results are shown in Figures 1(*a*) and 1(*b*) for electron densities of $1\cdot 1\times 10^{-5}$ and $2\cdot 2\times 10^{-5}$ cm^{-3} respectively. The derived extragalactic spectra for the steady-state theories can be compared with the experimental background spectrum, which has been plotted in Figure 2.

The spectral points are taken from the observations of many observers and are plotted for directions near to the galactic poles corresponding to minimum disk effects such as nonthermal emission or low frequency absorption. The experimental points at frequencies greater than 1000 MHz, which approximate closely to a 3°K black-body spectrum, are also included.

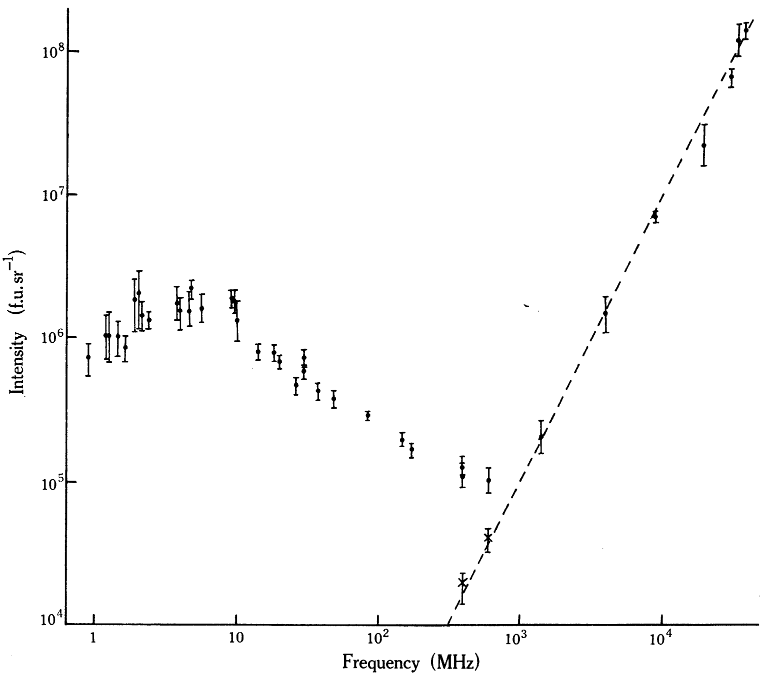

Fig. 2.—Observed total background spectrum for directions near to the galactic poles:

Observer	Frequency (MHz)	Observer	Frequency (MHz)
Ellis (1957)	0·9, 1·43, 2·13, 3·8, 4·4, 5·65, 9·13, 10·05	Yates (personal communication)	85
		Mills (1959)	85·5
Walshe, Haddock, and Schulte (1963)	1·225, 2·0	Howell and Shakeshaft (1967)	408, 610
Ellis (1964)	1·65, 2·4, 4·8, 4·6	Penzias and Wilson (1966)	1430
Yates and Wielebinski (1965)	14·1	Howell and Shakeshaft (1966)	1450
Yates and Wielebinski (1966)	14·1, 20, 30, 48·5, 85	Penzias and Wilson (1965)	4080
Shain and Higgins (1954)	18·3	Roll and Wilkinson (1966)	9380
Turtle *et al.* (1962)	26·3, 38, 178, 400	Welch *et al.* (1967)	2×10^4
Mathewson, Broten, and Cole (1965)	30	Thaddens and Clauser (1966)	$1{\cdot}15\times10^5$

Comparing Figures 1 and 2 we see that the expected extragalactic background intensity is approximately 1% of the observed intensity at 1 MHz and that this value is reduced to almost 0·1% at 1000 MHz. At even higher frequencies, the contribution to the background intensity in the steady-state models is insignificant and if it is shown conclusively that the 3°K black-body radiation is indeed extragalactic in origin, then it is difficult to imagine how any modification of the steady-state theory can fit the observed data.

At frequencies less than 1 MHz, we could expect an increase in the observed background intensity if galactic absorption is small and there is no low frequency cutoff in source spectra. The latter condition is unlikely to be met in view of the well-known mechanisms such as free–free absorption, synchrotron self-absorption, and low energy cutoff in the electron energy spectrum, all of which lead to a decrease in source intensity at decametric wavelengths. However, if an increase in observed background intensity is evident at a frequency less than 1 MHz, then absorption in the plane of the Galaxy must be considerably less than previous estimates, which have been based on the assumption of linear spectra in the halo and disk regions (Ellis and Hamilton 1966).

(b) "*Adiabatic*" *Cooling*

If it is assumed that the universe has cooled adiabatically, the gas expanding with $\gamma = 5/3$, then according to the laws of thermodynamics, $TV^{\gamma-1} = \text{constant}$, and the temperature must obey the law $T = T_0\,\omega^{-2}$. In an Einstein–de Sitter universe ($q_0 = \sigma_0 = 0\cdot5$) it follows with the aid of equations (27) and (29) that the limit of the optical depth is

$$\lim_{\omega\to 0}\left(\int_{\omega}^{1} K(\nu_0,\omega)\frac{\partial t}{\partial\omega}\,\mathrm{d}\omega\right) = 2\bar{g}K(\nu_0,1)\,,$$

where $\bar{g}$ is some suitable mean of the Gaunt factor. Hence we see that the optical depth converges to a finite limit, while according to equations (25) and (38) the free–free emission diverges to infinity. It follows that we cannot integrate equation (25) back to the infinite past, and a cutoff in emission must be introduced at a particular ω if the background radiation is to remain finite.

Now in evolving world models the thermal behaviour of the gas can be strongly influenced by heating and cooling mechanisms that destroy the adiabatic nature of the expansion. Ginzburg and Ozernoi (1966) have considered the kinetic temperature of the intergalactic gas in the Einstein–de Sitter model. They include in their analysis heating effects due to plasma oscillations excited by anisotropic cosmic rays and heat generation from ionization by "subcosmic rays". They also account for cooling due to free–free transitions, recombination radiation, and the expansion of the metagalaxy. A lower bound of 10^5 °K is estimated for the temperature of the gas at the present epoch by the solution of the cosmological energy equation derived from the first law of thermodynamics. Ginzburg and Ozernoi suggest that subcosmic rays consist of nonrelativistic protons with energy ~ 30 MeV and an energy density of about 10^{-15} erg cm^{-3}. However, it should be noted that, at present, there is no direct evidence for the existence of these hypothetical particles.

The differential equation describing the thermal properties of the gas has been derived here as a function of the expansion parameter ω. The equation can be written in the form

$$\mathrm{d}T/\mathrm{d}\omega = -\omega^{-2\cdot5}(AT^{\frac{1}{2}}+BT^{-\frac{1}{2}}-C)+D\omega^{-1\cdot5}+E\omega^{0\cdot5}-F\,.\,T\,.\,\omega^{-1}\,, \tag{42}$$

where the positive terms account for heating and the negative terms for cooling effects. The "constants" A, B, and F depend only on the present average mass density (taken here as $\rho_0 = 2 \times 10^{-29}$ g cm^{-3}) and fundamental constants, while C, D, and E are directly proportional to the matter and cosmic ray energy densities. For small ω, the first term in the equation dominates and as $\omega \to 0$, $T \to \text{const.} \times \omega^{-3}$. The differential equation has been solved by numerical methods for a series of values of T_0 in the range 10^4 to 10^7 °K, and for cosmic ray energy densities in the range 10^{-14} to 10^{-16} erg cm^{-3}. The possible solutions are shown in Figure 3, where T is plotted as a function of ω. Unless a cutoff in emission is introduced for some

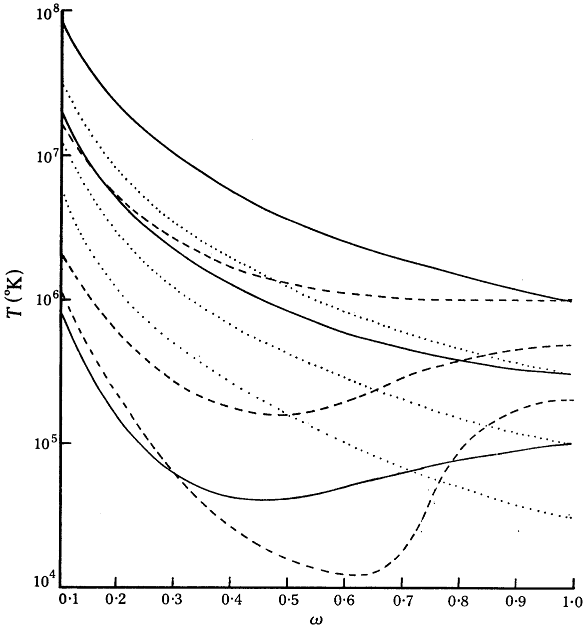

Fig. 3.—Plots of the temperature T as a function of the expansion parameter ω showing possible thermal histories in an adiabatically cooling universe. The full curves correspond to cosmic ray energy densities of 10^{-15}, the dotted curves to 10^{-16}, and the dashed curves to 10^{-19} erg cm^{-3}.

arbitrary ω, these thermal histories will lead to background intensities far in excess of the observed brightnesses. Indeed the free–free emission must diverge to infinity, although at a slower rate than for a simple adiabatic expansion. There are further difficulties associated with adiabatic expansions which will be mentioned briefly in Subsection (d) below. Expansion of this type cannot at any epoch attain a state of thermal equilibrium and this restricts the age of the universe.

In the derivation of the background radiation for the adiabatic model we have assumed a cosmic ray energy density of 10^{-15} erg cm^{-3} and equation (25) was integrated back to the epoch corresponding to a gas temperature of 10^9 °K. At temperatures much greater than this pair creation and annihilation processes would produce a thermal equilibrium distribution between particles and radiation. The type of expansion considered here is impossible under these initial conditions. The results are shown in Figure 4, and by comparison with Figure 1 we can see that the same general conclusions apply for this model as for the two models of the steady-state theory. The extragalactic background intensity for an adiabatically evolving universe will be approximately twice the intensity of a steady-state universe at low frequencies and, although the effects of free–free emission will become apparent at a significantly lower frequency, this model is also completely inadequate as an explanation of the microwave spectrum.

(c) *Constant Temperature Model*

The cosmological energy equation which can be written

$$\mathrm{d}(\epsilon R^3)/\mathrm{d}t + p\,\mathrm{d}R^3/\mathrm{d}t = 0, \tag{43}$$

where ϵ is the mean energy density, is easily derived from the Einstein field equations for a homogeneous, unbounded, isotropic medium. Equation (43) is usually accepted as the form of the first law of thermodynamics describing a universe undergoing adiabatic expansion. However, certain conceptual difficulties arise in the interpretation of equation (43) since it is implied that an expanding volume element does

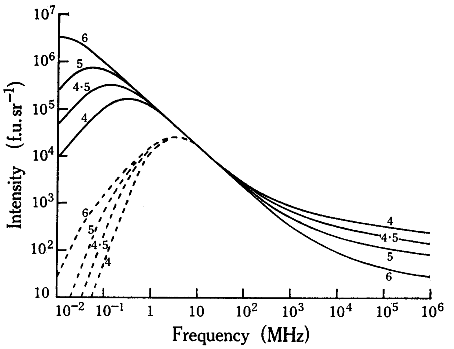

Fig. 4.—Expected extragalactic background spectra for an adiabatically cooling Einstein–de Sitter universe. The dashed curves indicate the possible low frequency spectra if the source average spectrum has a 5 MHz cutoff. The numbers on the curves are the logarithms of the temperatures of the intergalactic gas at the present epoch.

work on the surrounding gas, although symmetry considerations show that there cannot be a net flow of energy across a co-moving surface. Layzer (1966) believes equation (43) to be of a completely kinematical origin. Briefly, Layzer (1963, 1966) argues from the theory of particle dynamics in a homogeneous unbounded system and he finds two coupled equations for the mean kinetic energy per unit mass and the mean potential energy per unit mass. One equation is analogous to equation (43), while the other is the cosmogonic virial theorem. The total energy is negative, and if the effects of radiation are small then the kinetic and potential energies are approximately constant and as a consequence the kinetic temperature of the distribution must also remain constant.

The theory for a constant temperature universe as described by Layzer is valid only in the absence of heat sources, and in the numerical calculations for the background intensity it was assumed that the gas temperature remained constant throughout the entire history of the universe. This assumption will not significantly affect the high frequency spectrum, but the intensity at very low frequencies may be underestimated. The numerical results in Figure 5 are based on an Einstein–de Sitter universe in which the electron density is $1 \cdot 1 \times 10^{-5}$ cm^{-3} (i.e. 100% ionization) and by comparison with Figure 2 we see that the predicted intensity of the constant temperature model will exceed the total background intensity in the frequency range 10–10^4 MHz, while at higher frequencies it is considerably less than the observed intensity. We also note that radiation in the microwave region of the constant temperature model has a spectral index of 1·2, which is almost independent of either temperature or the degree of ionization of the gas. This value differs significantly from the spectral index of 2 for a Planck function, and we must conclude that the constant temperature model in its present form does not produce a satisfactory representation of the observations.

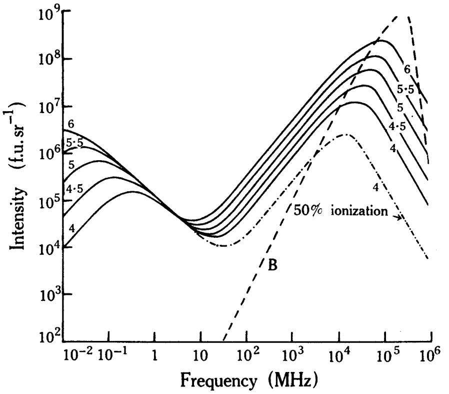

Fig. 5.—Expected extragalactic background spectra in a constant temperature Einstein–de Sitter universe. The full curves correspond to a gas that is completely ionized, while the dashed curve B represents the 3°K black-body spectrum.

(*d*) *Black-body Radiation*

The apparent detection of "relic" black-body radiation at 3·5°K by Penzias and Wilson (1965) has some important consequences, for it not only deals a severe blow to the steady-state theory, but it also implies that the universal plasma could not be as hot as a result of initial conditions. In evolutionary universes at early epochs, when the temperature was presumably high and the Thomson scattering depth large, energy exchanges between electrons and photons would ensure a Planck function. We would therefore find, at the present epoch, a gas which is cooler than the black-body radiation and this, of course, is a direct contradiction to the observations. According to Dicke *et al.* (1965), the gas remains in thermal equilibrium with the radiation until the onset of hydrogen recombination at approximately 4000°K, when the matter consequently cools faster than the radiation. Peebles (1965) believes that this stage of the expansion corresponds to the formation of galaxies. The subsequent release of nuclear energy with the conversion of gravitational energy into heat could be sufficient to increase the temperature of the remaining intergalactic gas as the epoch advances.

According to the calculations of Peebles (1966) on the primeval element abundances issuing from the "big bang", the mass abundance of helium is a function of the mean mass density in the universe and the present temperature of the fireball. For a density range 7×10^{-31} to $2\times10^{-29}\,\mathrm{cm}^{-3}$, the computed helium abundance is 27–30% by mass. The helium content of the universe is not known exactly, but a reasonable upper limit of 25% is consistent with present abundance observations. If the 3°K fireball radiation and low helium abundance is confirmed by future measurements, then the evolutionary universes based on general relativity must have an extremely low density (20 times less than the average density of matter in galaxies), or the fundamental concepts of general relativity theory itself must be invalid. In this paper, we will, for the lack of any further evidence, ignore the implications of the helium abundance and derive the expected background radiation on the basis of ordinary relativistic cosmology.

Weymann (1966) has examined the possible distortion of the Planck function due to inverse Compton scattering and free–free emission. He did not consider in detail the thermal history of the universal plasma after galaxy formation, but assumed an isothermal expansion at a temperature of 3×10^6 °K. In view of recent evidence, it appears that this estimate for the gas temperature is too high, and the consequences of a lower temperature will be an increase in free–free emissions

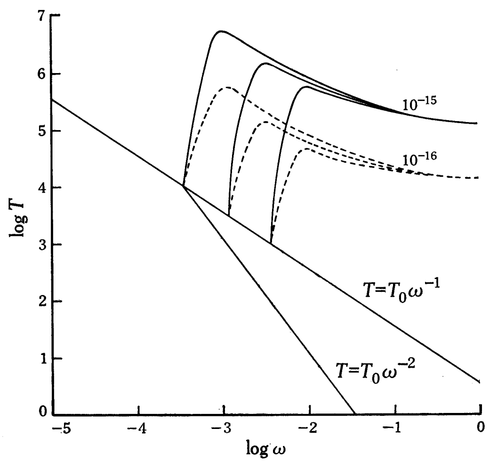

Fig. 6.—Possible thermal histories of an Einstein–de Sitter universe in which radiation is in equilibrium with matter. The temperature path of the gas depends on the epoch corresponding to the onset of heating and the cosmic ray energy density (taken as 10^{-15} and 10^{-16} erg cm^{-3} here). The lower straight line represents the thermal history of the gas in the absence of heating.

and a decrease in emissivity due to electron scattering. It should be noted that Weymann's analysis neglects the contribution of radio sources to the background intensity—a contribution that is certainly significant at frequencies less than 10^3 MHz. In the present analysis, an estimation is made of the temperature path of the gas, so enabling a more accurate calculation for the distortion of the black-body curve, and further the analysis is restricted to relatively low frequencies when the effects of Compton scattering are negligible.

If we accept "fireball" radiation as the interpretation of the microwave measurements, then we can draw some tentative conclusions concerning the temperature evolution of the plasma. Since the gas is tied to the radiation field, it must cool according to the law $T = T_0\omega^{-1}$, at least until galaxies begin to separate out. At some future stage, heating sufficient to ionize the gas at the present epoch must take place. If we assume that heating is due to cosmic rays and exploding galaxies, an estimate can be made for the temperature path of the gas.

The temperature will be again controlled by the differential equation (42), and we can specify boundary conditions corresponding to the time of onset of heating. The possible thermal histories of the intergalactic gas are plotted in Figure 6 for cosmic ray energy densities of 10^{-15} and 10^{-16} erg cm^{-3}.

The expected extragalactic background intensity has been calculated on the basis of a cosmic ray energy density of 10^{-15} erg cm^{-3} and a series of initial conditions depending on the epoch of galaxy formation and consequent heating of the gas. The results are shown in Figure 7, and it can immediately be seen that black-body radiation is completely acceptable as an explanation for the microwave background intensity. Upper limits can be set for the temperature (and therefore the epoch) at which heating of the gas sets in. Heating could not have occurred at an epoch earlier than that corresponding to a temperature of about 2×10^3 °K, and if the black-body spectral points at 408 and 610 MHz (Howell and Shakeshaft 1967) are included, then we can estimate an upper limit of 3×10^2 °K for the temperature of the gas at the time of onset of galaxy formation. All this implies that the heating process leading to ionization of the intergalactic gas must have originated more

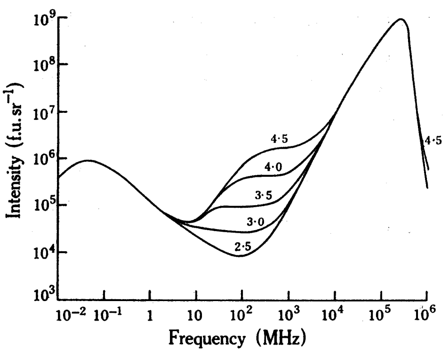

Fig. 7.—Possible background spectra for a universe filled with black-body radiation and a cosmic ray energy density of 10^{-15} erg cm^{-3}. The numbers on the curves are the logarithms of the temperatures when heating commenced leading to subsequent distortion of the Planck function at the present epoch.

recently than the epoch corresponding to a redshift of about 100. It must be noted here that the analysis of Howell and Shakeshaft assumed a black-body spectrum for the extragalactic component of the total background and for this reason their results are open to some doubt. A more exact estimate for the redshift could be provided if the extragalactic spectrum is experimentally determined at frequencies less than 400 MHz, although, of course, this is an extremely difficult procedure due to the very intense galactic radiation. The analysis has not been extended to frequencies greater than 10^6 MHz, but it is clear that distortion of the Planck function will be negligible between 10^3 and 10^6 MHz if, as suggested, heating has occurred more recently than the epoch for which $z \simeq 100$.

V. Discussion

This paper has been concerned with the properties of the expected extragalactic radio spectrum in both steady-state and evolutionary cosmologies as a function of the thermal history of the intergalactic ionized gas. For convenience in computation, an Einstein–de Sitter universe was assumed for evolving models and, although no attempt has been made to determine the background based on other world models, we can be sure, at least for the black-body case, that the background will not depend critically on the model parameters (i.e. on σ_0 and q_0). It can be shown that the present background intensity of a constant temperature universe is due almost entirely to radiation emitted at extremely early epochs. In this case, significant deviations could occur in the background spectra of Figure 5 for metrics differing from that of the Einstein–de Sitter model. The analysis has been restricted to frequencies below 10^6 MHz where emission from ionized hydrogen and ordinary radio galaxies provides the dominant contribution to the background intensity. At higher frequencies, the effects of other emission processes such as inverse Compton scattering (especially in relation to the black-body radiation) will become important.

The immediate conclusion of the numerical results is the obvious one that the microwave observations appear to rule out the possibility of steady-state cosmologies and also those evolutionary models that expand "adiabatically". There is some hope that a cosmology, other than the Einstein–de Sitter model, applied to a constant-temperature universe could account for the observed microwave spectrum. However, the present results indicate that the only satisfactory model is an expanding universe filled with black-body radiation, which at the present epoch, has a temperature of about 3°K. As a result of observations below 1000 MHz, distortion of the Planck function appears to be small and by comparison with the theoretical spectra of Figure 7 we can define an upper limit of redshift $z \simeq 100$ for the epoch corresponding to the formation of galaxies.

VI. Acknowledgment

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