THE MOMENTUM TRANSFER CROSS SECTION FOR ELECTRONS IN HELIUM DERIVED FROM DRIFT VELOCITIES AT 77°K

By R. W. Crompton,* M. T. Elford,* and A. G. Robertson*

[*Manuscript received May* 28, 1970]

Abstract

Drift velocities of electrons in helium at 76·8°K have been measured for $8\times10^{-20} \leqslant E/N \leqslant 2\times10^{-17}$ V cm². From these data, and the earlier measurements of Crompton, Elford, and Jory made at 293°K, the energy-dependent momentum transfer cross section has been determined for electrons with energies between 0·008 and 6 eV. The present cross section agrees with that of Crompton, Elford, and Jory to within 1%. The extension of the energy range to 8 meV permits a direct determination of the scattering length, for which a value of $1\cdot19\,a_0$ is obtained.

I. Introduction

Measurements of the cross section for elastic scattering of electrons by helium atoms have been made using both beam techniques (e.g. Golden and Bandel 1965) and swarm techniques (e.g. Crompton, Elford, and Jory 1967; hereafter referred to as CEJ). Although the calculation of the momentum transfer cross section from swarm data is a more complex procedure than the measurement of the total elastic scattering cross section by a beam technique, an examination of the limitations of each technique suggests that swarm techniques are currently capable of yielding data of higher accuracy for electrons of low energies. For energies less than a few tenths of an electron volt, swarm experiments provide the only available information (Crompton 1969).

Momentum transfer cross sections are derived from an analysis of data for one or more transport coefficients obtained over a wide range of mean swarm energies. The analysis is outlined in Section IV and discussed in more detail by Frost and Phelps (1964), Crompton and Jory (1965), and in CEJ. The present paper reports the application of this method to a new set of drift velocity data for electrons in helium at 76·8°K in the range $0\cdot008$ Td† $\leqslant E/N \leqslant 2\cdot0$ Td, where E/N is the ratio of electric field strength to gas number density. Since the lower limit of the energy range that may be investigated at a given temperature is determined by the thermal energy of the gas molecules, measurements made at 76·8°K provide cross section data with a lower limit of 0·008 eV compared with the 0·02 eV limit of CEJ whose data were obtained at 293°K.

II. Discussion of Experiment

The electrical shutter method of Bradbury and Nielsen (1936) was used to measure the electron drift velocities. The procedures to be followed to obtain maximum precision in the measurements have been discussed by Lowke (1963) and Elford

* Ion Diffusion Unit, Research School of Physical Sciences, Australian National University, P.O. Box 4, Canberra, A.C.T. 2600.

† 1 townsend (Td) = 1×10^{-17} V cm².

(1966). The drift tube described by Crompton, Elford, and McIntosh (1968) was used unmodified for some of the measurements but the intershutter distance was increased to 10 cm for one series of check measurements. This was achieved by inserting three additional guard electrodes identical with the central electrodes of the existing tube. The electron source has been described by Crompton and McIntosh (1968), the electrons being produced by α-particle ionization of the gas. A cold source of this kind is to be preferred for measurements at liquid nitrogen temperature since, at this temperature, small temperature gradients may lead to significant errors in the calculation of the molecular number density N from the gas pressure p.

Pressures were measured by a Texas Instruments quartz spiral manometer calibrated against a CEC type 6201 primary pressure standard. All pressures used in the experiments were close to calibration points, and are considered to be in error by less than $0\cdot1\%$.

A copper–constantan thermocouple attached to the guard ring immediately below the upper shutter was used to monitor the gas temperature. Over an 8 hr period the temperature changes indicated by the thermocouple were less than $0\cdot1$°K, while the measured temperature agreed with the calculated boiling point of the liquid nitrogen to better than $0\cdot1$°K. It was therefore assumed that the temperature of the gas throughout the apparatus was that of the liquid nitrogen bath. This temperature was calculated from the measured purity of the liquid nitrogen and the barometric pressure. Over a four day period the purity of the liquid nitrogen changed by less than $0\cdot2\%$. Such a change causes the boiling point of nitrogen to alter by less than $0\cdot03$°K. In measurements with the apparatus with a 10 cm shutter spacing a second thermocouple was attached to the guard ring immediately above the lower shutter. The pair of thermocouples indicated no temperature gradient within the tube. The uncertainty in the gas temperature is estimated to be less than $\pm0\cdot2\%$.

A correction has been applied to the length of the tube to account for contraction on cooling to liquid nitrogen temperature. Errors in the determination of the length of the tube, discussed by Crompton, Elford, and McIntosh (1968), and the finite thickness of the shutter wires which terminate the drift region cause an uncertainty of $\pm0\cdot15\%$ in the length of the drift region.

The gas used was Matheson Research Grade helium admitted from the cylinder through a Granville–Phillips variable leak valve and a u.h.v. tap. The use of a pressure regulator, with possible contamination from elastomer materials used in its construction, was thereby eliminated.

The drift tube, the associated vacuum system, and the gas handling apparatus were all designed for u.h.v. application. Although the ancillary vacuum equipment was baked at 200°C for a prolonged period prior to the experiments, the majority of the measurements were made without baking the drift tube. The decision not to bake the tube was taken on the grounds that errors arising from contamination of the gas samples were likely to be negligibly small, whereas there was some risk that errors could be introduced by baking as a result of distortion of the electrode structure or damage to the electrode surfaces. Calculations showed that, over the range of E/N covered by the present investigation, impurity levels of H_2 and N_2 of the order of 100 p.p.m. were required to give rise to errors greater than $0\cdot1\%$. These estimates were confirmed in a series of experiments in which drift velocities were measured in helium containing up to 700 p.p.m. of H_2 and N_2. Hydrogen and nitrogen were the most likely contaminants to be present as a result of outgassing since the experiments were performed at 77°K where the vapour pressures of CO_2 and H_2O are negligibly small. Even at the lowest pressures used (40 torr) a level of contamination of 100 p.p.m. would require a partial pressure of impurity of more than 10^{-2} torr, whereas the measured outgassing rates suggest background pressures more than two orders of magnitude lower than this.

As a final check on the adequacy of the gas handling techniques, two experiments were performed. In the first experiment, a check set of measurements was made after the drift tube had been baked at 175°C for more than 72 hr. These results agreed with those shown in Table 1 to within $0\cdot1\%$ wherever the electric field strengths were high enough to eliminate errors from contact potential differences. In the second test, helium was admitted to the drift tube which had not been baked, and was allowed to remain there for 24 hr at room temperature. A mass spectrometric analysis of the gas taken from the tube showed that the only detectable impurity was 10 p.p.m. of N_2, and there was evidence to suggest that even this level was predominantly attributable to background in the mass spectrometer. In any case the previous calculation and measurements showed that such a level would cause less than $0\cdot01\%$ change in the drift velocity. Thus, as was expected, there is no evidence to suggest that the purity of the gas taken from the apparatus differed significantly from the specification of the gas supplied, confirming that in experiments of this type adequate gas purity can be maintained without rigorous outgassing procedures that carry with them the risk of introducing more serious experimental errors.

III. Experimental Values of Drift Velocity

For most values of E/N, measurements were made at three or more pressures in each of several gas samples and at two drift distances. Each entry in Table 1 is a mean value of the measurements, which generally showed a scatter of less than $\pm0\cdot1\%$ from this value for E greater than about 10 V cm^{-1}. The lower entries are the drift velocities derived from the measured values of W' in the manner described in Section III(b) below.

(a) *Contact Potential Differences and Surface Effects*

In the majority of experiments necessitating the use of low field strengths small errors were observed, but it was noted that in many cases the magnitude of the error was not exactly inversely proportional to E. Furthermore, unlike the effects introduced by contact potential differences, which are usually comparatively reproducible under normal experimental conditions, these errors showed a time dependence in many of the experiments and were found to vary from one experimental run to another when the drift tube was warmed to room temperature between runs. A similar effect was observed by Crompton, Elford, and McIntosh (1968) when making measurements of D/μ, the ratio of diffusion coefficient to mobility, and was attributed to a surface phenomenon produced by charged particles striking the metal surfaces (e.g. Petit-Clerc and Carette 1968). It should be stressed that in the present experiments this effect was quite negligible for field strengths greater

Table 1

Experimental values of W' and drift velocity W for electrons in helium at 76·8°K

The upper entries are the measured values of W', the lower entries are the values of W derived from W' as explained in Section III(*b*)

E/N (Td)	W' and W (10^5 cm sec^{-1}) at Pressure (torr) of 700	600	500	400	300	250	200	150	100	80	60	50	40	Best Estimate W (10^5 cm sec^{-1})
0·00795	0·3457	0·3459	0·3462	0·3466	0·3471									0·345(5)
	0·3456	0·3457	0·3460	0·3464	0·3468									
0·01590	0·5707	0·5704	0·5702	0·5703	0·5705		0·5712							0·570(3)
	0·5705	0·5702	0·5700	0·5700	0·5701		0·5707							
0·02386	0·7376	0·7372	0·7367	0·7366	0·7367		0·7370		0·7389					0·736(7)
	0·7374	0·7371	0·7365	0·7363	0·7363		0·7364		0·7377					
0·03181	0·8751	0·8746	0·8741	0·8739	0·8738		0·8742		0·8763					0·873(9)
	0·8749	0·8744	0·8738	0·8736	0·8734		0·8735		0·8749					
0·03976	0·9923	0·9919	0·9912	0·9910	0·9908		0·9915		0·9936					0·991(1)
	0·9921	0·9917	0·9909	0·9906	0·9903		0·9908		0·9921					
0·04771	1·097	1·097	1·096	1·096	1·096		1·097		1·099					1·096
	1·097	1·097	1·096	1·096	1·096		1·096		1·097					
0·05567	1·192	1·191	1·191	1·190	1·190		1·191		1·193					1·190
	1·191	1·191	1·190	1·190	1·190		1·190		1·191					
0·06362	1·279	1·279	1·278	1·278	1·278		1·279		1·281					1·278
	1·279	1·279	1·278	1·278	1·277		1·278		1·279					
0·07157	1·360	1·359	1·359	1·359	1·359		1·359		1·361					1·359
	1·360	1·359	1·359	1·358	1·358		1·358		1·360					
0·07952	1·436	1·436	1·436	1·435	1·435	1·437	1·436		1·438					1·436
	1·436	1·436	1·435	1·435	1·435	1·436	1·435		1·436					
0·09543	1·576	1·576	1·575	1·575	1·575		1·575		1·578					1·575
	1·576	1·575	1·575	1·574	1·574		1·574		1·576					
0·1193	1·763	1·763	1·762	1·762	1·762	1·763	1·763		1·765					1·762
	1·763	1·762	1·762	1·762	1·761	1·762	1·762		1·763					
0·1431	1·930	1·930	1·929	1·929	1·928		1·929		1·932					1·929
	1·930	1·929	1·928	1·928	1·927		1·928		1·929					
0·1590		2·032	2·032	2·032	2·031	2·033	2·032		2·034					2·031
		2·032	2·031	2·031	2·031	2·032	2·031		2·032					
0·1988			2·266	2·266	2·266	2·266	2·267		2·270					2·265
			2·265	2·265	2·265	2·265	2·265		2·268					
0·2386				2·475	2·474	2·475	2·476	2·476	2·478					2·474
				2·474	2·473	2·474	2·474	2·474	2·476					

0·2783	2·665 2·664	2·666 2·664	2·666 2·664	2·667 2·665	2·669 2·666					2·665
0·3181	2·842 2·841	2·842 2·840	2·842 2·840	2·843 2·841	2·845 2·842			2·853 2·846		2·841
0·3499	2·975 2·973	2·974 2·973	2·974 2·973	2·976 2·974	2·978 2·975			2·985 2·978		2·974
0·3976		3·162 3·160	3·162 3·160	3·162 3·160	3·165 3·162	3·166 3·162	3·168 3·162	3·172 3·165		3·161
0·4771					3·453 3·449	3·454 3·449	3·455 3·449	3·459 3·452		3·450
0·5567					3·714 3·710	3·716 3·711	3·719 3·712	3·721 3·713		3·712
0·6362					3·958 3·953	3·963 3·958	3·963 3·956	3·960 3·952		3·955
0·7157					4·185 4·180	4·190 4·185	4·190 4·183	4·196 4·188	4·199 4·188	4·185
0·7952					4·404 4·399	4·405 4·399	4·406 4·400	4·410 4·401	4·415 4·403	4·400
0·9543							4·809 4·801	4·813 4·804	4·817 4·805	4·803
1·193							5·350 5·340	5·355 5·344	5·367 5·352	5·344
1·431								5·859 5 847	5·861 5·847	5·847
1·590							6·170 6·160	6·167 6·155	6·170 6·155	6·157
1·750								6·470 6·457	6·472 6·455	6·456
1·988									6·903 6·886	6·886

than about 10 V cm^{-1} and introduces no additional uncertainty into the final values of W, except for the lowest value of E/N.

(b) *Correction for the Effects of Electron Density Gradients within the Pulse*

The drift velocity W, defined by equation (2) in the following section, is derived from the effective transit time t measured in a time-of-flight experiment using a relation of the form

$$W = W'/\{1+(C(D/\mu)/V)\},$$

where V is the potential difference between the shutters and $W' = h/t$. The drift distance is h and the effective transit time t is the time interval between successive open times of the shutters when the gating frequency is adjusted to give maximum transmission through the drift tube.

The constant C is a coefficient whose magnitude depends on the mode of operation of the drift tube (Burch, personal communication), the gas, the ratio E/N, and the gas temperature (see e.g. CEJ). Burch has shown that under the normal conditions of operation used in the present experiments the value of C is small ($|C| < 1$) provided it is valid to assume that the energy distribution within the travelling pulses is uniform. Lowke and Parker (1969) have shown that the spatial variation of the energy distribution can, in certain circumstances, lead to values of $C > 1$ so that significant errors can arise unless $(D/\mu)/V$ is made small. In effect this requires that the measurements be made at sufficiently high gas number densities.

For the present results the value of C is taken to be $1\cdot5$ for all values of E/N. The work of Burch and of Lowke and Parker suggests that this is a reasonable value at least in the range of E/N for which the corrections to W' are greatest. In any case the corrections made to W' are generally less than $0\cdot1\%$, the largest being $0\cdot25\%$, so that with few exceptions errors of 100% in C are unimportant.

On semi-empirical grounds, CEJ also assumed a value of $1\cdot5$ for C, but they recognized the uncertainty involved, particularly in those cases for which their drift velocities were measured at only one pressure. Their data are slightly more sensitive to errors in C than are the present data.

A further possible source of error is suggested by the work of Kivel (1959) and O'Malley (1963) (see Frost and Phelps 1964). At sufficiently high values of N, and for electron swarms of sufficiently low energy, a slight dependence of the measured values of W on N might arise through the interaction of electrons with a number of atoms simultaneously, thereby producing an effective screening of the long range forces. It is predicted that this screening should introduce a correction to the measured values of W proportional to $N^{\frac{1}{3}}$ and that, at the maximum gas number densities used in our experiments, the effect might be measurable for electrons with energies below about $0\cdot01$ eV. Apart from the effect described in Section III(a), which does not follow the predicted relationship, no dependence of W on N was observed, suggesting that this effect is not present to any significant extent in these measurements. This is not altogether surprising because the mean energy of the electrons, even at the lowest value of E/N, is in the range where the effect should become very small (O'Malley 1963).

IV. Determination of the Momentum Transfer Cross Section

When inelastic processes may be neglected, the distribution function for electron energies may be written

$$f(\epsilon) = A \exp\left(-\int_0^{\epsilon} \{(ME^2e^2/6mN^2\epsilon q_{\mathrm{m}}^2(\epsilon))+kT\}^{-1}\,\mathrm{d}\epsilon\right), \tag{1}$$

where $\epsilon^{\frac{1}{2}} f(\epsilon)\,\mathrm{d}\epsilon$ is the probability that an electron has energy in the range ϵ to $\epsilon+\mathrm{d}\epsilon$, M and m are the molecular and electronic masses, $q_{\mathrm{m}}(\epsilon)$ is the energy-dependent momentum transfer cross section, k is the Boltzmann constant, T is the gas temperature, and e is the electronic charge. The constant A is obtained from the normalizing relationship

$$\int_0^{\infty} \epsilon^{\frac{1}{2}} f(\epsilon)\,\mathrm{d}\epsilon = 1\,.$$

Written in terms of this energy distribution function the formula for drift velocity becomes

$$W = -\frac{eE}{3N}\left(\frac{2}{m}\right)^{\frac{1}{2}} \int_0^{\infty} \frac{\epsilon}{q_{\mathrm{m}}(\epsilon)} \frac{\mathrm{d}f}{\mathrm{d}\epsilon}\,\mathrm{d}\epsilon\,. \tag{2}$$

Table 2

Momentum transfer cross section for electrons in helium

Energy ϵ (eV)	Cross Section $q_{\mathrm{m}}(\epsilon)$ (10^{-16} cm^2)	Energy ϵ (eV)	Cross Section $q_{\mathrm{m}}(\epsilon)$ (10^{-16} cm^2)
0·008	5·18*	0·25	6·27
0·009	5·19*	0·30	6·35
0·010	5·21	0·40	6·49
0·013	5·26	0·50	6·59
0·017	5·31	0·60	6·66
0·020	5·35	0·70	6·73
0·025	5·41	0·80	6·77
0·030	5·46	0·90	6·82
0·040	5·54	1·0	6·85
0·050	5·62	1·2	6·91
0·060	5·68	1·5	6·96
0·070	5·74	1·8	6·98
0·080	5·79	2·0	6·99
0·090	5·83	2·5	6·96
0·10	5·86	3·0	6·89
0·12	5·94	4·0	6·60*
0·15	6·04	5·0	6·26*
0·18	6·12	6·0	6·01*
0·20	6·16		

* These values are $\pm 5\%$, all other values are $\pm 2\%$.

The value of $q_{\mathrm{m}}(\epsilon)$ is determined, as described in CEJ, by adjusting some arbitrarily chosen cross section until the values of W calculated using equations (1) and (2) agree with the measured drift velocities to the degree justified by the accuracy of the measurements.

The assumptions involved in deriving expressions for the drift velocity have recently been re-examined by Cavalleri and Sesta (1968). In the Appendix we present a summary of calculations which indicate that for the present situation these assumptions introduce no significant error in the derived cross section.

Table 2 shows the derived cross section that gives agreement between the calculated drift velocities and those of Table 1 to within $0\cdot2\%$ everywhere and generally to better than $0\cdot1\%$. Using this cross section calculations were made of drift velocities at 293°K. As expected, these calculated values were within $0\cdot5\%$ of the values measured by CEJ for all values of E/N. The agreement between calculated values of D/μ and the CEJ values measured at 293°K is, with a single exception, within the 1% error limit placed on these data.

Over the common range of energy this cross section is in close agreement with that of CEJ, the maximum difference being less than 1%. However, because of the lower temperature used in the present experiments it has been possible to extend the low energy limit of the cross section. The high energy limit has been determined by taking into account the drift velocity data of CEJ.

(*a*) *Energy Range of the Cross Section*

The mean energies of the swarms used in the determination of the momentum transfer cross section ranged from about $0\cdot01$ eV for the lowest value of E/N at $76\cdot8$°K to about $1\cdot5$ eV at the highest value of E/N (used by CEJ) at 293°K. If the cross section is changed by 2% for *all* energies the calculated drift velocities change by at least 1%, the error limit placed on the experimental data. At the lowest value of E/N, this change is $1\cdot6\%$. An accuracy claim of $\pm2\%$ on the momentum transfer cross section is therefore justified over the range of energies approximately bounded by the mean energies of the swarms at the limits of the range of measurement. However, the width of the energy distribution in a swarm makes the drift velocity sensitive to the momentum transfer cross section for a range of energies on either side of the mean energy of the swarm. We have therefore examined in some detail the energy limits which should be placed on the cross section. The discussion is in terms of the lower limit, which is of chief interest in this paper, but the technique applies equally well to the upper limit.

Two methods of fixing the limit have been employed. First, a 2% step in the cross section was moved in the direction of increasing energy until a 1% change in the calculated value of W at the lowest value of E/N was observed. Any deviation as large as 2% at a higher energy than this limit would cause a disagreement between the calculated and measured values of W greater than the error limit placed on W and would therefore be corrected. Using this technique a claim of $\pm2\%$ on q_{m} over the range $0\cdot017$ to 2 eV appears to be justified.

A second technique has also been applied which makes use of the separation which may be effected between random and systematic errors. The 1% error limit placed on the drift velocity data is dominated by the systematic component. The accuracy with which the value of W at the lowest value of E/N can be measured obviously sets the lower limit of the range in which the cross section can be derived with a given accuracy. This measurement, which is the least accurate of the set, has a "systematic error" of about $0\cdot7\%$ and a "random error" of about $0\cdot4\%$ (the latter value is $0 \cdot 1\%$ for most values of E/N). It should be pointed out that the random error limit is determined by the greatest deviation shown by the measurements made at the higher pressures from the mean values given in Table 1. These are the measurements which most influence the "best estimate" values. The systematic error limit is the arithmetic sum of the errors in W produced by the maximum error considered possible in the measurements of pressure, temperature, frequency, drift length, and in the parameter C.

The systematic errors, since they affect all values of E/N, will cause a vertical shift in the cross section without significantly changing its shape. The $0 \cdot 7\%$ systematic error bar on W at the lowest value of E/N is equivalent to a 1% shift in q_{m}. The $0 \cdot 4\%$ random error may therefore account for a further 1% error in q_{m} to make up the total error bar of 2%. To determine the energy limit, a 1% step in q_{m} was therefore moved to increasing energies until a $0 \cdot 4\%$ change in the calculated value of W was observed. This procedure indicates that the values of $q_{\mathrm{m}}(\epsilon)$ given in Table 2 may be considered correct to $\pm 2\%$ in the range $0 \cdot 01$ to 3 eV. Above 3 eV and below $0 \cdot 01$ eV the accuracy falls to $\pm 5\%$. A similar analysis applied to the data of CEJ shows that their cross section may be considered accurate to $\pm 2\%$ for energies down to about $0 \cdot 03$ eV.

V. Discussion

(*a*) *Comparison with Theoretical Cross Sections*

Many of the important experimental and theoretical determinations of the momentum transfer cross section have been considered in CEJ. As the present cross section (Table 2) agrees closely with that of CEJ their comments on these cross sections require no further amplification. Since the publication of their paper, however, a number of new calculations of electron–helium scattering have been published. The first two of these to be considered, Hoeper, Franzen, and Gupta (1968) and Bransden and McDowell (1969), are similar in that the phase shifts are derived from experimental data.

Hoeper, Franzen, and Gupta (1968) used the differential scattering measurements of Ramsauer and Kollath (1932) to calculate the phase shifts of the first four partial waves. These phase shifts were then used to calculate the momentum transfer cross section (curve B in Fig. 1). This cross section differs by up to 10% from the present one (Table 2 and curve A in Fig. 1).

Bransden and McDowell (1969) used considerably more experimental data in the fitting procedure and were thus able to impose more constraints on their solution. They compared experimental values of the total, diffusion, and elastic differential cross sections with values calculated from an assumed set of phase shifts, the phase shifts being adjusted to give the best overall fit to the experimental data. The validity of the conclusions drawn by Bransden and McDowell is limited because error limits of $\pm 10\%$ were assigned to all the experimental data used without discussion or justification. The disagreement between the momentum transfer cross section calculated by Bransden and McDowell and the present results is as large as 15% in the region of overlap and is well outside experimental error.

In the case of electron–helium scattering, derivations of phase shifts from experimental data would now seem to be unnecessary as calculations based on interaction potentials are available. In the low energy regime these calculations, two of which are discussed below, give better agreement with the most precise recent experimental cross sections than the cross sections obtained by either Hoeper, Franzen, and Gupta or Bransden and McDowell.

The calculation of Callaway *et al.* (1968) uses the extended polarization potential. This is a more complete formalism based on the earlier work of La Bahn and Callaway (1966), who showed that agreement between theory and experiment is considerably improved by the inclusion of non-adiabatic interaction terms. Figure 1 shows that this calculated cross section (curve C) closely resembles the present one in shape but lies about 5% lower throughout the energy range. The earlier dynamic exchange calculation of La Bahn and Callaway (1966; curve D in Fig. 1) is in closer agreement with the measured cross section. Callaway *et al.* attribute the differences between their two calculations to slight changes produced in the exchange terms of the potential function when the specifically velocity-dependent interactions used previously are replaced by an additional central distortion potential.

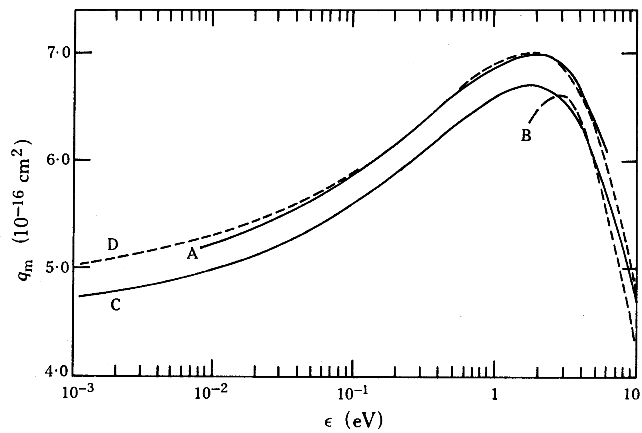

Fig. 1.—Energy-dependent momentum transfer cross section $q_m(\epsilon)$: curve A, present determination from drift velocity experiments; B, Hoeper, Franzen, and Gupta (1968) with data from Ramsauer and Kollath (1932); C, Callaway *et al.* (1968); D, La Bahn and Callaway (1966).

The first results of an expansion method of calculating the scattering phase shifts for helium in this energy range have been reported by Michels, Harris, and Scolsky (1969). The ground state target atom was represented by two different approximations but in the energy range under consideration the results were found to be insensitive to which of the two was used. From their diagram it appears that for energies above 1 eV their cross section lies only a few per cent below ours; however, the discrepancy is of the order of 10% below 0·5 eV. The scattering length of 1·145 is 4% lower than our experimental value. Thus the two most recent theoretical calculations give cross sections somewhat below our experimental result. It was this fact that caused us to examine once more the possible sources of systematic error in our work. However, a more rigorous examination of the possible effect of impurities (Section II) and of the assumptions made in the theory used to analyse the data

(Appendix) has shown that the errors assigned to our cross section do not require modification.

(b) *Scattering Length*

Since the present momentum transfer cross section extends to $0\cdot008$ eV, the scattering length for electrons in helium can be determined directly by simple extrapolation. The value is found to be $1\cdot19\,a_0$ (a_0 is the Bohr radius). The extrapolation is over such a small energy range that an overall error limit of $\pm2\%$ is justified. It has been assumed in this extrapolation that there is no marked structure in the cross section between $0\cdot008$ eV and zero energy.

An alternative method of obtaining the scattering length is the use of modified effective range theory. The momentum transfer cross section is given by this theory as (O'Malley 1963)

$$q_{\mathrm{m}} = 4\pi\{A^2+(4\pi/5a_0)\alpha Ak+(8/3a_0)\alpha k^2\ln(ka_0)+Ck^2+\ldots\},$$

where k is the electron wave number and α is the electric polarizability (taken as $1\cdot36\,a_0^3$ for helium). A and C are considered as adjustable parameters when fitting the above expression by a least squares fit to the experimental curve. The parameter A is to be identified as the scattering length. Previous workers have used this theory to extrapolate experimental cross sections to much lower energies and to obtain the scattering length but there has been no investigation of whether this procedure is valid. Since the present cross section extends over nearly three orders of magnitude from $0\cdot008$ eV, an investigation of the validity of such an extrapolation using modified effective range theory is possible. Three different energy ranges were chosen and a fit made over each range in turn. The theory was then used to extrapolate the cross section to energies outside the fitted energy range, the extrapolated curve and the value of A being compared with that obtained experimentally.

The following energy ranges were considered:

(1) *$0\cdot008 < \epsilon < 0\cdot1$ eV*

If a fit is made to only that part of the cross section between $0\cdot008$ eV and $0\cdot1$ eV, A is found to be $1\cdot18\,a_0$. However, the predicted cross section at higher energies gives a rather poor fit to the experimental curve.*

(2) *$0\cdot2 < \epsilon < 2$ eV*

If the fit is made over the mid energy range ($0\cdot2 < \epsilon < 2$ eV) of the present cross section the scattering length is found to be $1\cdot21\,a_0$. At energies less than $0\cdot2$ eV the predicted cross section is found to be up to 6% higher than the experimental curve. This difference is much greater than the estimated error of the experimentally derived cross section.

(3) *$0\cdot008 < \epsilon < 2$ eV*

When a wider energy range is used ($0\cdot008 < \epsilon < 2$ eV) a value of $1\cdot19\,a_0$ for A is obtained but the fitted curve over this energy range deviates by up to 4% from the experimental cross section. This difference is also greater than the estimated error of the experimental cross section.

* We are indebted to Dr. H. H. Michels whose analysis of our cross section by a similar fit to the modified effective range formula prompted us to perform these calculations.

From the fits to the data within the three energy ranges above it can be seen that the value of A and the predicted cross section depend on the energy range over which the fit is made. In particular, case (2) indicates that an extrapolation from $0 \cdot 2$ eV to lower energies is subject to considerable error. Such an extrapolation was made by Golden (1966) using the data of Golden and Bandel (1965). These data had a lower energy limit of $0 \cdot 3$ eV and the value of the scattering length obtained ($1 \cdot 15 a_0$) must be therefore regarded with caution. Two other values of A have been reported. O'Malley (1963) used the data of Ramsauer and Kollath (1929, 1932) to obtain a value of $1 \cdot 19 a_0$ but the agreement of this value with the present value must be regarded as fortuitous in view of the comments above on the extrapolation procedure and the uncertainty of the data. Frost and Phelps (1964), using a simple extrapolation of a cross section derived from swarm data, obtained a value of $1 \cdot 18 a_0$ which is in good agreement with the present value.

We conclude with the comment that although the derivation of the cross section for momentum transfer from an analysis of electron transport coefficients is necessarily more complex than the derivation of the total scattering cross section from single beam experiments, nevertheless the evidence presented in this paper supports the view that such analyses of swarm experiments have provided the most accurate data at present available for low energy electron scattering in helium.

VI. Acknowledgments

One of us (A.G.R.) wishes to acknowledge the financial assistance received from an A.N.U. Research Scholarship and from the N.S.W. Education Department.

VII. References

Ballerio, R., Bonalumi, R., and Cavalleri, G. (1969).—*Energia nucl., Milano* **16**, 455.
Bradbury, N. E., and Nielsen, R. A. (1936).—*Phys. Rev.* **49**, 388.
Bransden, B. H., and McDowell, M. R. C. (1969).—*J. Phys. B.* **2**, 1187.
Callaway, J., La Bahn, R. W., Pu, R. T., and Duxler, W. M. (1968).—*Phys. Rev.* **168**, 12.
Cavalleri, G. (1969).—*Phys. Rev. Lett.* **23**, 907.
Cavalleri, G., and Sesta, G. (1968).—*Phys. Rev.* **170**, 286.
Cavalleri, G., and Sesta, G. (1969).—*Phys. Rev.* **177**, 434.
Crompton, R. W. (1969).—*Adv. Electronics Electron Phys.* **27**, 1.
Crompton, R. W., Elford, M. T., and Jory, R. L. (1967).—*Aust. J. Phys.* **20**, 369.
Crompton, R. W., Elford, M. T., and McIntosh, A. I. (1968).—*Aust. J. Phys.* **21**, 43.
Crompton, R. W., and Jory, R. L. (1965).—Proc. 4th Int. Conf. on the Physics of Electronic and Atomic Collisions, Quebec, p. 118.
Crompton, R. W., and McIntosh, A. I. (1968).—*Aust. J. Phys.* **21**, 637.
Davidson, P. M. (1954).—*Proc. phys. Soc.* B **67**, 159.
Elford, M. T. (1966).—*Aust. J. Phys.* **19**, 629.
Frost, L. S., and Phelps, A. V. (1964).—*Phys. Rev.* **136**, A1538.
Golden, D. E. (1966).—*Phys. Rev.* **151**, 48.
Golden, D. E., and Bandel, H. W. (1965).—*Phys. Rev.* **138**, A14.
Hoeper, P. S., Franzen, W., and Gupta, R. (1968).—*Phys. Rev.* **168**, 50.
Huxley, L. G. H. (1960).—*Aust. J. Phys.* **13**, 578.
Kivel, B. (1959).—*Phys. Rev.* **116**, 1484.
La Bahn, R. W., and Callaway, J. (1966).—*Phys. Rev.* **147**, 28.
Lowke, J. J. (1963).—*Aust. J. Phys.* **16**, 115.
Lowke, J. J., and Parker, J. H. (1969).—*Phys. Rev.* **181**, 302.

Michels, H. H., Harris, F. E., and Scolsky, R. M. (1969).—*Phys. Lett.* A **28**, 467.
O'Malley, T. F. (1963).—*Phys. Rev.* **130**, 1020.
Petit-Clerc, Y., and Carette, J. D. (1968).—*Vacuum* **18**, 7.
Ramsauer, C., and Kollath, R. (1929).—*Annln Phys.* **3**, 536.
Ramsauer, C., and Kollath, R. (1932).—*Annln Phys.* **12**, 529.

Appendix

In deriving equation (2) the assumption is made that the distribution function can be adequately represented by a two-term expansion in spherical harmonics. Although it has been assumed hitherto that such an expansion leads to negligible error in relating the drift velocity to the momentum transfer cross section, no quantitative estimate of the error has been made. In view of the claim we have made for the accuracy of the cross section derived in this paper it is necessary to attempt to make such an estimate.

If equation (2) is expressed in terms of a velocity distribution function $f(c)$, defined such that $4\pi c^2 f(c)\,\mathrm{d}c$ is the probability that an electron has a velocity between c and $c+\mathrm{d}c$, and the integration on the right-hand side is then performed by parts we have (Davidson 1954)

$$W=\frac{4\pi Ee}{3m}\int_0^\infty c^2\left(\frac{2\lambda_{\mathrm{m}}(c)}{c}+\frac{\mathrm{d}\lambda_{\mathrm{m}}(c)}{\mathrm{d}c}\right)f(c)\,\mathrm{d}c\,, \tag{A1}$$

where $\lambda_{\mathrm{m}}(c)$ is the mean free path for momentum transfer.

The same formula is obtained by using a free path treatment (Huxley 1960) in which the assumption is made that the fractional change in velocity, $\Delta c/c$, of the electron between successive collisions is small. In deriving this formula, Huxley showed that the drift velocity of the class of electrons which traverse free paths with speed c is given by

$$w(c)=\frac{Ee}{3m}\left(\frac{2\lambda_{\mathrm{m}}(c)}{c}+\frac{\mathrm{d}\lambda_{\mathrm{m}}(c)}{\mathrm{d}c}\right)$$

(see also Davidson 1954). Integration over the distribution of electron speeds then results in the formula (A1). In evaluating the formula the function $f(c)$ has been taken as the first term of the two-term expansion in spherical harmonics. The fact that the same formula for W is obtained in each case indicates that the approximations made in the two theories are equivalent (Cavalleri and Sesta 1968).

Recently, Cavalleri and Sesta (1968, 1969) have developed a more rigorous free path theory which avoids the assumption $\Delta c/c \ll 1$. Furthermore they do not make the usual assumption that the inelastic collision frequency is small compared with the elastic collision frequency, although this is of no relevance to the present problem. The formula for drift velocity derived by these authors is

$$W_c=\int_0^\infty c^2 w_0(c_0)f_0(c_0)\,T(c_0)\,\mathrm{d}c_0\Big/\int_0^\infty c^2 f_0(c_0)\,T(c_0)\,\mathrm{d}c_0\,, \tag{A2}$$

where $T(c_0)$ is the mean free time between collisions of electrons with initial speed c_0 immediately after a collision and $f_0(c_0)$ is the distribution function for these initial speeds. $w_0(c_0)$ is the drift velocity of the class of electrons with initial speed c_0 where $w_0(c_0)$ is given by

$$w_0(c_0) = \frac{\displaystyle\int_0^\infty \sin\theta_0 \left\{\int_0^\infty (c_0\cos\theta_0 + at)\exp\left(-\int_0^t (c/\lambda(\tau))\,\mathrm{d}\tau\right)\mathrm{d}t\right\}\mathrm{d}\theta_0}{\displaystyle\int_0^\infty \sin\theta_0 \left\{\int_0^\infty \exp\left(-\int_0^t (c/\lambda(\tau))\,\mathrm{d}\tau\right)\mathrm{d}t\right\}\mathrm{d}\theta_0}. \qquad \text{(A3)}$$

In this formula, θ_0 is the angle between $\boldsymbol{E}$ and $\boldsymbol{c}_0$, $a = eE/m$,

$$c(c_0, \theta_0, \tau) = \{(c_0\cos\theta_0 + a\tau)^2 + c_0^2\sin^2\theta_0\}^{\frac{1}{2}}$$

is the speed at time τ after a collision of an electron with initial speed c_0 in the initial direction θ_0, and $\lambda(\tau)$ is the mean free path of electrons with velocity $c(c_0, \theta_0, \tau)$.

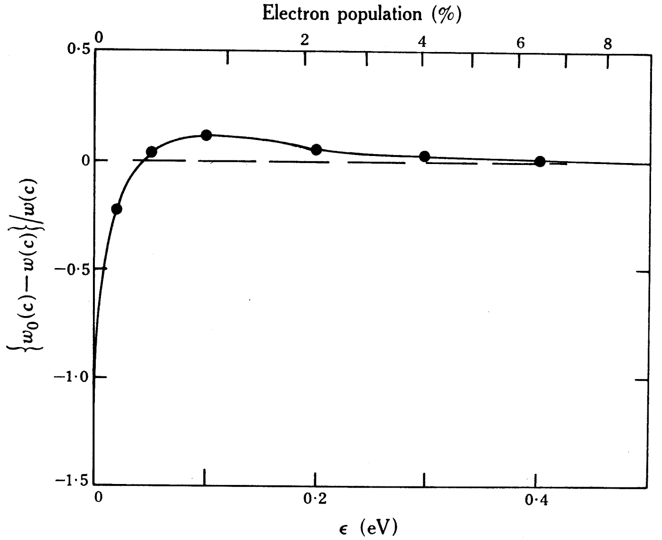

Fig. 2.—Fractional difference between $w_0(c)$ and $w(c)$ as a function of energy ϵ for $E/N = 3\cdot64$ Td. The scale on the upper abscissa indicates the percentage of the electron population with energies less than the corresponding energy on the lower abscissa.

Equation (A3) is derived on the assumption that the scattering is isotropic. To generalize this equation to cover nonisotropic scattering $\lambda(\tau)$ is replaced by the mean free path for momentum transfer given by $1/Nq_{\mathrm{m}}(c)$ (Cavalleri and Sesta 1968).

In order to calculate W_c from the formula (A2) a knowledge of $f_0(c_0)$ is required. This function has been calculated only for the case λ_{m}/c = constant (Ballerio, Bonalumi, and Cavalleri 1969), so that unfortunately it is still necessary to make some approximation in applying the formula even though it is formally rigorous. Cavalleri (1969) has shown, however, that to a good approximation $f_0(c_0)T(c_0)$ can be replaced by $Tf(c)$, where T is a normalizing constant and $f(c)$ is again the first term of the usual two-term expansion. It follows that

$$W_c = 4\pi \int_0^\infty c^2 w_0(c) f(c)\, \mathrm{d}c\,.$$

Since W_c is expected to be a higher order approximation to the drift velocity than W of formula (A1), the order of magnitude of the error incurred in making the usual approximations can be calculated. The method of performing the calculation has been to determine first $w_0(c)$ and $w(c)$ and thus determine the relative error $\phi(c) = \{w_0(c) - w(c)\}/w(c)$ as a function of c. The fractional error in W, that is,

$$(4\pi/W) \int_0^\infty \phi(c)\, w(c) f(c)\, c^2\, \mathrm{d}c$$

has then been calculated.

The error incurred in using formula (A1) is expected to be a maximum when the largest number of electrons in the swarm receive a significant increase to their speed along a free path. In the present case a simple argument shows that this occurs where $(E/N)/(D/\mu)$ is a maximum, i.e. at the maximum value of E/N used in our experiments. Calculations of $w_0(c)$, $w(c)$, and $f(c)$ were therefore made at $E/N = 3\cdot 64$ Td, the highest values used by CEJ. The curve showing $\{w_0(c) - w(c)\}/w(c)$ as a function of electron energy at this value of E/N is shown in Figure 2. In this case it is found that only about 6% of the electrons have energies below $0\cdot 4$ eV, i.e. have sufficiently small velocities that they contribute significantly to any difference between W_c and W. Integration of the error curve as described above then shows that the error in W introduced by making the usual assumption that $\Delta c \ll c$ is of the order of $0\cdot 1$%.

A similar calculation performed at the lowest value of E/N used in the experiments ($0\cdot 008$ Td) gave the somewhat larger estimated error of $0\cdot 25$%. For the reasons stated above, a smaller error would be expected and we attribute this inconsistency to less satisfactory integration in this instance. In view of other possible sources of error in the estimate it did not seem profitable to extend the calculations further.

In conclusion we wish to stress again that the estimates we have made are only approximate since the estimation relies on the use of an approximation for $f_0(c_0)$ which is least valid in the region of interest. Nevertheless, it would appear that even a large error in $f_0(c_0)\,T(c_0)$ would not sufficiently alter the fraction of the electron population with small velocities to produce a significant change in the calculated value of W_c.