ON THE THEORY OF TYPE II AND TYPE III SOLAR RADIO BURSTS

II.* ALTERNATIVE MODEL

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Abstract

It is argued that observational data on type III bursts point to the exciting agent for a burst being a bunch of fast electrons which experience no strong two-stream instability. Ways in which the two-stream instability might be suppressed are discussed. A model for type III bursts based on a bunch of electrons passing through the corona and generating electron plasma waves incoherently is explored. It is found that the emission at the fundamental frequency results from the coherent scattering of electron plasma waves into electromagnetic waves. Coherent scattering of electron plasma waves into electron plasma waves is found to be the major effect allowing emission at the second harmonic to arise from the coalescence of electron plasma waves into electromagnetic waves. Reasonable agreement with observation is found for 10^{33}–10^{34} electrons per bunch.

I. INTRODUCTION

In Part I (Melrose 1970; present issue pp. 871–84) a number of difficulties encountered by existing models for type II and type III solar radio bursts were indicated. In the present paper we attempt to formulate a model, primarily for type III bursts, which overcomes the objections raised in Part I. In this formulation we adopt the most direct interpretation of the observations and explore possible ways in which the observations might be explained theoretically. The notation of Part I is used throughout.

The observations are discussed in Section II. In brief these indicate that type III bursts, which occur in groups usually associated with a single flare or flare-like event, are excited by the passage of bunches of electrons passing through the solar corona. If each burst is associated with a bunch of electrons then energetic considerations impose severe limits on the loss of energy which the bunch suffers due to any coherent emission of l-waves (electron plasma waves); typically energy losses by the bunch through coherent emission of l-waves in excess of about 10 times the energy loss through incoherent emission of l-waves are untenable. This, and other observational data, indicate that, if the exciting agent is a bunch of electrons, then any two-stream instability must be strongly suppressed.

The suppression of the two-stream instabilitity is considered in Section III, where we explore one possible way in which a suppression might occur. In the remainder of this article we simply assume that any two-stream instability is suppressed to the extent that the energy lost by the bunch through coherent emission

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of l-waves is not substantially greater than the energy lost due to incoherent emission of l-waves.

Thus the basic model for type III bursts which is explored here is one in which the exciting agent is a bunch of electrons which emit l-waves incoherently. Any coherent emission in detailed calculations is ignored but the effects of coherently emitted l-waves are considered briefly.

The conversion of l-waves into t-waves, which can escape to be observed at Earth, occurs through scattering by plasma particles, which leads to emission at the fundamental frequency, and by coalescence of two l-waves, which leads to emission at the second harmonic. The scattering by plasma particles can be either coherent or incoherent. The coherent process converting l-waves into t-waves is only important if the growth time for the t-waves is much less than their escape time. We find this to be the case. Emission at the second harmonic requires either scattered particles or scattered l-waves (see Part I). In Section IV we examine various processes which allow emission at the second harmonic to arise, and find that the most important effect is the coherent scattering of l-waves into l-waves by plasma particles.

The conclusion that coherent scattering processes are important has the unfortunate practical implication that the radiated power cannot be calculated in detail, as can be done when all processes are incoherent, for example. Limitations on the power radiated through these processes are considered in Section V, where it is shown that the results are compatible with observation.

II. Interpretation of Observational Data

We consider the implications of some widely accepted interpretations of observational data on possible theories for type III bursts. We concentrate on five features of the observations.

(a) *Electron Bunches*

The most direct interpretation of the exciting agent for type III bursts is that each burst is due to the passage through the corona of a bunch of fast electrons ejected in the explosive phase of a solar flare or flare-like event. The observed velocity of propagation is then identified as the velocity of the electrons in the bunch; the velocity spread is necessarily small for the electrons in the bunch to remain together. Type III bursts occur in groups with about 10 bursts per group; consequently a series of bunches is required. Because the bunches rise to very high levels in the corona they must travel along field lines which are "open", i.e. the field lines must extend high into the corona.

Wild, Smerd, and Weiss (1963) have discussed this interpretation. A major piece of evidence in favour of bunches of electrons acting as the exciting agent relates to the onset of type III bursts and X-ray emission occurring within a second or so of the flash phase of the flare. The electrons are thought to be accelerated, within a second or so (Sturrock and Coppi 1966) during the development of the instability associated with the flare. These electrons lead to X-ray emission (see e.g. de Jager 1967) while those which can escape to the corona along appropriately directed field lines presumably cause the type III emission. de Jager (1967) estimated that about 10^{35} electrons per explosive event are involved (as did Wild, Smerd, and Weiss 1963).

The arguments against the exciting agent being fast ions rather than fast electrons rest largely on the implausibility of ions rather than electrons being accelerated in such a short time. There are models for solar flares in which the energy is stored in energetic ions in "magnetic bottles" and released in the flash phase (Schatzman, personal communication). Even granted such a flare mechanism (see Sweet 1969 for a review of various mechanisms) it is hard to see how a bunch of nearly monoenergetic protons could be released.

Another piece of evidence which points to the exciting agent being a bunch of electrons is the absence of type III bursts with streaming velocities u less than about $0\cdot 2\,c$ (see Kundu 1965 and references therein). The point is that scattering via Coulomb interactions with the plasma particles randomizes the motion of a bunch of electrons with $u \ll 0\cdot 2\,c$ before the bunch can propagate any significant distance through the corona. To see this, consider a distribution of electrons all with speed u and a distribution of pitch angles α (the angle between the velocity of an electron and the magnetic field direction) described by $\Phi(\alpha)$ with

$$\int_0^\pi \mathrm{d}\alpha\, \sin\alpha\, \Phi(\alpha) = 1\,.$$

According to Sivukhin (1966) $\Phi(\alpha, t)$ evolves according to

$$\frac{\partial \Phi}{\partial t}(\alpha, t) = \nu_0 \left(\frac{V_\mathrm{e}}{u}\right)^3 \frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left(\sin\alpha \frac{\partial\Phi}{\partial\alpha}(\alpha, t)\right), \tag{1}$$

where

$$\nu_0 = (e^2 \omega_\mathrm{p}^2 / m_\mathrm{e}\, V_\mathrm{e}^3) \ln \Lambda \tag{2}$$

and $\ln\Lambda$ (≈ 20 in the corona) is the Coulomb logarithm. By expanding in spherical harmonics, i.e. by writing

$$\Phi(\alpha, t) = \sum_{n=0}^{\infty} \Phi_n(t)\, \mathrm{P}_n(\cos\alpha)\,,$$

$$\Phi_n(t) = \tfrac{1}{2}(2n+1) \int_{-1}^{+1} \mathrm{d}(\cos\alpha)\, \mathrm{P}_n(\cos\alpha)\, \Phi(\alpha, t)\,,$$

the solution of (1) for given initial conditions reduces to

$$\Phi(\alpha, t) = \sum_{n=0}^{\infty} \Phi_n(t=0)\, \mathrm{P}_n(\cos\alpha) \exp\{-n(n+1)\nu_0 (V_\mathrm{e}/u)^3 t\}\,. \tag{3}$$

For $\Phi(\alpha, t=0) = \delta(\alpha)/\sin\alpha$, for example, one finds that the initial streaming motion is substantially dispersed, e.g. $\Phi(\alpha = \tfrac{1}{2}\pi) \gtrsim \tfrac{1}{2}\Phi(\alpha = 0)$ for

$$t \gtrsim \tfrac{1}{3}\nu_0^{-1} (u/V_\mathrm{e})^3\,, \tag{4}$$

i.e. after the bunch has travelled a distance

$$ut \gtrsim \frac{m_\mathrm{e} u^4}{3e^2 \omega_\mathrm{p}^2 \ln\Lambda} \approx 3\times 10^{10} \left(\frac{10^9 (\mathrm{cm}^{-3})}{n_\mathrm{e}}\right) \left(\frac{u}{0\cdot 2\,c}\right)^4 \ \mathrm{cm}\,. \tag{5}$$

With $n_e \gtrsim 10^9\,\mathrm{cm}^{-3}$ at the point where the bunch originates, it is apparent that bunches with $u \ll 0\cdot 2\,c$ cannot propagate high into the corona. Conversely for $u > 0\cdot 2\,c$ and $n_e \approx 10^9\,\mathrm{cm}^{-3}$ initially the bunch escapes to higher levels in the corona, where n_e is smaller, and the dispersion of the bunch occurs relatively high in the corona. No relevant limitation on u would apply if the bunch were to consist of ions.

(b) *Energetic Considerations*

Energetic considerations impose two limitations on any model based on bunches of electrons as the exciting agents for type III bursts. Firstly the total kinetic energy in the group of bunches associated with any flare-like event must be less than the total energy released in such an event. Secondly the total energy lost per bunch in the passage through the corona must be less than the total kinetic energy per bunch.

The energetics of solar flares have been reviewed by Parker (1963), Sturrock and Coppi (1966), Sweet (1969), and other authors. The total energy released per flare appears to be about 10^{32} erg. If there are N electrons per bunch, 10 bunches per flare, and if each electron has an energy of 100 keV, then the total kinetic energy released in electrons is 10^{32} erg for $N \approx 10^{38}$ electrons per bunch. One expects there to be far fewer electrons per bunch than this extreme upper limit allows.

The rate of energy loss per electron includes the effects of Coulomb interactions with the background plasma particles and that of incoherent emission of l-waves. These two effects cause an electron with velocity u to slow down according to

$$\frac{\mathrm{d}E}{\mathrm{d}t} = m_e u\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{e^2\omega_p^2}{u}\left\{\ln\Lambda + \ln\left(\frac{u}{V_e}\right)\right\}, \tag{6}$$

where the term $\ln\Lambda$ comes from the Coulomb interactions and the term $\ln(u/V_e)$ from the incoherent emission of l-waves. Setting $\mathrm{d}u/\mathrm{d}t = -u/t_s$ to define a slowing down time t_s, we have

$$t_s = \nu_0^{-1}(u/V_e)^3, \tag{7}$$

where we neglect $\ln(u/V_e)$ ($\approx 3\cdot 5$ for $u = 0\cdot 3\,c$) compared to $\ln\Lambda$ (≈ 20). Comparison of (7) with (4) indicates that in a typical type III burst, in which scattering effects are not entirely negligible, a small but significant fraction of the kinetic energy per electron is lost due to Coulomb interactions.

The implication is that other energy losses cannot greatly exceed the rate of energy loss per electron given by (6) or (7). Any significant coherent emission of l-waves leads to an energy loss which exceeds that lost through incoherent emission of l-waves. In view of the discussion of the scattering effect of the Coulomb interactions given above, it would appear that any energy loss through coherent emission of l-waves can be no greater than about 10 times that lost through incoherent emission of l-waves.

It is expected that a bunch of electrons with $u \gg V_e$ will cause a two-stream instability to develop. In this instability l-waves are coherently emitted with an e-folding growth time much shorter than the time in which the bunch passes any fixed point. If such a two-stream instability develops then the energy lost by the bunch in coherently emitting l-waves exceeds that lost by incoherent emission by an enormous factor. The energetic constraints deduced above imply that any two-stream instability must be very strongly suppressed. The processes which may lead to such a suppression are discussed in Section III below.

Another feature of the energetics is the power radiated in electromagnetic waves. Wild (1950; see also Kundu 1965), finds that the time integrated intensity per burst averaged over many bursts varies as

$$E_f = 6\times10^{-16}\{10^{-8}(\mathrm{Hz}^{-1})f\}^{-3\cdot5} \quad \mathrm{erg\,cm^{-2}\,Hz^{-1}}, \tag{8}$$

where $f = \omega_{\mathrm{p}}/2\pi$. Integrating (8) over frequency with a cutoff at $f = f_0$ and assuming that the radiation in the corona is emitted isotropically (not all of which can escape), the energy radiated per burst reduces to ($R_{\mathrm{E}} = 1$ A.U.)

$$4\pi R_{\mathrm{E}}^2\int_{f_0} \mathrm{d}f\,E_f \approx 7\times10^{19}\{10^{-8}(\mathrm{Hz}^{-1})f_0\}^{-2\cdot5} \quad \mathrm{erg}. \tag{9}$$

Thus the total energy radiated per burst is of the order of 10^{20}–10^{21} erg. (With a duration of about 10 sec per burst the instantaneous power is 10^{19}–10^{20} erg sec^{-1} typically.) With 100% efficiency in converting the kinetic energy of electrons into electromagnetic waves we would require $N = 10^{28}$ electrons. Thus we have extreme limits on N corresponding to $10^{28} \ll N \ll 10^{38}$. Even with each extreme limit satisfied by several orders of magnitude (as one expects) the actual value of N remains poorly determined by energetic considerations. With 10 bursts per explosive event the estimates by de Jager (1967) on the X-ray emission and by Wild, Smerd, and Weiss (1963) (who do not present their reasoning) suggest that about 10^{34} electrons per bunch is plausible.

(c) *Collisional Damping*

The observed duration of type III bursts at fixed frequency is characteristic of the time for collisional relaxation in the corona (see Kundu 1965 and references therein). Thus the duration of the bursts at a given frequency, which frequency determines the local electron number density, is of the order of the self-collision time ($\sim \nu_0^{-1}$) for electrons at that density and at the coronal temperature. This observation indicates that after the passage of the bunch of electrons the l-waves generated by the electrons remain behind to be collisionally damped. Perkins and Salpeter (1965) found the effective absorption coefficient (the inverse of the e-folding time for energy decay) for collisional damping of l-waves to be given by

$$\gamma_{\mathrm{c}} = \tfrac{1}{3}\sqrt{(2/\pi)}\nu_0, \tag{10}$$

where ν_0 is given by (2). The observations indicate that the l-waves persist at a fixed point for a time of order γ_{c}^{-1} after the passage of the bunch.

This observation further indicates that any two-stream instability must be strongly suppressed (see Part I). Tidman, Birmingham, and Stainer (1966) point out that one expects l-waves to be Landau damped after the passage of a collisionless shock front in which a two-stream instability occurs.

The observation that the damping time for l-waves is γ_{c}^{-1} is automatically explained if it is assumed that a bunch of monoenergetic electrons excites l-waves only incoherently. An electron with velocity u excites l-waves with phase velocities $v_\phi \leqslant u$. However, a bunch of monoenergetic electrons cannot reabsorb l-waves with $v_\phi < u$ because the absorption coefficient (see equations (I10), where the prefix I indicates Part I) depends on the derivative $\partial f(v)/\partial v$ of the distribution function at $v = v_\phi$; for $f(v) \equiv 0$ for $v < u$ reabsorption is strictly zero. Tidman and Dupree (1965) pointed out that a distribution of suprathermal electrons with a velocity gap leads to a very high effective temperature for l-waves with phase velocities corresponding to this gap (see their Section 3, Case (ii)). Thus for $v_\phi < u$ the l-waves, once generated by the fast electrons, can be only either collisionally damped or Landau damped by thermal electrons. Landau damping by thermal electrons gives an absorption coefficient

$$\gamma_{\mathrm{L}} = \sqrt{(\tfrac{1}{2}\pi)}\,\omega_{\mathrm{p}}(v_\phi/V_{\mathrm{e}})^3 \exp\{-\tfrac{1}{2}(v_\phi/V_{\mathrm{e}})^2\}. \tag{11}$$

For the range of phase velocities

$$aV_{\mathrm{e}} \lesssim v_\phi < u, \qquad a^2 = 2\ln\{\sqrt{(\tfrac{1}{2}\pi)}\,a^3\omega_{\mathrm{p}}/\gamma_{\mathrm{c}}\} \approx 2\ln\Lambda, \tag{12}$$

Landau damping by thermal electrons can be neglected in comparison to collisional damping. The l-waves in the range (12) persist for a time γ_{c}^{-1}.

(*d*) *Ratio of Intensities*

For type III bursts the ratio of the intensities at the fundamental and second harmonic is usually greater than or comparable to unity. Any model for the emission must account for the observed ratio and also for the absence of observable higher harmonics.

As emphasized in Part I, nonthermal emission at the second harmonic is possible only if the two coalescing l-waves both come from nonthermal distributions of such waves. Now a unidirectional bunch of electrons with velocity u generates l-waves with wavenumber k at an angle, called the Čerenkov angle,

$$\Theta_{\mathrm{C}} = \cos^{-1}(\omega_{\mathrm{p}}/ku) < \tfrac{1}{2}\pi, \qquad \omega_{\mathrm{p}}/k \gtrsim V_{\mathrm{e}}, \tag{13}$$

relative to the streaming direction. It is impossible for any two l-waves so generated to coalesce into a t-wave. There must be either scattered electrons generating l-waves with given k over a wide cone of angles or scattered l-waves for any significant emission at the second harmonic to occur.

Let us consider the scattering of electrons briefly. The scattering described by (1) leads to electrons with pitch angles $\alpha \sim \frac{1}{2}\pi$ only when a substantial dispersion in angles of propagation has occurred for all electrons, i.e. the scattering described by (1) leads to no significant spread in the angular distribution of l-waves with given k until the unidirectional character of the bunch is nearly destroyed. Now (1) takes into account only the effect of distant encounters; close encounters cause individual electrons to be scattered through a large angle (with little change in speed). The rate at which close encounters occur is given by (Spitzer 1956)

$$\nu_1 \approx (\nu_0/4\ln\Lambda)(V_{\mathrm{e}}/u)^3,$$

so that one expects there to be about

$$N_{\mathrm{s}} \approx (\nu_1/\gamma_{\mathrm{c}})N \sim (N/\ln \Lambda)(V_{\mathrm{e}}/u)^3 \tag{14}$$

scattered electrons left behind in the region where l-waves remain excited by the bunch. We examine the scattering of the l-waves, generated directly by the bunch, into l-waves in Section IV and find this scattering to be a more important effect in allowing emission at the second harmonic than is the presence of scattered electrons.

The approximate equality of the observed intensities at the two harmonics both in type II and type III bursts may well indicate that the emission processes, at the two harmonics, are constrained to lead to approximately the same intensity. By this is meant that the intensity at the second harmonic may not be approximately equal to that at the fundamental due to the fortuitous appearance of a parameter of order unity (as occurs in the models studied by, for example, Ginzburg and Zhelezniakov 1958; Wild, Smerd, and Weiss 1963; and Tidman 1965) but may be due to a constraint imposed on both radiation processes. If we accept this idea then the only possible constraint would appear to be reabsorption. The scattering of l-waves into t-waves can proceed only until the effective temperature of the t-waves approaches that of the l-waves. If the emission at the second harmonic involves scattered l-waves then the effective temperature of the t-waves produced by the coalescence process is limited by the minimum of the effective temperatures of the scattered l-waves and the unscattered l-waves; the effective temperature of the scattered l-waves can approach but not exceed that of the unscattered l-waves. Thus if the emissions at both the fundamental and second harmonic approach maximum efficiency then the effective temperatures both approach that of the initially generated l-waves. Any model in which this occurs (e.g. the model of Kaplan and Tsytovich 1967) automatically accounts for the approximate equality of the intensities at the two harmonics. This condition of maximum efficiency is not quite achieved in the model presented below. It proves useful to express the emitted radiation in terms of a fraction of the maximum possible effective temperature (of the emitted l-waves); the observations require that this fraction be comparable for the fundamental and second harmonic.

(e) *Backward Emission*

Smerd, Wild, and Sheridan (1962) pointed out that the emission at the second harmonic in type II bursts appears to originate from a lower level in the corona than does the fundamental; similar, but inconclusive, evidence indicates that this effect also applies to type III bursts. These authors accounted for the observation by firstly pointing out that if the second harmonic is emitted preferentially in the backward direction, i.e. towards the photosphere, then the radiation escapes after being reflected (refracted) from a lower level in the corona, and secondly by arguing in favour of backward emission. Their arguments in favour of backward emission were based on emission due to combination scattering off thermal fluctuations, which was shown in Part I to be a negligible emission process, and so require some modification to be acceptable.

We defer discussion of preferential backward emission of the second harmonic until we identify the spectra of l-waves which determine the emission at the second harmonic.

III. Suppression of Two-stream Instability

If we are to retain the basic assumption that the exciting agent for type III bursts is a bunch of fast electrons then both energetic considerations and the inference that the l-waves are collisionally damped require that any two-stream instability must be very effectively suppressed. One expects a stream of electrons with number density $n_{\mathrm{s}} \ll n_{\mathrm{e}}$, streaming velocity $u \gg V_{\mathrm{e}}$, and velocity spread Δu to cause l-waves to grow exponentially with the maximum growth rate at $k \approx \omega_{\mathrm{p}}/u$ given by

$$|\gamma| \approx (\pi/2e)^{\frac{1}{2}}\omega_{\mathrm{p}}(n_{\mathrm{s}}/n_{\mathrm{e}})(u/\Delta u)^{2}. \tag{15}$$

According to Shapiro (1963) the end result of the instability is that the stream loses three-quarters of its initial kinetic energy with one-third of the initial kinetic energy density remaining in l-waves. Evidently some process or processes must halt the exponential growth of l-waves long before this end result is approached if type III bursts are due to such a stream of electrons.

The maximum growth of l-waves in a two-stream instability occurs for l-waves with $v_{\phi} \approx u$ and directed along the streaming direction. Any process which causes these l-waves to be either damped or scattered can halt the exponential growth if the rate of damping or scattering approaches the growth rate (15). Kaplan and Tsytovich (1967) invoked the scattering of l-waves into l-waves to suppress the two-stream instability. The rate at which the l-waves are scattered is proportional to the energy density in l-waves and so, when this energy density reaches a value at which the scattering rate balances the growth rate (15), the exponential growth ceases. However, as discussed in Part I, this particular suppression process becomes effective only when the energy density in l-waves reaches such a high level that the model comes into conflict with observation (and with the energetic considerations discussed in Section II(b), as may be readily confirmed).

Another class of processes which can lead to the effective removal of l-waves and thereby suppress the instability involves s-waves. For example, the process $l+s \rightarrow l$ leads to a scattering of l-waves provided that the $\boldsymbol{k}$ vectors allow the process to occur (see equation (I22) and following comments). This process requires s-waves with $k_s < \lambda_{\mathrm{De}}^{-1}$; the scattering rate is proportional to the energy density in such s-waves. There is another class of processes which involve s-waves with $k_s > \lambda_{\mathrm{De}}^{-1}$. In this case scattering by the plasma particles can lead to the scattering of an l-wave into an s-wave, the simultaneous absorption of an s-wave and an l-wave or, as always, the reverse processes. Tsytovich (1966) discussed these processes and pointed out that when the l-waves and s-waves are oppositely directed the net effect is one of absorption for both l-waves and s-waves.

As pointed out in Part I, there are reasonable grounds to expect that s-waves might be generated through a current instability. Because the stream of fast electrons involves a net motion of negatively charged particles the (more mobile) thermal electrons stream in the opposite direction to maintain charge neutrality. If U is the streaming velocity of the thermal electrons then the vanishing of the net current requires that U be given by

$$U = (n_{\mathrm{s}}/n_{\mathrm{e}})u. \tag{16}$$

A current instability develops and causes s-waves to be coherently emitted for $U > v_{\mathrm{s}}$, where v_{s} is defined in (I4g). The maximum growth rate for the s-waves is at $k_s \approx \omega_{\mathrm{p}}/U$ where

$$|\gamma| \approx \tfrac{1}{2}\pi_{\mathrm{i}} = \tfrac{1}{2}\omega_{\mathrm{p}}(m_{\mathrm{e}}/m_{\mathrm{i}})^{\frac{1}{2}}. \tag{17}$$

The s-waves generated in this current instability for $U \gtrsim v_{\mathrm{s}}$ have $k_s > \lambda_{\mathrm{De}}$ and are directed oppositely to the l-waves generated in the two-stream instability.

According to Tsytovich (1966), under these conditions the damping of the l-waves is described by an effective absorption coefficient given by

$$\begin{aligned}\gamma &\sim \pi\omega_{\mathrm{p}}\frac{W^s}{n_{\mathrm{e}}\,m_{\mathrm{e}}\,V_{\mathrm{e}}^2}\frac{1}{k_s\,\lambda_{\mathrm{De}}}\\ &\approx \pi\omega_{\mathrm{p}}\frac{W^s}{n_{\mathrm{e}}\,m_{\mathrm{e}}\,V_{\mathrm{e}}^2}\left(\frac{n_{\mathrm{s}}}{n_{\mathrm{e}}}\right)\left(\frac{u}{V_{\mathrm{e}}}\right),\end{aligned} \tag{18}$$

where W^s is the energy density in s-waves and where we use $k_s \approx \omega_{\mathrm{p}}/U$ and (16). The effective absorption coefficient for the s-waves is given by

$$\begin{aligned}\gamma &\sim \pi\pi_{\mathrm{i}}\frac{W^l}{n_{\mathrm{e}}\,m_{\mathrm{e}}\,V_{\mathrm{e}}^2}\frac{k_l}{k_s^2\,\lambda_{\mathrm{De}}}\\ &\approx \pi\omega_{\mathrm{p}}\frac{W^l}{n_{\mathrm{e}}\,m_{\mathrm{e}}\,V_{\mathrm{e}}^2}\left(\frac{n_{\mathrm{s}}}{n_{\mathrm{e}}}\right)\left(\frac{u}{V_{\mathrm{e}}}\right)\left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)^{\frac{1}{2}},\end{aligned} \tag{19}$$

where W^l is the energy density in l-waves and where we use $k_s \approx \omega_{\mathrm{p}}/U$, $k_l \approx \omega_{\mathrm{p}}/u$, and (16).

It would appear that if the s-waves grow faster than the l-waves, i.e. from (15) and (17) if

$$\left(\frac{n_{\mathrm{s}}}{n_{\mathrm{e}}}\right)\left(\frac{u}{\Delta u}\right)^2 \leqslant \left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)^{\frac{1}{2}}, \tag{20}$$

then the presence of the s-waves suppresses the two-stream instability for, from (15) and (18),

$$\frac{\pi W^s}{n_{\mathrm{e}}\,m_{\mathrm{e}}\,V_{\mathrm{e}}^2}\left(\frac{u}{V_{\mathrm{e}}}\right) \gtrsim \left(\frac{u}{\Delta u}\right)^2. \tag{21}$$

To these must be added the condition, from (16),

$$U = (n_{\mathrm{s}}/n_{\mathrm{e}})u > v_{\mathrm{s}} = V_{\mathrm{e}}(m_{\mathrm{e}}/m_{\mathrm{i}})^{\frac{1}{2}}, \tag{22}$$

for the s-waves to grow at all. From (20) and (22) we infer that for $\Delta u \sim u$, the s-waves grow and do so faster than the l-waves for

$$\frac{V_{\mathrm{e}}}{u}\left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)^{\frac{1}{2}} \lesssim \frac{n_{\mathrm{s}}}{n_{\mathrm{e}}} \lesssim \left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)^{\frac{1}{4}}. \tag{23}$$

With $(m_{\mathrm{e}}/m_{\mathrm{i}})^{\frac{1}{4}} \approx 1/43$ and $V_{\mathrm{e}}/u \approx 1/20$ for a typical type III burst the condition

(23) is not particularly restrictive. The condition (21) then implies that the two-stream instability is suppressed when the energy density in s-waves exceeds about 1% of the thermal energy density in particles.

This semiquantitative discussion provides a *prima facie* case for the existence of a particularly effective suppression of the two-stream instability, for $n_s \gtrsim 10^{-3} n_e$. The continued growth of s-waves would lead to a prohibitive energy loss (the current instability draws its energy ultimately from the streaming motion of the fast electrons) if allowed to continue indefinitely; however, the scattering of s-waves into s-waves is a very effective process and will limit the instability, especially for U close to v_s. The development of the current instability has been discussed by Kadomstev (1965), Sagdeev and Galeev (1969), and many others.

The discussion of the suppression of the two-stream instability is not pursued further here. The processes outlined above indicate that there are more effective ways of suppressing the two-stream instability than that invoked by Kaplan and Tsytovich (1967); the present discussion serves to point out that such processes exist but not to demonstrate that they are as effective as required for the problem in question. In the remainder of this article we assume that either these or other processes do adequately suppress the two-stream instability.

IV. Spectrum of l-waves

In this section we consider the spectrum of l-waves likely to result from the passage of a bunch of electrons through the corona. The bunch itself generates l-waves incoherently and by any residual coherent emission which we assume to be weak. Scattered electrons also generate l-waves incoherently; the energy deposited in l-waves by scattered electrons is smaller by the ratio N_s/N (see (14)) than that deposited by the bunch itself through incoherent emission. The l-waves once generated are scattered by the plasma particles; scattering with conversion into t-waves leads to emission at the fundamental while, as shown below, scattering into l-waves is the only significant source of l-waves leading to emission at the second harmonic.

(a) *Incoherently Emitted l-waves*

Consider a bunch of N electrons all with speed u and a pitch angle distribution $\Phi(\alpha)$ (see Section II) sharply peaked near $\alpha = 0$. According to equations (I8) with (I1) for $k \ll \lambda_{\mathrm{De}}^{-1}$ the power radiated incoherently per electron into the range $\mathrm{d}^3\boldsymbol{k}/(2\pi)^3$ at $\boldsymbol{k}$ is given by

$$P(\boldsymbol{k}) = (2\pi)^2 \left(\frac{e^2 \omega_{\mathrm{p}}^2}{k^3 u}\right) \delta(\cos\Theta_{\mathrm{C}} - \omega_{\mathrm{p}}/ku)\,, \tag{24}$$

where the Čerenkov angle Θ_{C} is relative to the direction of the velocity of the electron. For a bunch of electrons distributed with axial symmetry the l-waves generated by the bunch are also distributed with axial symmetry. Let ψ be the angle between $\boldsymbol{k}$ and the magnetic field (net streaming, i.e. $\alpha = 0$) direction. Then if we write the energy density in l-waves generated incoherently by the bunch in the form

$$W^l = 2\pi \int_0^\pi \mathrm{d}\psi \sin\psi \int \frac{\mathrm{d}k\, k^2}{(2\pi)^3} T^l(k,\psi)\,, \tag{25}$$

the effective temperature reduces to

$$T^l(k,\psi) = (2\pi)^2 \frac{Ne^2\omega_\mathrm{p}^2}{Ak^3u^2}\frac{\sin|\psi-\Theta_\mathrm{C}|}{\sin\psi}\Phi(|\psi-\Theta_\mathrm{C}|), \qquad k \gtrsim \omega_\mathrm{p}/u, \tag{26}$$

where A is the surface area of the bunch, Au the volume swept up by the bunch per unit time, and Θ_C is determined by (13). Integrating (25) over the range of phase velocities (12) we find

$$W^l = (Ne^2\omega_\mathrm{p}^2/Au^2)\ln(u/aV_\mathrm{e}), \tag{27}$$

which is identified as the energy density in incoherently generated l-waves remaining behind to be collisionally damped after the passage of the bunch.

(b) *Coherently Emitted l-waves*

Any residual coherent emission leads to a spectrum of l-waves with $k \approx \omega_\mathrm{p}/u$, $\Delta k \approx (\omega_\mathrm{p}/u)(\Delta u/u)$, and an energy density which is expected to be a small fraction of the local thermal energy density in particles. If this fraction is η then for the coherently emitted l-waves one has

$$T^l(k,\psi) = (2\pi)^2\eta n_\mathrm{e} m_\mathrm{e} V_\mathrm{e}^2\left(\frac{u}{\omega_\mathrm{p}}\right)\left(\frac{\Delta u}{u}\right)\frac{\delta(\psi)}{\sin\psi}, \tag{28}$$

with $k \approx \omega_\mathrm{p}/u$, $\Delta k \approx (\omega_\mathrm{p}/u)(\Delta u/u)$. We concentrate on the incoherently emitted l-waves because the fraction η in (28) is unknown.

(c) *Scattering of l-waves*

For any significant emission at the second harmonic to arise there must be either scattered electrons or scattered l-waves. Scattered electrons produce l-waves with an energy density given roughly by N_s/N times that given by (27), with N_s given by (14). The scattering of l-waves into l-waves can be either incoherent or coherent. An examination of incoherent scattering alone shows that this acting on the spectrum (26) for a time γ_c^{-1} produces an energy density in scattered l-waves well in excess of that produced by scattered electrons; we ignore the influence of the scattered electrons.

Incoherent scattering can be neglected if coherent scattering plays any significant role. Coherent scattering is the dominant effect if the growth rate of the scattered waves greatly exceeds γ_C. From (I15) this growth rate is given by

$$\frac{1}{N^l}\frac{\partial N^l}{\partial t} = \sum_\alpha n_\alpha \int \mathrm{d}^3\boldsymbol{p} \int \frac{\mathrm{d}^3\boldsymbol{k}_l'}{(2\pi)^3} N^{l'} w_\alpha^{ll'}\hbar(\boldsymbol{k}_l-\boldsymbol{k}_l')\cdot\frac{\partial f_\alpha}{\partial \boldsymbol{p}}, \tag{29}$$

where we sum over all species of particles. The probability $w_\alpha^{ll'}$ in (29) contains a δ function which imposes the condition

$$\omega^l-\omega^{l'} = (\boldsymbol{k}_l-\boldsymbol{k}_l')\cdot\boldsymbol{v}. \tag{30}$$

If we take f_α to be a Maxwellian distribution then the right-hand side of (29) is only positive, i.e. only corresponds to coherent scattering, for

$$\omega^l - \omega^{l'} \approx (3V_e^2/2\omega_p)(k_l^2 - k_l'^2) < 0, \tag{31}$$

where we use equations (I1). The inequality (31) requires that the wavenumber k of the scattered l-wave be less than that of the unscattered l-wave, i.e. the phase velocity of the l-waves increases as a result of coherent scattering.

Tsytovich (1966) compares the effects of the scattering by plasma electrons and by plasma ions. Plasma electrons scattering l-waves with

$$(m_i/m_e)^{\frac{1}{4}} \lesssim v_\phi^l/V_e \lesssim (m_i/m_e)^{\frac{1}{2}},$$

which is the range of interest here, lead to a growth rate of the order of

$$|\gamma| \sim \omega_p \frac{W^l}{n_e m_e V_e^2}\left(\frac{m_e}{m_i}\right)^2\left(\frac{v_\phi^l}{V_e}\right)^3, \tag{32}$$

with the change in v_ϕ^l of the order of v_ϕ^l in each scattering process. Scattering of l-waves in the same range of phase velocities by plasma ions leads to a small change in v_ϕ^l with approximate reversal of the direction of the $\boldsymbol{k}$ vector in each scattering process. From (30) we find, for the change Δk in k due to the scattering,

$$\Delta k/k \lesssim V_i v_\phi^l/3V_e^2 \tag{33}$$

in this case and then the growth rate from (29) and (I16) is of the order of

$$|\gamma| \sim \omega_p \frac{W^l}{n_e m_e V_e^2}\left(\frac{\Delta k}{k}\right)\left(\frac{V_e^2}{V_i v_\phi^l}\right). \tag{34}$$

For $v_\phi^l \approx u$ numerical values allow $\Delta k/k \approx 1$ in (33) and then (34) predicts that the scattering is the more effective for smaller values of v_ϕ^l; however, for $v_\phi^l \ll u$, (34) with (33) is roughly independent of v_ϕ^l. For $v_\phi^l \approx u \approx 0\cdot 3\,c$ the growth rate due to scattering by ions (34) exceeds that due to electrons (32).

Coherent scattering is important only if the exponential growth proceeds for many e-folding times. This requires $|\gamma|/\gamma_c \gg 1$. From (2), (10), (27), (33), and (34) this condition reduces to

$$\frac{|\gamma|}{\gamma_c} \sim \frac{N}{A}\frac{\omega_p V_e}{n_e u^2 \ln \Lambda} \gg 1. \tag{35}$$

Inserting numerical values the inequality (35) requires

$$\frac{N}{A} \gg 3\times 10^{13}\left(\frac{n_e}{10^9(\mathrm{cm}^{-3})}\right)^{\frac{1}{2}}\left(\frac{10^6(^\circ\mathrm{K})}{T}\right)^{\frac{1}{2}}\left(\frac{u}{c}\right)^2 \quad \mathrm{cm}^{-2}. \tag{36}$$

With $u \approx 0\cdot 3c$ and $A \approx 10^{20}\,\mathrm{cm}^2$ (the observed area of emission) this requires $N \gg 3\times 10^{32}$ electrons per bunch. This condition is almost certainly satisfied and so the scattering is coherent.

There is no obvious way in which the spectrum of scattered l-waves can be determined. However, we can state that the effective temperature of the scattered distribution of l-waves will be comparable to that of the unscattered l-waves. A similar conclusion applies if one considers the scattering of the coherently generated l-waves described by (28).

V. Emitted Radiation

In this section we consider the emission of electromagnetic waves due to the presence of the electron plasma waves whose spectra are discussed in the previous section. We concentrate on the incoherently emitted l-waves because of the uncertainty in the estimation of any quantities associated with the coherently emitted l-waves.

(a) *Emission at the Fundamental*

Emission at the fundamental is assumed to be due to the scattering of l-waves into t-waves by plasma particles; this scattering can be either incoherent or coherent. (Although s-waves appear in processes invoked in Section III, the process $l+s \to t$ is not allowed for these s-waves because their wavenumbers satisfy $k_s > \lambda_{\mathrm{De}}^{-1}$.)

Incoherent scattering always occurs. We estimate the power radiated due to this process for the spectrum of l-waves described by (26) with $\Phi(\alpha) = \delta(\alpha)/\sin\alpha$ and with the emitting volume identified as $Au\gamma_{\mathrm{c}}^{-1}$. Consider the emission of t-waves at an angle θ to the streaming direction. Owing to the axial symmetry about this direction, the integral over azimuthal angle in equation (I13) with (I16) reduces to the integral

$$\begin{aligned}(2\pi)^{-1}\int_0^{2\pi} \mathrm{d}\phi\,|\boldsymbol{\kappa}_t\times\boldsymbol{\kappa}_l|^2 &= \tfrac{1}{2}(1+\cos^2\theta)-\tfrac{1}{2}\cos^2\psi\,(3\cos^2\theta-1)\\ &= f_1(\theta,\psi)\,. \end{aligned}\tag{37}$$

The power radiated in the range $\mathrm{d}\theta$ at θ then reduces to

$$\begin{aligned}P_{\omega\mathrm{p}}(\theta) = {}&\frac{Au}{\gamma_{\mathrm{c}}}\frac{\surd 3\,e^2\omega_{\mathrm{p}}\,V_{\mathrm{e}}}{8\pi^2 c}\int_{\omega_{\mathrm{p}}/u}^{\omega_{\mathrm{p}}/aV_{\mathrm{e}}}\mathrm{d}k\,k^3\\ &\times\int_0^{\pi}\mathrm{d}\psi\,\sin\psi\sin\theta\,f_1(\psi,\theta)\,\{T^l(k,\psi)/m_{\mathrm{e}}c^2\}\,,\end{aligned}\tag{38}$$

which is the appropriate generalization of (I31) from spherical symmetry to axial symmetry. Inserting (26) as modified, using (2), (10), and (37), and retaining only the lowest order terms in aV_{e}/u, we find

$$\begin{aligned}P_{\omega\mathrm{p}}(\theta) &= \frac{Au}{\gamma_{\mathrm{c}}}\frac{\surd 3\,e^2\omega_{\mathrm{p}}^2\,V_{\mathrm{e}}}{8\pi^2 c}\frac{4\pi^2 N e^2\omega_{\mathrm{p}}^2}{Au^2 m_{\mathrm{e}}c^2}\int_{\omega_{\mathrm{p}}/u}^{\omega_{\mathrm{p}}/aV_{\mathrm{e}}}\mathrm{d}k\,\sin\theta\,f_1(\psi=\Theta_{\mathrm{C}},\theta)\\ &\approx \frac{3\surd(3\pi/2)}{4}\frac{N}{a\ln\Lambda}\frac{e^2\omega_{\mathrm{p}}^2}{u}\left(\frac{V_{\mathrm{e}}}{c}\right)^3\left(1+\cos^2\theta\right)\\ &\approx 1\cdot 2\times 10^{-19}N\frac{c}{u}\left(\frac{T_{\mathrm{e}}}{10^6(^\circ\mathrm{K})}\right)^{3/2}\left(\frac{\omega_{\mathrm{p}}/2\pi}{10^8(\mathrm{Hz})}\right)^2\\ &\quad\times\tfrac{3}{4}(1+\cos^2\theta)\quad \mathrm{erg\,sec}^{-1}\,.\end{aligned}\tag{39}$$

To achieve a power of 10^{19}–10^{20} erg sec^{-1} (see (9) for a duration of about 10 sec with $u = 0{\cdot}3\,c$, $T_e = 2\times10^6$ °K, and $\omega_p/2\pi = 10^8$ Hz), we require $N = 10^{37}$–10^{38} electrons per bunch. As anticipated in Part I, this process requires a prohibitively excessive number of electrons.

However, if the scattering of l-waves into t-waves is coherent then with the same spectrum of l-waves a much greater power output can result. The maximum conceivable power output corresponds to that where the effective temperature of the t-waves approaches that of the l-waves. Tsytovich (1966) considered the coherent scattering process and found that for $(v_\phi^l/V_e) > (m_i/m_e)^{\frac{1}{3}} \approx 10$, which is the case here, the rate at which l-waves were converted into t-waves was comparable to the rate at which l-waves were scattered into l-waves.

Even though we show that the coherent scattering of l-waves into l-waves is an important effect, the fact that the rate of conversion of l-waves into t-waves by coherent scattering is comparable to that for scattering of l-waves into l-waves does not necessarily mean that the conversion of l-waves into t-waves is as effective as that of l-waves into other l-waves. This is because t-waves propagate at group velocities very much greater than the group velocity of any l-waves; propagation effects can limit any exponential growth of t-waves simply because the t-waves can propagate into a region where the change in the electron number density, and so the plasma frequency, causes the wavenumber k_t to change sufficiently for any continued exponential growth to be impossible. Thus the condition $|\gamma| \gg \gamma_c$ imposed in (35) is to be replaced by $|\gamma| \gg \max\{\gamma_c, (\Delta t)^{-1}\}$, where Δt is the time in which propagation effects limit any exponential growth.

Roberts (1959) argued that the electron density in the region of emission must be inhomogeneous for the fundamental to escape. Let us suppose that these inhomogeneities consist of regions of linear dimension L each of which is homogeneous. According to (I29), with $k_l \sim \omega_p/u$ corresponding to the most efficient wavenumbers for coherent scattering, the group velocity of the t-waves is

$$v_g^t = k_t c^2/\omega_p \approx \sqrt{3}\,(V_e/u)c. \tag{40}$$

The time Δt is then of the order of

$$\Delta t = 3^{-\frac{1}{2}}(u/V_e)L/c. \tag{41}$$

In place of (35) we then require

$$|\gamma|\,\Delta t \sim \frac{NL}{A}\,\frac{e^2\omega_p^3}{n_e m_e V_e^3 uc} \gg 1 \tag{42}$$

for the coherent scattering of l-waves into t-waves to be important. The value of L is unknown. If there exist homogeneous regions with $L \sim 10^{10}$ cm then we have $\Delta t \sim 3$ sec $\sim \gamma_c^{-1}$ and the restriction (42) is comparable to that given by (35). Thus, provided that there exist homogeneous regions with dimensions not too much smaller than 10^{10} cm in linear dimensions, such regions allow coherent scattering of l-waves into t-waves to be approximately as effective as the coherent scattering of l-waves into l-waves.

Assuming this to be the case the surface area of the radiating region is of the order of $L^2 \sim 10^{20}\,\mathrm{cm}^2 \sim A$. Assuming maximum efficiency, the effective temperature of the emitted t-waves is of the order of that of the l-waves with $k \sim \omega_p/u$, that is, from (26),

$$[T_{\omega p}]_{\max} \approx (2\pi)^2 \frac{N}{A}\frac{e^2 u}{\omega_p} \approx \frac{3N}{A}\frac{u}{c}\left(\frac{10^8(\mathrm{Hz})}{\omega_p/2\pi}\right) \;{}^\circ\mathrm{K}. \tag{43}$$

With $u \approx 0\cdot3\,c$ the maximum possible effective temperature (43) exceeds that observed (up to $10^{12}\,{}^\circ$K) for $N/A > 10^{12}$, e.g. for $N > 10^{32}$ electrons per bunch for $A \approx 10^{20}\,\mathrm{cm}^2$.

A more plausible estimate would correspond to $L < 10^{10}$ cm in (42). On taking into account the difficulty which the fundamental has in escaping from the corona (see Kundu 1965 and references therein), a value $N \approx 10^{34}$ electrons per bunch would appear to be reasonable. In making a better estimate one would need to have some knowledge of the inhomogeneities in the electron density because the efficiency of the coherent scattering, the surface area of the emission, and the ease with which the t-waves can escape all depend on the detailed structure of the region of emission. Our rough estimate with 10^{34} electrons per bunch requires that the effective temperature of the escaping t-waves correspond to about 1% of the maximum possible. This is little more than a guess on the efficiency of the various processes involved, e.g. a value $N = 10^{33}$ electrons per bunch would be about as plausible.

The frequency dependence of the emission corresponds to a time-integrated intensity varying according to

$$\frac{L^2 k_t^2 T_{\omega p}}{\gamma_c} \propto \left(\frac{L^2}{A}\right)\omega_p^{-1}. \tag{44}$$

This ignores the fact that the t-waves can probably escape more easily from higher levels in the corona. Comparing (44) with the observed $\omega_p^{-3\cdot5}$ dependence (see equation (8)), it is clear that L and/or the probability of the t-waves escaping from the corona must increase as n_e (and so ω_p) decreases, i.e. must increase at higher levels in the corona. This is plausible.

These rough estimates indicate that the model is compatible with the observations on the emission at the fundamental. We might add that if the bunch passes through a highly inhomogeneous region of the corona with $L \ll 10^{10}$ cm then the efficiency of coherent emission drops off drastically, e.g. the emission can drop to that corresponding to incoherent scattering (39); a fading out and recommencement of the emission in a type III burst is sometimes observed.

(b) *Emission at the Second Harmonic*

Detailed analysis of the emission at the second harmonic is complicated by several effects. Firstly the spectrum of scattered l-waves is poorly determined both in its k dependence and angular dependence. Secondly the important l-waves contributing to the emission at $2\omega_p$ have $k \sim \omega_p/u$ and so do not satisfy the condition $v_\phi^l \ll c$ used as a simplifying assumption in (I32). In order to proceed we examine different aspects of the emission at $2\omega_p$ by making different simplifying assumptions.

The overall power radiated can be estimated by assuming that the scattered l-waves are isotropically distributed with an effective temperature given by

$$[T_l(k)]_{sc} = \xi(2\pi)^2 Ne^2\omega_p^2/Ak^3u^2, \tag{45}$$

i.e. by assuming that the scattering does not significantly change the k values but isotropizes a fraction ξ of the l-waves in the spectrum (26). If we further assume that the approximations made in (I32) apply, then the resulting power radiated is underestimated if l-waves with $k \sim \omega_p/u$ dominate the emission; the angular distribution is very poorly described by making this approximation when l-waves with $k \sim \omega_p/u$ dominate the emission.

From (I17) with (I20) and (I32) for the axially symmetric case the integral over azimuthal angle reduces to

$$(2\pi)^{-1}\int_0^{2\pi} d\phi\,|\boldsymbol{\kappa}_t\cdot\boldsymbol{\kappa}_l|^2|\boldsymbol{\kappa}_t\times\boldsymbol{\kappa}_l|^2$$

$$= \cos^2\psi\cos^2\theta + \tfrac{1}{2}\sin^2\psi\sin^2\theta - (\cos^4\psi\cos^4\theta + 3\cos^2\psi\sin^2\psi\cos^2\theta\sin^2\theta + \tfrac{3}{8}\sin^4\psi\sin^4\theta)$$

$$= f_2(\psi,\theta). \tag{46}$$

The power radiated in the range $d\theta$ at θ from a volume $Au\gamma_c^{-1}$ neglecting reabsorption then reduces to

$$P_{2\omega_p}(\theta) = \frac{Au}{\gamma_c}\frac{3\sqrt{3}\,e^2\omega_p^2}{4\pi^2 c}\int_{\omega_p/u}^{\omega_p/aV_e} dk\,k^2\int_0^{\pi} d\psi\,\sin\psi\sin\theta\,f_2(\psi,\theta)$$

$$\times\{T^l(k,\psi)/m_e c^2\}\{T^l(k,\pi-\psi)/m_e c^2\}, \tag{47}$$

which is the appropriate generalization of (I33).

The spectrum (26) in (47) leads to no emission at $2\omega_p$ for $\Phi(\alpha)$ sharply peaked near $\alpha = 0$. The spectra (26) and (45) lead to

$$P_{2\omega_p}(\theta) = \xi\frac{Au}{\gamma_c}\frac{3\sqrt{3}\,e^2\omega_p^2}{4\pi^2 c}\left(\frac{4\pi^2 Ne^2\omega_p^2}{Aum_e c^2}\right)^2\int_{\omega_p/u}^{\omega_p/aV_e} dk\,k^{-4}\sin\theta\,f_2(\psi = \Theta_C,\theta)$$

$$\approx \frac{9\sqrt{(3\pi/2)}}{5}\frac{Ne^2\omega_p^2}{m_e c^2 u\ln\Lambda}\left(\frac{V_e}{c}\right)^3\left[\xi\frac{4\pi^2 Ne^2 u}{A\omega_p}\right]\sin\theta\cos^2\theta(1-\tfrac{5}{7}\cos^2\theta), \tag{48}$$

where we retain only the lowest order term in aV_e/u. The spectrum (45) with itself leads to isotropic emission determined by (I33) and comparable to (48) for $\xi \approx 1$.

The maximum effective temperature of the t-waves emitted at the fundamental corresponds to the quantity in square brackets in (48) with $\xi = 1$. If we express (48) in terms of the effective temperature of the emission at $2\omega_p$ from a surface area A we find

$$T_{2\omega_p} \approx \frac{6\pi^3\sqrt{\frac{3}{2}}}{5}\frac{Ne^2}{Am_e u\omega_p\ln\Lambda}\left(\frac{V_e}{c}\right)^3\left[\xi\frac{4\pi^2 Ne^2 u}{A\omega_p}\right], \tag{49}$$

where we ignore angular factors. Reabsorption is unimportant if $T_{2\omega_p}$ is less than the quantity in square brackets, i.e. if the numerical coefficient multiplying this quantity is less than unity (otherwise reabsorption limits $T_{2\omega_p}$ to the effective temperature of the scattered l-waves, i.e. limits this coefficient to unity). Inserting numerical values (49) reads

$$T_{2\omega_p} \approx 10^{-17}\frac{N}{A}\frac{c}{u}\frac{10^8(\mathrm{Hz})}{\omega_p/2\pi}\left(\frac{T_e}{10^6(^\circ\mathrm{K})}\right)^{3/2}\left(\xi\frac{4\pi^2Ne^2u}{A\omega_p}\right). \tag{50}$$

With $T_e \approx 2\times10^6\,^\circ$K, $u \approx 0\cdot3\,c$, $\omega_p/2\pi \approx 10^8$ Hz, and $N/A \approx 10^{14}$, $T_{2\omega_p}$ is about 1% of its maximum possible value. As already pointed out, the approximations made may lead to an underestimation of $T_{2\omega_p}$.

Now we expect the scattering of l-waves into l-waves to be very effective and so have $\xi \approx 1$. The emission at $2\omega_p$ then gives an effective temperature $T_{2\omega_p}$ which is a few per cent of the maximum possible effective temperature described by (43). As already argued the actual effective temperature of the emission at ω_p is likely to be substantially less than the maximum value (43). Clearly, comparable effective temperatures at the two harmonics are consistent with the model for $N/A \approx 10^{14}$ electrons cm^{-2} per bunch. The fact that $T_{2\omega_p}$ is only a few per cent of the maximum possible value implies that any emission at $3\omega_p$ due to the process $t+l \to t$ can be no greater than a few per cent of that at $2\omega_p$ for $N/A \approx 10^{14}$.

In view of the dependence of equation (43) on N/A and (50) on $(N/A)^2$ the model predicts that the more intense the burst the higher should be the ratio of the intensity at the second harmonic to that at the fundamental. There does not appear to be any discussion of the relevant correlation in the literature.

The qualitative effect of backward or forward asymmetry in the emission at $2\omega_p$ from axially symmetric distributions of l-waves is excluded by our use of (I32) which, according to (46), imposes backward–forward symmetry. This is a spurious result of the approximations made. If we consider the coalescence of l-waves $\boldsymbol{k}_1$ directed along the streaming direction with any other l-waves $\boldsymbol{k}_2$, then

$$\boldsymbol{k}_t = \boldsymbol{k}_1+\boldsymbol{k}_2\,, \qquad k_t = \sqrt{3}\,(\omega_p/c) < k_1, k_2\,, \tag{51}$$

imply that the emission is in the forward (backward) direction, i.e.

$$\begin{aligned} &\boldsymbol{k}_t\,.\,\boldsymbol{k}_1 > 0\,, \qquad \text{for} \qquad k_2 < k_1 \\ &(\boldsymbol{k}_t\,.\,\boldsymbol{k}_1 < 0\,, \qquad\qquad\quad\; k_2 > k_1)\,. \end{aligned}$$

Because the magnitude of the $\boldsymbol{k}$ vectors decreases during the scattering process one expects $k_2 < k_1$ and so forward emission to arise from the coalescence of l-waves from the scattered and unscattered distributions. However, because the scattering of l-waves with $k_1 \approx \omega_p/u$ leads to l-waves with $k_2 < \omega_p/u$, it is possible for two scattered l-waves, which must be propagating in the backward direction, to coalesce into a t-wave which must also be propagating in the backward direction. (There is no restriction requiring that k remain greater than ω_p/c in the scattering process.) Because the conversion of l-waves into t-waves becomes particularly effective for $k_l \sim \omega_p/c$ this latter coalescence process may prove the more important and so lead to a net backward emission.

Owing to the present identification of the coherent scattering processes as the dominant effects, it has not proved possible to treat the emission in detail. However the discussion of the present section indicates that a bunch of 10^{33}–10^{34} electrons emitting l-waves incoherently suffices in accounting for the overall observed properties of type III bursts. If the energy density in coherently emitted l-waves were greatly to exceed that found for the incoherently emitted waves (27) then an embarrassingly intense emission of electromagnetic waves would be predicted.

VI. Conclusions

The model proposed here indicates that a bunch of 10^{34} electrons can account for the observed emission in type III bursts provided that any two-stream instability can be adequately suppressed. Although arguments have been given in favour of such a suppression in Section III, the arguments there are heuristic. The validity of the present model, and indeed any model based on the emission from a bunch of electrons, rests on the existence of some such suppression process. Note that with $N/A = 10^{14}$ electrons cm^{-2} the condition $n_{\mathrm{s}} \gtrsim 10^{-3} n_{\mathrm{e}}$ required in Section III is satisfied for a length of the bunch less than about 10^{8} cm; this is much less than $u\gamma_{\mathrm{c}}^{-1}$ which is the depth from which the emission of t-waves occurs.

Although the present analysis has shown that coherent scattering processes are a dominant effect, no attempt has been made to treat these in any detail, due simply to the complexity of the analysis. Kaplan and Tsytovich (1967) also found that coherent scattering processes were important, and treated these processes by making gross approximations on the angular dependence of the scattering. Such an approach allows reasonable quantitative estimates provided that the region of emission is optically thick. This proviso is not adequately satisfied in the model presented above.

In Part I it was pointed out that models developed for type III bursts may be applicable to type II bursts; the argument rests on the supposition that the slowly moving shock front associated with type II bursts leads to the ejection of fast streams of electrons. In the present model the important parameter is not the number of electrons per bunch but the number per surface area in the bunch. If this number remains of the order of 10^{13}–10^{14} electrons cm^{-2} then there are reasonable grounds for applying the present model to type II bursts.

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