THE X-RAY BACKGROUND IN ISOTROPIC WORLD MODELS

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Summary

This paper is an attempt to describe the diffuse X-ray background in terms of Compton radiation from cosmic ray electrons in intergalactic space. Similarities between the X-ray and radio source spectra suggest that fast electrons escape more or less freely from radio galaxies. It is assumed that the time scale of electron injection is small compared with the characteristic time of evolution of the universe. The electrons are considered to lose energy through Compton scattering (due to the presence of the universal black-body radiation at 3°K) and by expansion of the coordinate system.

A least squares fit of the derived spectra to the experimental X-ray spectrum provides useful information on the epoch at which electron injection commences. Normalization requires either non-equilibrium conditions to exist within radio sources or the number of sources in unit coordinate volume to be a strong function of epoch. The results are consistent with a universe in a state of rapid expansion.

I. Introduction

The diffuse X-ray background depends for its explanation on an efficient mechanism for the conversion of cosmic ray electron energies into X-ray quanta. It is generally believed that inverse Compton scattering (i.e. the production of high energy photons from the collisions of fast electrons with thermal photons) is responsible for the X-ray radiation. The question of the origin of these scattering processes has been investigated in some detail by different authors. Gould (1965) and Felten and Morrison (1966) have examined the possibility that the Compton interaction of of relativistic electrons in the halo with black-body photons or the stellar radiation field could produce the observed isotropic background. However, the halo model predicts a flux that is two orders of magnitude less than that required by the observations and the shape of the derived spectrum does not provide a good match to the experimental spectrum. Further, there is no evidence of strong anisotropy in the X-ray background and there are therefore convincing reasons why we must exclude the halo model as a possible explanation of the observed flux.

Difficulties are also encountered if it is assumed that the X-ray background is the sum of contributions from the halos of all external galaxies. Felten and Morrison have shown that, unless the X-ray or γ-ray emission in the average galaxy is much greater than that in our own, the integrated radiation from external galaxies will be several orders of magnitude less than the observed X-ray background.

Gould and Burbidge (1966) have suggested that the diffuse flux is the result of radiation from discrete X-ray sources in external galaxies but, if our Galaxy is typical, it appears (Oda 1966) that the expected flux will be only 1% of the observed

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value. Bergamini, Londvillo, and Setti (1967) have proposed that the X-ray background is due to the inverse Compton interaction of fast electrons with the universal black-body radiation in strong radio galaxies. They find an equation for the ratio of the expected radio and X-ray background intensities and allow for an evolution in radio source population. Further, they assume that the radio background at 178 MHz is about 25% of the observed intensity and, with a few other reasonable assumptions, conclude that the X-ray background can be interpreted as due to emission from strong extended galaxies if the magnetic field strength is of the order of 1 μG. However, their assumption of the extragalactic radio background intensity appears to be excessive by at least one order of magnitude (Payne 1969), and consequently the required field strength is less than is acceptable from radiofrequency observations and equipartition arguments.

We are led finally to consideration of the possible origin of the diffuse X-ray flux in intergalactic space through the interaction of cosmic ray electrons with the 3°K black-body radiation. The difficulty in this model is the provision of a reliable estimate for the rate of injection of relativistic electrons from galaxies. The problem has been dealt with briefly by Felten and Morrison (1966), who consider a model in which electrons are supplied by explosive events in strong radio sources. If the total energy content of relativistic particles in these objects is about 10^{58} erg and 1% of this energy is in fast electrons that are lost to intergalactic space in a time of approximately 10^6 years, then the calculated X-ray intensity is two orders of magnitude less than that required by the observations. However, Burbidge (1962) suggests that the usual assumption of equipartition between particle and magnetic field energies is hardly tenable for sources subject to frequent disruptive events. This argument is used by Felten and Morrison (1966) who point out that electrons, sufficient to produce the observed X-ray background, can be injected into space by galaxies in which a non-equipartition condition exists.

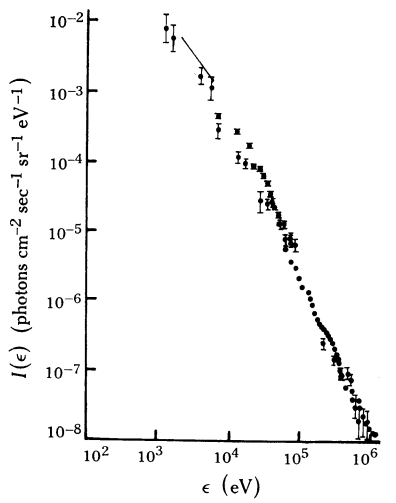

Fig. 1.—Spectrum of the diffuse background of cosmic X-rays.

The analysis of Felten and Morrison (1966) does not include the effects of the space–time geometry, and also assumes an electron energy spectrum that is constant, independent of epoch. In the present paper, allowance is made for a non-Euclidean geometry and the electron spectrum is essentially dynamic through its dependence on epoch.

The experimental X-ray background that must be compared with theoretical models is plotted in Figure 1 from results of Gould (1967) and recent measurements by Henry *et al.* (1968) at low energies. The spectrum of the diffuse flux in the range 1 keV to 1 MeV has been derived from data furnished by balloon, rocket, satellite, and space-probe experiments. The evidence so far indicates that the X-ray background is isotropic, at least to the limits of accuracy ($\sim 10\%$) of the observations. The data may be fitted by a power law of the form

$$I(\epsilon) = K\epsilon^{-(m+1)},$$

where ϵ is the photon energy and m the index for the energy–intensity spectrum obtained by multiplying the photon numbers by ϵ. A least squares fit over the whole observational range (1 keV to 1 MeV) yields a value of $m = 1 \cdot 25 \pm 0 \cdot 18$, while fitting over the steeper part of the spectrum yields $m = 1 \cdot 42 \pm 0 \cdot 12$. The value obtained for m is important, not only for the definition of the energy distribution of the cosmic ıay electrons responsible for the X-ray flux but also in providing a clue to the possible origin of these fast electrons.

II. Electron Energy Losses in Metagalactic Space

In this section, the actual injection processes of the relativistic electrons are ignored and our attention is confined to the problem of electron energy losses in intergalactic and metagalactic space. The cosmic ray electrons may dissipate their energy by the inverse Compton effect, synchrotron and bremsstrahlung radiation, expansion of the coordinate system, and the ionization of atomic hydrogen. The spatial density of intergalactic atomic hydrogen is extremely small and measurements indicate (Goldstein 1963; Davies 1964) a maximum value of about 10^{-7} cm^{-3}, while some evidence (Gunn and Peterson 1965) suggests a value as low as 10^{-9} cm^{-3}. Therefore we may immediately neglect ionization losses since they are approximately proportional to the concentration of atomic hydrogen and are accordingly exceedingly small. The density of intergalactic ionized hydrogen, at the present epoch, may be taken to be about 10^{-5} cm^{-3} and, when compression of the plasma is allowed for, this implies that the characteristic time for energy losses by bremsstrahlung radiation is much greater than the characteristic time H_0^{-1} of evolution of the universe. Also, the present analysis is concerned with the latter stages of the expansion of the universe when galaxy formation has been completed and the plasma density is fairly small. In this case, bremsstrahlung losses will be insignificant and can be neglected in the following calculations.

Now before we can progress to a discussion of the remaining types of energy losses, it is necessary to define the space–time metric. The interval ds between two events in the four-dimensional continuum is defined as usual by the Robertson–Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{R^2(t)}{c^2}\left(\frac{\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)}{(1+\frac{1}{4}kr^2)^2}\right), \tag{1}$$

where (r, θ, ϕ) are three-dimensional co-moving radial coordinates, t is the universal time, R is the radius of curvature which is a function of t only, and k is the curvature constant having values of $+1$, 0, or -1 depending on whether the space–time is closed, flat, or open. With the aid of Einstein's equations, this metric yields a fundamental equation describing the expansive properties of the universe which may be written in the form (e.g. McVittie 1965)

$$\mathrm{d}\omega/\mathrm{d}t = H_0\{(2\sigma_0/\omega) + (q_0+1-3\sigma_0) + (\sigma_0 - q_0)\omega^2\}^{\frac{1}{2}}, \tag{2}$$

where

$$\omega = R/R_0, \qquad \sigma_0 = 4\pi G\rho_0/3H_0^2, \qquad \text{and} \qquad q_0 = -(R_0 H_0^2)^{-1}\,\mathrm{d}^2R_0/\mathrm{d}t_0^2,$$

σ_0 being the density parameter, q_0 the acceleration factor, ρ_0 the mean density of matter, and R_0 the value of R at the present epoch. It is now possible to consider the electron energy losses in more detail.

(a) *Energy Losses by Expansion*

The problem of propagation of cosmic rays through intergalactic space has been examined in some detail by Ginzburg and Syrovatsky (1964). They find that these fast electrons essentially pervade the whole of metagalactic space in a time that is short compared with the characteristic time of evolution of the universe. In these circumstances "redshift" losses by expansion of coordinates will be relevant and according to Ginzburg and Syrovatsky the electron energy loss may be written as

$$\mathrm{d}E/\mathrm{d}t = -(V/R)E = -(R^{-1}\,\mathrm{d}R/\mathrm{d}t)E\,, \tag{3}$$

and using $\omega = R/R_0$ this becomes

$$\mathrm{d}E/\mathrm{d}t = -(\omega^{-1}\,\partial\omega/\partial t)E\,.$$

Substituting $\mathrm{d}t = (\partial t/\partial\omega)\mathrm{d}\omega$, this may be rewritten in the simple form

$$\mathrm{d}E/\mathrm{d}\omega = E/\omega\,, \tag{4}$$

which defines the energy of an electron as a function of the expansion parameter ω only.

(b) *Inverse Compton Losses*

The inverse Compton process involves the production of a high energy photon from the collision of a relativistic electron with a low energy photon. The properties of the generated power depend on the energy and velocity distributions of electrons and photons. The problem has been treated in some detail by Feenberg and Primakoff (1948), who derive a rather complicated formula for the total power dissipated by a single fast electron. However, for an isotropic distribution of relativistic electrons interacting with thermal photons, the formula reduces (e.g. Ginzburg and Syrovatsky 1964; Felten and Morrison 1966) to the simple equation

$$\mathrm{d}E/\mathrm{d}t = -\tfrac{4}{3}\sigma_{\mathrm{t}}\,c\rho(E/m_0\,c^2)^2\,, \tag{5}$$

where σ_{t} is the total Thomson cross section and ρ the photon energy density. The temperature of the universal black-body radiation at the present epoch is close to 3°K, and this corresponds to an energy density $\rho_0 = 0\cdot38\ \mathrm{eV\,cm^{-3}}$. Also, as the universe expands, the cosmological redshift serves to adiabatically cool the black-body radiation while preserving a Planck function. In the absence of interaction with matter, the temperature is proportional to ω^{-1} and the photon energy density at an epoch defined by ω may be written as

$$\rho = \sigma T^4 = \rho_0\,\omega^{-4}\,, \tag{6}$$

where σ is the Boltzmann constant. Therefore, from equation (5), the energy loss by photon scattering is

$$\mathrm{d}E/\mathrm{d}\omega = -BE^2\omega^{-4}\,\partial t/\partial\omega\,, \tag{7}$$

where

$$B = \tfrac{4}{3}c\sigma_{\mathrm{t}}\,\rho_0/(m_0\,c^2)^2$$

and $\partial t/\partial\omega$ is obtained from equation (2). It is clear from equation (7) that the inverse Compton loss is a strong function of the expansion parameter and will be most important at early epochs.

(c) *Synchrotron Losses*

Arguments based on energy equipartition between magnetic field and matter (Ginzburg and Syrovatsky 1961) yield a value of $H \simeq 0\cdot 5$ μG for the intergalactic magnetic field strength. It is natural to assume that the energy density $H^2/8\pi$ varies in the same way as the gas energy density. In evolutionary cosmologies, $\rho = \rho_{\mathrm{p}}\,\omega^{-3}$ and if the random motion is approximately constant then $H = H_{\mathrm{p}}\,\omega^{-3/2}$, where H_{p} and ρ_{p} are the present values for the magnetic field strength and the matter density respectively. Now the synchrotron loss of a single electron moving in a magnetic field with perpendicular component $H_{\perp}$ can be written as (Oort and Walraven 1956)

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{2e^4c}{3(m_0c^2)^4}H_{\perp}^2E^2. \tag{8}$$

Converting the energy loss to a function of the expansion parameter ω, and assuming that the magnetic field is isotropic on the average ($H_{\perp}^2 = \frac{2}{3}H^2$),

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = -\frac{\sigma_{\mathrm{t}}}{6\pi}cH_{\mathrm{p}}^2\left(\frac{E}{m_0c^2}\right)^2\omega^{-3}\frac{\partial t}{\partial\omega}. \tag{9}$$

The ratio of the total power generated by photon scattering to that generated by synchrotron emission is, by equations (7) and (9),

$$P_{\mathrm{c}}/P_{\mathrm{s}} = \rho_0(H_{\mathrm{p}}^2/8\pi)^{-1}\omega^{-1} \simeq 60\omega^{-1} \tag{10}$$

if the magnetic field strength $H_{\mathrm{p}} \simeq 0\cdot 5$ μG. Synchrotron emission in metagalactic space is therefore seen to be small compared with the emission by inverse Compton scattering and will be neglected in this analysis.

III. Electron Energy Spectrum

It is the purpose of this section to obtain an equation for the quasi-equilibrium electron energy spectrum at any epoch. There are only two remaining energy losses that must be considered in the following analysis: losses by Compton scattering, and expansion of coordinates. Under these conditions, the differential equation describing the electron energy as a function of the expansion parameter will be

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = -\left(\frac{E}{\omega}+\frac{BE^2}{\omega^4}\frac{\partial t}{\partial\omega}\right). \tag{11}$$

The solution to this equation with boundary conditions at injection of $E = E_0$ and $\omega = \omega_0$ is

$$E(\omega) = \frac{E_0\,\omega_0}{\omega}\left(1+BE_0\,\omega_0\int_{\omega_0}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{-1}, \tag{12}$$

which again demonstrates the dependence of electron energy on the chosen world model.

We now turn our attention to the problem of the electron energy spectrum in metagalactic space. An assembly of electrons with a number density $N(E,\omega)\,\mathrm{d}E\,\mathrm{d}\omega$ will have an energy distribution determined by the equation

$$\frac{\partial N(E,\omega)}{\partial\omega}+\frac{\partial}{\partial E}\left(\frac{\mathrm{d}E}{\mathrm{d}\omega}N(E,\omega)\right)=0\,, \tag{13}$$

which is the continuity equation for electrons in (E,ω) space without a source function and N is the number of electrons in an arbitrary region of the ordinary three-space. Equation (13) has been solved for initial conditions $N(E,\omega_0)\mathrm{d}E = KE^{-\gamma}\,\mathrm{d}E$ corresponding to an instantaneous injection spectrum at $\omega=\omega_0$. The solution is

$$N(E,\omega)=KE^{-\gamma}\left(\frac{\omega_0}{\omega}\right)^{\gamma-1}\left(1-BE\omega\int_{\omega_0}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{\gamma-2}. \tag{14}$$

Suppose now that from suitable galaxies electrons are injected at a constant rate $qE^{-\gamma}$. The number of electrons with energy E escaping from a single galaxy in time $\mathrm{d}t_0$ will be $qE^{-\gamma}\,\mathrm{d}t_0$, and it follows from equation (14) that the energy spectrum is

$$N(E,\omega)=V(\omega)\,qE^{-\gamma}\int_{t_{0\mathrm{m}}}^{t}\mu(\omega_0)\left(\frac{\omega_0}{\omega}\right)^{\gamma-1}\left(1-BE\omega\int_{\omega_0}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{\gamma-2}\mathrm{d}t_0\,, \tag{15}$$

where V is the volume of the region under consideration and μ is the spatial density of sources. It is assumed here that the galaxies emitting fast electrons form a single class of objects with the quantity μ representing their weighted mean density. If, as in ordinary evolutionary cosmology, the number η of sources per unit coordinate volume is constant, then $\mu=\mu_0\,\omega_0^{-3}$ where μ_0 is the local density at the present epoch. However, if the source population varies in such a way that η is a smooth function of epoch, the density may be defined by $\mu=\mu_0\,\omega_0^{-x}$ where in general x will be greater than three. Hence, converting equation (15) to a function of the expansion parameter ω and defining N^* as the number of electrons per unit proper volume, we have finally

$$N^*(E,\omega)=\mu_0 qE^{-\gamma}\int_{\omega_{0\mathrm{m}}}^{\omega}\omega_0^{-x}\left(\frac{\omega_0}{\omega}\right)^{\gamma-1}\left(1-BE\omega\int_{\omega_0}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{\gamma-2}\frac{\partial t_0}{\partial\omega_0}\,\mathrm{d}\omega_0\,. \tag{16}$$

The minimum value $\omega_{0\mathrm{m}}$ for the lower limit of integration is determined not only by E and ω but also by the properties of the injection spectrum. This will be discussed in more detail in the next section. It is sufficient here to point out that $\omega_{0\mathrm{m}}$ must satisfy the relation

$$BE\omega\int_{\omega_{0\mathrm{m}}}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\leqslant 1\,. \tag{17}$$

IV. X-ray Background

In this section an expression is derived for the X-ray emission per unit proper volume which leads to the equation for the X-ray background intensity through the general equation for the background radiation in isotropic world models. The total power generated by a single electron by photon scattering can be written, for the epoch ω, as

$$P(E,\omega) = \tfrac{4}{3}c\sigma_{\mathrm{t}}\,\rho_0\,\omega^{-4}(E/m_0\,c^2)^2\,, \tag{18}$$

which follows immediately from equation (17). The spectral distribution of the radiation involves difficult integrations over electron energies, photon energies, and the scattering angles of the Compton interactions. However, the spectral power must possess the property

$$P(E,\omega) = \int_0^\infty P(E,\omega,\nu)\,\mathrm{d}\nu \tag{19}$$

and, since we are concerned here with a continuum of cosmic ray electron energies ranging over several orders of magnitude, the problem may be simplified by collapsing the emission spectrum into a δ-function at its peak or characteristic frequency. Thus, we may write, in a similar manner to Felten and Morrison (1966),

$$P(E,\omega,\nu) = P(E,\omega)\,\delta(\nu-\nu_{\mathrm{c}})\,, \tag{20}$$

where ν_{c} is the characteristic emission frequency for electrons of energy E at an epoch corresponding to ω.

Now it is easily verified (e.g. Ginzburg and Syrovatsky 1964) that, in a local inertial frame and for an isotropic distribution of electron velocities, the average energy of a recoil photon following a Compton interaction with a relativistic electron is

$$\bar{\epsilon}' = \tfrac{4}{3}(E/m_0\,c^2)^2\bar{\epsilon}\,, \tag{21}$$

where $\bar{\epsilon}$ is the mean photon energy. The properties of black-body radiation produce $\bar{\epsilon} = 2\cdot 7\;kT$ and therefore equation (21) yields

$$\bar{\epsilon}' = 3\cdot 6(E/m_0\,c^2)^2\,kT\,. \tag{22}$$

Then, allowing for the adiabatic expansion of the photon gas, the characteristic frequency is

$$\nu_{\mathrm{c}} = 3\cdot 6(E/m_0\,c^2)^2(kT_0/h)\omega^{-1}\,, \tag{23}$$

when T_0 is the temperature of the black-body radiation at the present epoch. Furthermore, the received frequency $\nu_{\mathrm{c}0}$ is independent of the epoch of emission and depends solely on the electron energy. This follows from the redshift relation $\nu_{\mathrm{c}} = \nu_{\mathrm{c}0}\,\omega^{-1}$ which exactly compensates the relation $T = T_0\,\omega^{-1}$ for the photon temperature. Following Felten and Morrison (1966), the X-ray emission per unit proper volume may now be written in the form

$$j(\nu,\omega) = \int_{m_0c^2}^\infty P(E,\omega,\nu)\,N^*(E,\omega)\,\mathrm{d}E = \int P(E,\omega)\,\delta(\nu-\nu_{\mathrm{c}})\,N^*(E,\omega)\frac{\partial E}{\partial\nu_{\mathrm{c}}}\,\mathrm{d}\nu_{\mathrm{c}} \tag{24}$$

and from equation (23) and the properties of the δ-function the emission coefficient is

$$j(\nu,\omega) = P(E_\nu,\omega)\,N^*(E_\nu,\omega)\,h(m_0\,c^2)/7\cdot 2\,E_\nu\,kT_0\,\omega^{-1}\,, \tag{25}$$

where the emission frequency ν is equal to the frequency ν_0 measured by the origin observer and equation (22) yields

$$E_\nu = m_0 c^2 (h\nu_0/3\cdot 6\,kT_0)^{\frac{1}{2}}. \tag{26}$$

Combining equations (18), (25), and (26), we find

$$j(\nu_0,\omega) = \tfrac{2}{3}\sigma_t c\rho_0\,\omega^{-3}(h/3\cdot 6\,kT_0)^{3/2} m_0 c^2 \nu_0^{\frac{1}{2}} N^*(E_\nu,\omega). \tag{27}$$

Recalling that $N^*(E,\omega)$ is the number of electrons with energy E per unit proper volume per unit energy, we replace in equation (16) $q = q'(m_0 c^2)^{\gamma-1}$, and from equation (27) the formula for the emission coefficient is

$$j(\nu_0,\omega) = \tfrac{2}{3}\sigma_t\,\rho_0\,\omega^{-3}\left(\frac{h}{3\cdot 6\,kT_0}\right)^{\frac{1}{2}(3-\gamma)} \nu_0^{\frac{1}{2}(1-\gamma)}\,\mu_0\,q' \int_{\omega_{0\mathrm{m}}}^{\omega} f(\omega_0)\frac{\partial t_0}{\partial\omega_0}\,\mathrm{d}\omega_0, \tag{28}$$

where q' has units sec^{-1} and

$$f(\omega_0) = \omega_0^{-x}\left(\frac{\omega_0}{\omega}\right)^{\gamma-1}\left(1 - BE_\nu\,\omega\int_{\omega_0}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{\gamma-2}, \tag{29}$$

with the electron energy E_ν being determined through equation (26). The general equation (Payne 1969) for the extragalactic background intensity in isotropic world models is

$$I_\nu = \frac{c}{4\pi}\int_{\omega\mathrm{min}}^{1} j(\nu\omega^{-1},\omega)\omega^3\frac{\partial t}{\partial\omega}\,\mathrm{d}\omega \tag{30}$$

when absorption in the intergalactic medium is negligible. So, finally, equations (28) and (30) yield the rather complicated equation for the X-ray background intensity

$$I(\nu_0) = A_1 A_2^{3-\gamma}\nu_0^{\frac{1}{2}(1-\gamma)}\,\mu_0\,q'\int_{\omega\mathrm{min}}^{1}\frac{\partial t}{\partial\omega}\,\omega^3\left(\int_{\omega_{0\mathrm{m}}}^{\omega} f(\omega_0)\frac{\partial t_0}{\partial\omega_0}\,\mathrm{d}\omega_0\right)\mathrm{d}\omega, \tag{31}$$

where

$$A_1 = \sigma_t c^2\rho_0/6\pi \qquad \text{and} \qquad A_2 = (h/3\cdot 6\,kT_0)^{\frac{1}{2}}.$$

The constants A_1 and A_2 are determined by the properties of the black-body radiation. If the temperature of the thermal photons at the present epoch is $T_0 = 3\,^\circ\mathrm{K}$, then $A_1 = 1\cdot 946\times 10^{-26}$ and $A_2 = 2\cdot 108\times 10^{-6}$, where both quantities are in MKS units. Equation (31) involves an integration over three variables and it is necessary to introduce some simplifying assumptions before this can be accomplished by numerical techniques. If, as must be the case, there is a maximum electron energy E_m for the injection spectrum, then the lower limit $\omega_{0\mathrm{m}}$ of the inner integral of equation (31) will depend not only on ω and E but also on the value of E_m. This introduces severe complications into the equations since $\omega_{0\mathrm{m}}$ is then the solution of the equation

$$\frac{E}{E_\mathrm{m}} = \frac{\omega_{0\mathrm{m}}}{\omega}\left(1 + BE_\mathrm{m}\,\omega_{0\mathrm{m}}\int_{\omega_{0\mathrm{m}}}^{\omega}\frac{1}{\omega'^5}\frac{\partial t'}{\partial\omega'}\,\mathrm{d}\omega'\right)^{-1}. \tag{32}$$

In order to avoid these extra difficulties, we have assumed a linear injection spectrum defined over all energies, and in the ensuing computations the maximum initial energy will be infinite. The value of ω_{0m} will then be determined by substituting E_m equal to infinity in equation (32), which then yields

$$\int_{\omega_{0m}}^{\omega} \frac{1}{\omega'^5} \frac{\partial t'}{\partial \omega'} \mathrm{d}\omega' = \frac{1}{B\omega E}. \tag{33}$$

It follows from equations (32) and (33) that, for $E_m \gg E$, the errors introduced into the calculations by assuming an infinite energy range will not be significant.

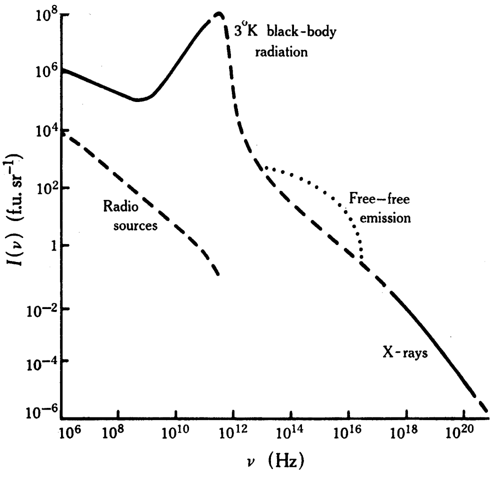

Fig. 2.—Qualitative features of the measured and estimated background intensities for the observable region of the electromagnetic spectrum. The solid lines indicate measured regions of the spectrum and the dashed lines are theoretical predictions. (Note that 1 f.u. $= 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$.)

At high photon energies, the X-ray intensity may be overestimated, but at relatively low energies the calculations should approximate to the real situation. The lower cutoff frequency of the X-ray background will correspond to the low energy cutoff for the electron spectrum which, in the absence of ionization losses, is close to the electron rest mass energy. The actual energy at which this cutoff occurs does not concern us here since we can arbitrarily limit our derived spectrum by the low frequency limit of the observations and this will not affect the equations.

The experimental radio and X-ray background spectra are shown schematically in Figure 2. The solid lines indicate measured regions of the spectrum, while the dashed lines are extrapolations or theoretical predictions. The positions of the high and low frequency cutoffs for both spectra will depend on the exact form of the appropriate electron energy spectrum and are shown only roughly in the diagram.

The intensity contribution from discrete radio sources depends on the particular world model and the source average luminosity and mean spectral index. The Parkes catalogue for radio sources in the declination zones 0° to +20° and −20° to 0° (Day *et al.* 1966; Shimmins *et al.* 1966) provides the most comprehensive set of data on spectral indices. Previous analyses of the spectral index distribution have relied upon data obtained from surveys made with different instruments and different observational techniques. The Parkes surveys have the important advantage of the same instrument being used to observe sources at the three frequencies 408, 1410, and 2650 MHz. The Parkes data imply a mean spectral index $\alpha = 0\cdot 88$ with a variance on this value of about $0\cdot 10$. Therefore, according to the relation $\gamma = 2\alpha+1$, the differential energy spectrum for the electrons responsible for the radio emissions must have an index $\gamma_r = 2\cdot 76$. On the other hand, the X-ray background spectrum (Fig. 1) implies an electron population having $\gamma_x \simeq 3\cdot 8$, which differs from γ_r by a value close to unity. Furthermore, numerical computations in the following section show that inveı se Compton and expansive energy losses can account for a change of unity in γ. We therefore assert that X-ray photons are produced in intergalactic space by the interaction of fast electrons with thermal photons and that the electron population has an original electron spectrum close to the average spectrum in radio sources. It is unlikely, of course, that the energy spectrum would remain completely unaltered with the escape of electrons from the radio source into intergalactic space. In all probability, there will be a tendency for an excess of high energy electrons to be produced by the injection process, which will lead to a flattening of the initial electron spectrum. Comparison of the radio and X-ray background spectra indicates that the change in index produced by this effect is less than the uncertainties involved in defining the slope of the X-ray spectrum. Hence we propose a model in which explosive and disruptive events in radio galaxies are the source of relativistic electrons responsible for radio frequency emission, and in which these electrons escape more or less unimpeded into intergalactic space. The initial electron energy distribution may therefore be assumed, for all intents and purposes, to have a spectral index of $\gamma_x = 2\cdot 76$.

Also included in Figure 2 is the possible effect on the background intensity of free–free emission from intergalactic ionized hydrogen. The amount of bremsstrahlung emission depends on the density and temperature of the plasma, but more critically it depends on the epoch at which heating of the gas occurs. The apparent lack of distortion in the black-body curve at low frequencies (Payne 1969) provides an upper limit for the redshift z' corresponding to this epoch. The calculations indicate $z' \simeq 100$ and that heating of the gas produces an expansion which is practically isothermal. Under these circumstances, the free–free emission intensity will be less than 100 f.u. sr^{-1} (10^{-24} W m^{-2} Hz^{-1} sr^{-1}), assuming reasonable values of 2×10^{-5} cm^{-3} for the electron density and 10^5 °K for the gas temperature. The gas becomes optically thick at a frequency of approximately 3×10^{16} Hz, and the resulting spectrum is indicated in Figure 2 by the dotted line. Recent observations of soft X-rays by Henry *et al.* (1968) support the argument for a background contribution due to free–free emission from a dense (10^{-5} to 10^{-6} cm^{-3}) intergalactic plasma. After allowing for probable effect of interstellar absorption, they find an X-ray flux at $0\cdot 27$ keV (6×10^{16} Hz) of about 3 photons cm^{-2} sec^{-1} sr^{-1} eV^{-1}. However, there is alternative evidence (Bowyer, Field, and Mack 1968) that apparently contradicts these results, and until further independent measurements are made the background intensity will remain poorly defined at low energies. If the soft X-ray flux is indeed of the order of that reported by Henry *et al.* then the contribution to the background intensity due to bremsstrahlung emission will dominate the contributions from other processes, and it follows that the predictions of the present analysis can only find direct application at photon energies greater than about 300 eV.

V. Results

(a) *Model Definition*

The numerical integration of equation (31) will prove too difficult if we revert to the usual differential equation (2) for the expansion factor. It is possible, however, to analyse several world models that have simple expansion functions. These may be listed as:

Model	Dirac	Einstein–de Sitter	Milne	Page	de Sitter
R/R_0	$(t/t_0)^{\frac{1}{3}}$	$(t/t_0)^{\frac{2}{3}}$	t/t_0	$(t/t_0)^2$	$\exp\{H_0(t-t_0)\}$
$\mathrm{d}\omega/(H_0\,\mathrm{d}t)$	ω^{-2}	$\omega^{-0\cdot 5}$	1	$\omega^{0\cdot 5}$	ω

So, in the interests of a fast calculation procedure, the conditions defining models have been simplified by assuming that, in general,

$$\mathrm{d}\omega/\mathrm{d}t = H_0\,\omega^n\,, \tag{34}$$

where the value of n now determines the evolutionary path of the model. This means that models examined by this method will always be in a state of expansion and other models such as those of the oscillating type cannot have definition. The method is still instructive, however, and by suitable choice of the parameter n equation (34) can closely simulate the expansive period of an oscillating universe.

It is now a simple matter to transform equation (29) into the function

$$f(\omega_0) = \omega_0^{-x}\left(\frac{\omega_0}{\omega}\right)^{\gamma-1}\left\{1-\frac{BE_\nu\,\omega}{n'}\left(\omega_0^{-n'}-\omega^{-n'}\right)\right\}^{\gamma-2}, \tag{35}$$

where $n' = n+4$. The equation (31) now degenerates into an integration over two variables which may be evaluated by numerical methods. The lower limit for the inner integral (corresponding to infinite initial electron energy) is, by equation (33),

$$\omega_{0\mathrm{m}} = \left(n'/BE\omega + \omega^{-n'}\right)^{-1/n'}. \tag{36}$$

The three equations (31), (35), and (36) are sufficient for the derivation of theoretical spectra as functions of n and ω_{min}.

(b) *Derived Spectra*

Equation (31) has been numerically integrated for different world models defined by the parameter n and for a series of values of ω_{min} corresponding to the epoch at which the electron injection phase begins. For reasons discussed in Section IV, we take $\gamma = 2\cdot 76$ for the index of the initial electron energy spectrum. The theoretical spectra have been normalized to fit the observed intensity at a photon energy of 1 MeV and for each value of n the integration procedure determined the value of ω_{min} for a best fit condition to the spectral points of Figure 1. The expected X-ray background spectra for $n = -2\cdot 0$ are plotted in Figure 3, and the result of electron energy losses due to the inverse Compton effect and coordinate expansion is seen to be a steepening of the spectra at high photon energies. A "break" in the X-ray spectrum occurs at a photon energy E_{b} that is shifted progressively to higher photon energies as ω_{min} is decreased. At energies much greater than E_{b}, the spectral index increases by $0\cdot 5$, implying a change of unity in γ for the energy distribution of the electrons responsible for the X-radiation.

The spectra in Figure 3 have been extended to a low photon energy of 1 eV, but this procedure is probably unrealistic in view of the uncertainties in the electron spectrum at low energies. Although experimental evidence is so far restricted to only a few of the strong radio sources, it appears that the average source spectrum has a low frequency cutoff in the range 1–10 MHz, and this could be produced by any

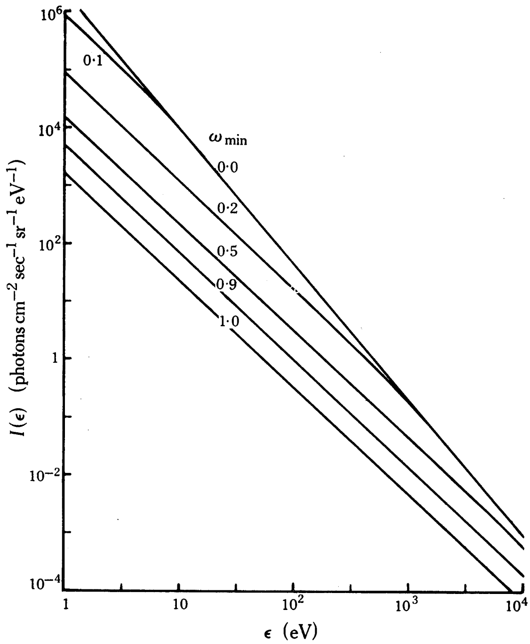

Fig. 3.—Theoretical X-ray spectra for $n = -2\cdot0$ and various values of the expansion parameter $\omega_{\min}$ at commencement of electron injection.

number of processes, including thermal absorption, synchrotron self-absorption, and a cutoff in the energy spectrum of the relativistic electrons generated within the source. Experimental data do not yet permit a separation of these effects, but in the last case the electron energy cutoff E_c is given by the equation for synchrotron emission (e.g. Oort and Walraven 1956) as

$$\nu_c = 1\cdot61\times10^{-11}H_\perp E_0^2 \quad \text{MHz}, \tag{37}$$

where E_c is expressed in electron-volts and $H_\perp$ is the perpendicular component of the magnetic field strength. The average field in strong radio sources is about 5×10^{-5} G and substituting, for example, $\nu_c = 5$ in equation (37), it follows from equation (26) that the X-ray background spectrum should have a "break" at a photon energy of about 30 eV. These rough calculations serve to underline the difficulties involved in making any precise determination of the soft X-ray flux.

Results for other values of n are summarized in Table 1, which includes for each n the normalization constant $\mu_0\times q'$ equal to the total number of high energy electrons injected per unit time and per unit proper volume.

The results in Table 1 show quite clearly the dependence of $\omega_{\min}$ (which defines the epoch at commencement of electron injection) on the index n defining the particular world model. Those universes that are slowly expanding (positive values of n) require that electron injection begins at large values of the expansion parameter corresponding to small redshifts. Now the process of electron dissipation into

TABLE 1

RESULTS OF NORMALIZATION OF DERIVED SPECTRA TO FIT EXPERIMENTAL DATA

n	0·5	0·0	−0·5	−1·0	−2·0	−5·0	−10·0
$\omega_{\min}$	0·88	0·85	0·81	0·74	0·65	0·51	0·35
$\mu_0 \times q'$ ($\times 10^{-19}$)							
$x = 3 \cdot 0$	4·72	3·96	3·21	2·78	2·45	2·18	2·04
$x = 5 \cdot 5$	0·67	0·58	0·51	0·45	0·39	0·36	0·34

intergalactic space is presumably associated with the radio emitting phase of a galaxy's evolution, and, excluding quasi-stellar objects, some radio sources are known to have redshifts greater than 0·3. Using the relation $z = \omega^{-1} - 1$, this determines an upper limit of $\omega_{\min} \simeq 0 \cdot 75$ for the expansion factor at the commencement of cosmic ray injection from radio galaxies. On the basis of these rough estimates, world models defined by equation (34), and having $n > -1 \cdot 0$, cannot account for the shape of the X-ray background spectrum since they require an excessively large value of $\omega_{\min}$. Alternatively, universes that are rapidly expanding at the present epoch (negative values of n) have values of $\omega_{\min}$ that are reasonably compatible with the general properties of the observed spatial distribution of radio sources. Comparison of equations (2) and (34) shows that negative values of n are consistent with an oscillating universe, and the results presented in Table 1 therefore lend support to recent evidence indicating a closed oscillating world model.

We now turn to the consideration of the predictions and consequences of the normalization constants listed in Table 1. Recalling that q' is the number of electrons injected per unit time, the total power P emitted in the form of cosmic ray electrons will be

$$P = \int_{E_1}^{E_2} q'(m_0 c^2)^{\gamma-1} E^{-\gamma+1} \, \mathrm{d}E \,, \tag{38}$$

where E_1 and E_2 are the minimum and maximum values of the appropriate energy range.

Let β be the ratio of the total heavy particle energy E_{p} to the total electron energy E_{e} within a radio galaxy. The value of β is somewhat uncertain, but Burbidge (1959) uses $\beta = 100$ on the assumption that the energy E_{p} is gradually transferred to secondary electrons. Integrating between electron energies corresponding to frequencies of 10^7 and 10^{10} Hz of the radio spectrum and assuming $\beta = 100$, Maltby, Matthews, and Moffet (1963) have obtained values for the total energy content (including magnetic field and particles) in 24 strong radio sources. If energy equipartition exists between field and particles, the energy contained in relativistic electrons is about 4×10^{57} erg. Furthermore, if radio sources dissipate their electron energy in a time of approximately 10^6 years, then the power output as cosmic ray electrons will be approximately $P_0 = 1\cdot5\times10^{44}$ erg sec^{-1}. The results of Maltby, Matthews, and Moffet yield an average field strength $H \simeq H_\perp = 4\times10^{-5}$ G and, substituting $\nu_{c1} = 10^7$ and $\nu_{c2} = 10^{10}$ in equation (37), we find $E_1 = 1\cdot2\times10^8$ and $E_2 = 4\times10^9$ eV. It then follows after integration of equation (38) that $P = q'\times1\cdot6\times10^{-8}$ erg sec^{-1}, and from the derived values of $\mu_0\times q'$ in Table 1 the average power output from radio galaxies may be calculated if μ_0 is known.

The spatial distribution of powerful radio sources is not well understood, but there are evidently only three of them (Virgo A, Centaurus A, and Fornax A) within a radius of approximately 20 Mpc. This implies a local source density at the present epoch of $\mu_0 \simeq 4\times10^{-72}$ m^{-3}. If the proper density of radio sources depends only on the geometrical properties associated with the expansion in relativistic cosmologies, then $x = 3$ in equation (29) and it follows from the results in Table 1 and equation (38) that for $n = 0\cdot5$ the output power is $P = 1\cdot9\times10^{45}$ erg sec^{-1}, while the extreme value of $n = -10\cdot0$ yields $P = 8\times10^{44}$ erg sec^{-1}. These estimates are at least an order of magnitude greater than the total power derived above by assuming energy equipartition between magnetic field and particles. According to these arguments, the rate of electron injection from radio galaxies having constant number–density in coordinate volume is insufficient to account for the X-ray flux.

The energy problem in radio galaxies has been studied by Burbidge (1962) and he proposes a model for rapidly evolving sources in which the magnetic field strengths in regions of radio emission are less than the equilibrium values. If we allow a field H_1, instead of the equipartition value H, then the total particle energy will be increased in the ratio $(H/H_1)^{3/2}$. Substituting $H_1 \simeq 1$–2 μG yields a value for the total power dissipated in relativistic electrons which is of the same order as the values of P required to account for the X-ray intensity.

It is not necessary, however, to rely on a non-equilibrium condition between field and particles to obtain satisfactory values for the injected power. A numerical analysis by Longair (1966) has shown that radio source counts may be explained in terms of the evolution of powerful radio sources. The results indicate a source density distribution given by $\mu \simeq \mu_0\,\omega^{-5\cdot5}$ in an Einstein–de Sitter universe. If we assume arbitrarily that the evolution parameter $x = -5\cdot5$ in equation (29), then the derived values for the normalization constants (listed in Table 1) imply a decreased output power of $P = 3\times10^{44}$ erg sec^{-1} for $n = 0\cdot5$ and $P = 1\cdot3\times10^{44}$ erg sec^{-1} for $n = -10\cdot0$. In this case, the emitted power for values of n less than about $-2\cdot0$ are in close agreement with the power P_0 calculated from other independent data. The required *severity* in the evolution of the source density depends on n and is least when n is large and negative. It is obvious, however, that for reasonable values of n the source density must be a fairly strong function of epoch. It should be noted that, although these estimates are based on a spatial density evolution, the quantitative results will not be altered by assuming an evolution in source luminosity. This follows directly from equation (29), provided of course that the cosmic ray injection power is proportional to the source luminosity.

VI. Conclusions

This analysis has shown that the apparent decrease in X-ray intensity at photon energies below 10 keV may be satisfactorily explained in terms of inverse Compton radiation in rapidly expanding universes. The difficulties involved in accounting for the intensity of the X-ray flux can be most easily circumvented by assuming either (1) the magnetic field strength in radio sources is less than the equilibrium value, or (2) the spatial density of sources in coordinate volume is a strong function of epoch. Existing knowledge of radio source physics is not sufficient to determine, with any certainty, which of these effects is dominant. There is little direct evidence to support condition (1). On the other hand, if the universe is evolving, then the large-scale properties of radio galaxies are also likely to evolve. This is implied by the radio source counts which require a rather strong evolutionary function. Condition (2) therefore appears most likely, but the supporting evidence is weak.

VII. Acknowledgment

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