MULTILEVEL ANALYSIS OF THE p-WAVE ^{12}C–α ELASTIC SCATTERING PHASE SHIFT BETWEEN 9·16 AND 12·11 MeV*

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A previous analysis (Clark, Sullivan, and Treacy 1968) of the ^{12}C–α elastic scattering data indicated that a single-level fit to the p-wave phase shifts (Fig. 1) was satisfactory near the resonance energy, but that a multilevel analysis would be required to fit the phase shifts across the full energy range. In this note such an analysis is considered and in particular, an α-particle width is assigned to the 1^- levels in ^{16}O at excitation energies of 7·116 and 9·59 MeV. A quantitative knowledge of the reduced width of the 7·116 MeV level is of importance in astrophysical calculations, since the level is suspected to have a large α-particle reduced width (Fowler and Hoyle 1964) and consequently, although energy bound by 46 keV, its influence on the production rate of ^{16}O in stellar interiors may be substantial.

For reasons previously outlined (Clark, Sullivan, and Treacy 1968), only the one-channel case of elastic scattering was considered in the present analysis. The analytic form of the phase shift in terms of level parameters is given by (Lane and Thomas 1958)

$$\delta_1 = \beta_1 - \phi_1 + \omega_1 , \tag{1}$$

where the resonant phase shift

$$\beta_1 = \tan^{-1}\{RP/(1-RS)\} \tag{2}$$

and

$$R = \sum_{\lambda} \{\gamma_{1\lambda}^2/(E_{1\lambda}-E)\} . \tag{3}$$

The energy eigenvalue and the reduced width of the level λ are denoted by $E_{1\lambda}$ and $\gamma_{1\lambda}^2$ respectively. The hard sphere phase shift, the Coulomb phase shift, the penetration factor, and the shift factor are denoted by ϕ_1, ω_1, P, and S respectively.

The phases were fitted with the expression (1) using a nonlinear least-squares fitting programme. The procedure consisted of finding a minimum in a multidimensional surface, $X^2(E_{1\lambda}, \gamma_{1\lambda}^2)$, by assuming a quadratic approximation to the surface in the neighbourhood of the minimum. The quantity $X^2(E_{1\lambda}, \gamma_{1\lambda}^2)$ was defined in terms of the data points $\delta_1^{\exp}$, their respective errors ϵ, and the function δ_1, discussed above, by

$$X^2(E_{1\lambda}, \gamma_{1\lambda}^2) = (n-m)^{-1} \sum [\{\delta_1^{\exp} - \delta_1(E_{1\lambda}, \gamma_{1\lambda}^2)\}/\epsilon]^2 , \tag{4}$$

where n represents the number of data points and m the number of free parameters. The fitting procedure requires the selection of a starting set of parameters in the region of the "correct" solution.

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Initially a three-level fit using a channel radius a_2 of 5·3 fm was made to the p-wave phase shift. For the present work, effects of the 13·1 MeV level, whose α-width is only about 40 keV, were negligible. In addition to the 1^- levels at 9·59 and 12·44 MeV excitation in ^{16}O, a background level was included. A background phase shift, resulting from the effects of distant levels, may be accommodated in R-matrix theory by the inclusion of an extra level in the analysis. Subsequently it was shown that the parameters describing the background could be adjusted to include the influence of the 12·44 MeV level on the p-wave phase shift. The inclusion of the background level in the analysis was justified by the fact that the X^2 value obtained with the two-level approximation was a factor of 2·5 smaller than that obtained with the one-level approximation. The level parameters used to describe the 9·59 MeV level and the background level were denoted by E_{11}, γ_{11}^2 and E_{21}, γ_{21}^2 respectively. The best fit to the δ_1 data, with phase shifts calculated using the two-level approximation, is shown in Figure 1. Acceptable fits to the data were obtained with a wide range of parameter values describing the background level.

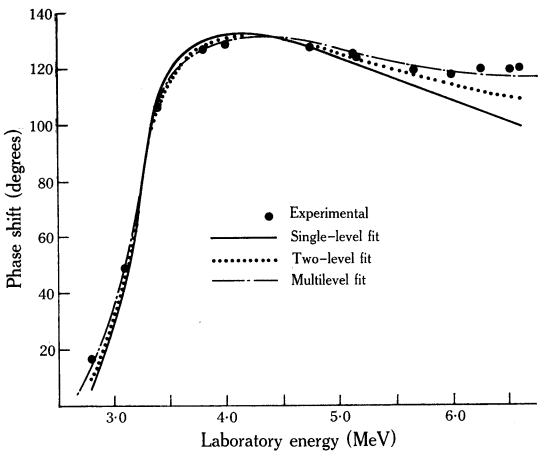

Fig. 1.—Single-, two-, and three-level fits to the experimental p-wave phase shifts derived by fitting the angular distributions of α-particles elastically scattered from ^{12}C (Clark, Sullivan, and Treacy 1968) in the energy range 9·16–12·11 MeV excitation in ^{16}O.

To improve the fit to the p-wave phase shift data, the two-level analysis was extended to a three-level one by a consideration of the effect of the 1^- level at 7·116 MeV excitation in ^{16}O. As the resonance energy E_{r1} of the level was well known (Browne and Michael 1964) the eigenvalue energy E_{31} of the level could be calculated using the relation

$$E_{r1} = E_{31} - \gamma_{31}^2 S(E_{r1}), \tag{5}$$

provided the reduced width of the level γ_{31}^2 and the shift factor S were known. The shift factor, which is non-zero for negative energy channels and charged particles, was calculated using a WKB approximation as outlined by Lane and Thomas (1958). Initially γ_{31}^2 was optimized while the other parameters were kept fixed at their initial values. The initial parameter values were taken from the optimum two-level fit to the data, together with the estimate of γ_{31}^2 made by Loebenstein *et al.* (1967). The "final" level parameters were obtained by allowing E_{11}, γ_{11}^2, E_{21}, γ_{21}^2, and γ_{31}^2 to vary simultaneously. E_{r1} was accurately known and so kept fixed through-

out the fitting routine. The quality of the fits, for the case where the channel radius a_1 is 5·3 fm, is shown in Table 1. The three-level analysis is justified by a factor of three decrease in the value of X^2 that occurs when the effect of the 7·116 MeV level is included. Acceptable fits, defined rather arbitrarily as fits where $X^2 \leqslant 1$, were obtained over a wide range (40–80 MeV) of E_{21} values with γ_{31}^2 varying by about $\pm 10\%$ of its mean value in the range. Similarly E_{11} and γ_{11}^2 vary by $\pm 15\%$ from their mean value in the range.

TABLE 1

LEVEL PARAMETERS OBTAINED FROM FITS TO p-WAVE PHASE SHIFTS

The shifts are over the energy range 9·16–12·11 MeV excitation in ^{16}O. A three-level approximation was used with a channel radius $a_1 = 5 \cdot 3$ fm

E_{11}	γ_{11}^2	E_{21}	γ_{21}^2	E_3†	γ_{31}^2	X^2
1·12	1·02	18·35	3·39	−0·99	0·26	1·2
1·17	0·99	28·90	5·42	−0·81	0·31	0·8
1·23*	0·93*	49·24*	9·05*	−1·95*	0·53*	0·7*
1·21	0·95	58·01	11·87	−1·99	0·54	0·8
1·05	1·13	79·41	23·38	−2·23	0·60	0·9
0·95	1·25	99·73	33·87	−3·02	0·83	1·2

* Used as a starting set of values in minimizing X^2 for various channel radii (see Fig. 2).

† As discussed in the text, this is not a free parameter.

That the overall three-level fit is superior at higher energies may appear rather surprising, in view of the remoteness of the 7·12 MeV level. It is clear, however, from Figure 1, that the latter level affects the fit considerably near the low-energy "knee" of the data, permitting the fit higher up to be accommodated better by the background. In addition, the width of the 7·12 MeV level is quite large over the range of the data.

TABLE 2

OPTIMUM 1^- LEVEL PARAMETERS OBTAINED FROM THREE-LEVEL ANALYSIS OF p-WAVE PHASE SHIFTS

The shifts are over the energy range 9·16–12·11 MeV excitation in ^{16}O. $a_1 = 5 \cdot 3$ fm

E_α (lab) (MeV)	Excitation (MeV)	J^π	r (c.m.) (MeV)	γ^2 (c.m.) (MeV)	θ_α^2
—	7·116	1^-	—	$0 \cdot 53^{+0 \cdot 28}_{-0 \cdot 14}$	0·71
3·24±0·04	9·59	1^-	0·91	0·93±0·05	1·2

Table 2 contains the optimum level parameters ($a_1 = 5 \cdot 3$ fm) assigned to the 1^- levels contributing to the ^{12}C(α, α) process in the region investigated. The parameters describing the broad background level are not included. The corresponding calculated phase shifts that gave the best fits to the δ_1 data are shown in Figure 1. The error assigned to a particular parameter was chosen to correspond to the

variation in the parameter that was necessary to produce a 50% change in the value of X^2 for the fit, the other parameters being kept fixed at their true values. The deterioration that a 50% change in X^2 produces in the fit would be visible.

The reduced width γ_{31}^2 also lacks sensitivity to changes in the channel radius. This is illustrated in Figure 2, where γ_{31}^2 is plotted as a function of the channel radius a_1. The curve was obtained by minimizing X^2 for various channel radii, the set of values marked with an asterisk in Table 1 being used as starting values.

Summarizing, it may be said that the inclusion of the bound level produces an improved fit to the p-wave phase shift data, but unfortunately, the assigned reduced width of the level has an estimated possible error of 50%.

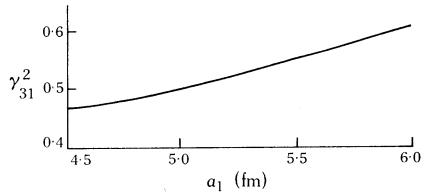

Fig. 2.—Plot of the calculated reduced width of the 7·116 MeV level, γ_{31}^2, as a function of the channel radius a_1.

Loebenstein *et al.* (1967) extracted from their ^{6}Li(^{12}C, d)^{16}O data a value for the dimensionless reduced width* of $\theta_\alpha^2 = 0{\cdot}1 \pm 0{\cdot}04$ for the 7·116 MeV level in ^{16}O. This result was based on a Hauser–Feshbach type calculation. A calculation made by Stephenson (1966) and based on a model of ^{16}O proposed by Brown and Green (1966) predicted a value $\theta_\alpha^2 = 0{\cdot}08 \pm 0{\cdot}04$. Fowler and Hoyle (1964) estimated a value of $\theta_\alpha^2 = 0{\cdot}78$ by using a cluster model of ^{16}O. The value assigned to the dimensionless reduced width from the present analysis, within the limits $0{\cdot}7 < X^2 < 1{\cdot}1$, was

$$\theta_\alpha^2 = 0{\cdot}71^{+0{\cdot}37}_{-0{\cdot}18}.$$

This value is considerably larger than that assigned by Stephenson (1966) and Loebenstein *et al.* (1967), but is in agreement with that of Fowler and Hoyle (1964). The reason for these inconsistencies in the results is not obvious.

References

Brown, G. E., and Green, A. M. (1966).—*Nucl. Phys.* **75**, 401.
Browne, C. P., and Michael, I. (1964).—*Phys. Rev.* **134**, B133.
Clark, G. J., Sullivan, D. J., and Treacy, P. B. (1968).—*Nucl. Phys.* A **110**, 481.
Fowler, W. A., and Hoyle, F. (1964).—*Astrophys. J. Suppl. Ser.* **91**, 201.
Lane, A. M., and Thomas, R. G. (1958).—*Rev. mod. Phys.* **30**, 257.
Loebenstein, H. M., Mingay, D. W., Winkler, H., and Zaidins, C. S. (1967).—*Nucl. Phys.* **91**, 481.
Stephenson, G. J. (1966).—*Astrophys. J.* **146**, 950.

* Defined as the ratio γ^2/γ_0^2, where γ_0^2 is the "single alpha particle width" $3\hbar^2/2Ma^2$.