DISTURBANCE DAILY VARIATION DURING THE IGY

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[*Manuscript received March* 31, 1970]

Abstract

The disturbance daily variation field *SD*, was studied for the three seasons using IGY data from 50 observatories. Equivalent external and internal electric current systems which could account for the *SD* field were obtained by the method of spherical harmonic analysis. The external current systems were found to be only approximately symmetrical about the geomagnetic equator. The internal current systems, although less intense, had a similar pattern to the corresponding external current systems thus suggesting that the internal current systems are induced by the corresponding external current systems.

I. Introduction

The disturbance daily variation *SD* is defined by Chapman (1964) to be $S_{\mathrm{d}}-S_{\mathrm{q}}$ or $S_{\mathrm{a}}-S_{\mathrm{q}}$ and denoted by SD_{d} and SD_{a} respectively, where S_{q}, S_{d}, and S_{a} are the solar daily variations for international quiet, international disturbed, and all days respectively. In the present work only SD_{d} will be used to represent *SD*.

A worldwide external electric current system to represent the disturbance daily variation was proposed by Chapman (1935) for an idealized Earth for which the geomagnetic and geographic axes coincide. This current system is symmetric about the equator and current loops above the auroral zone remain above while current loops below the auroral zone remain below. The above features are also observed in Burdo's external electric current system as given in Yanowsky (1953). It differs mainly from that of Chapman in that account has been taken of the non-coincidence of the geomagnetic and geographic axes.

Chapman's current system was inferred from a study of the morphology of the disturbance daily variation. The method used in computing Burdo's current system was not revealed by Yanowsky and he did not give the reference to Burdo's work. Namikawa (1957) obtained an external electric current system for the disturbance daily variation for lower latitudes (i.e. latitudes between $\pm 60°$) by graphical integration.

A further investigation of the worldwide *SD* field has been undertaken here in order to obtain as good a quantitative knowledge as possible. Much work (see e.g. Matsushita and Campbell 1967) has already been undertaken in the study of *SD* and disturbance phenomena. It is to be hoped that the increased knowledge will help in elucidating the mechanisms which give rise to the *SD* variations. The method of spherical harmonic analysis (SHA) has been used in order to obtain the internal as well as the external current systems.

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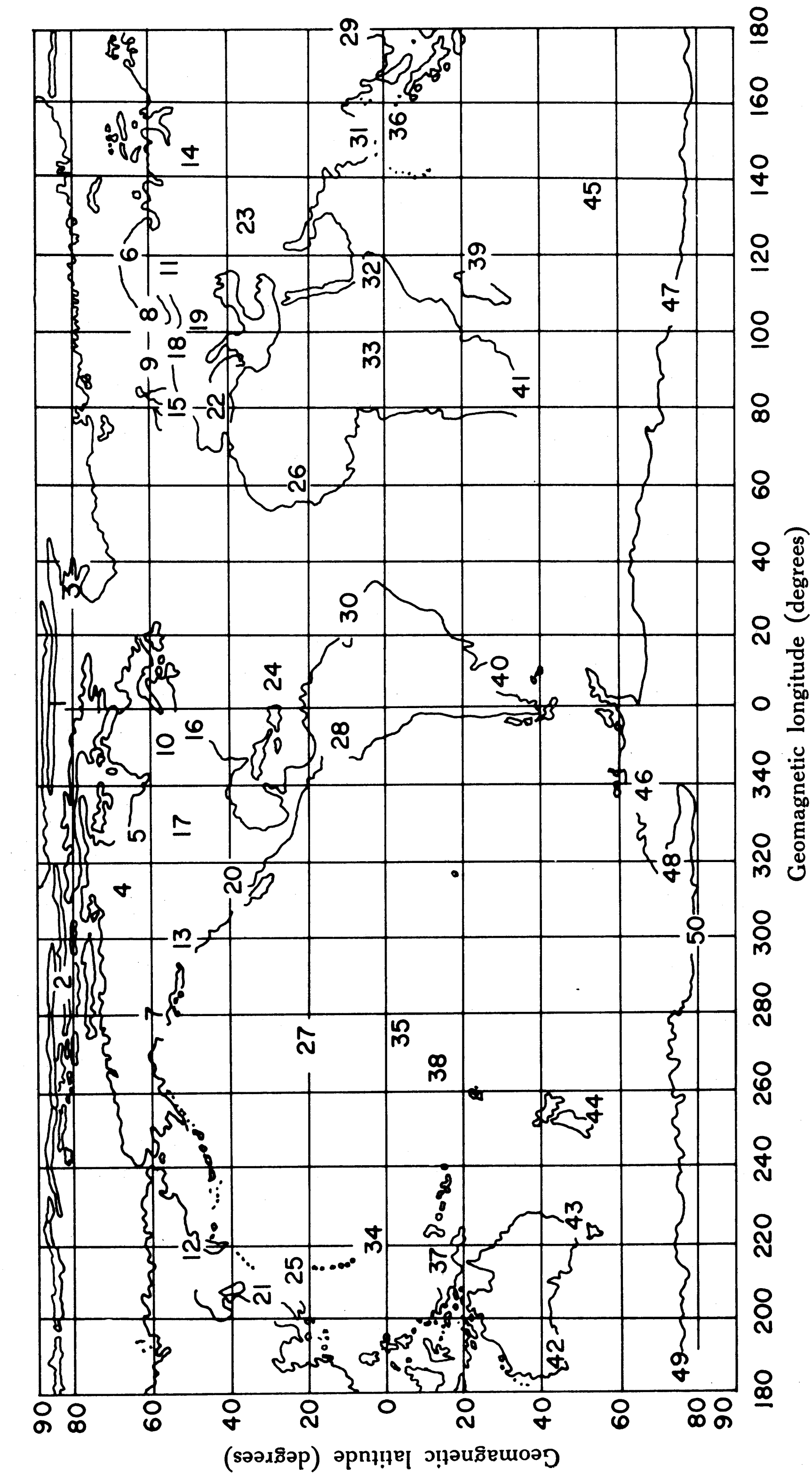

Fig. 1.—World map in geomagnetic coordinates showing the distribution of the magnetic observatories whose data are used in the present work.

TABLE 1

LOCATION OF OBSERVATORIES

No.	Observatory	Geographic Latitude	Geographic Longitude	Geomagnetic Latitude	Geomagnetic Longitude	ψ
1	Thule	N. 77° 29′	W. 69° 10′	N. 88·95°	357·81°	2·0°
2	Resolute Bay	N. 74 42	W. 94 54	N. 82·9	289·3	45·4
3	Godhavn	N. 69 14	W. 53 31	N. 79·8	32·5	−17·6
4	Baker Lake	N. 64 18	W. 96 00	N. 73·7	315·1	18·9
5	Fort Churchill	N. 58 30	W. 94 12	N. 68·7	322·7	13·5
6	Tromso	N. 69 40	E. 18 57	N. 67·2	116·8	−30·8
7	Sitka	N. 57 04	W. 135 19	N. 60·0	275·4	21·4
8	Dombas	N. 62 04	E. 09 07	N. 62·3	100·1	−24·8
9	Lerwick	N. 60 08	W. 01 11	N. 62·5	88·6	−23·6
10	Agincourt	N. 43 47	W. 79 18	N. 55·0	347·0	3·6
11	Nurmijarvi	N. 60 31	E. 24 39	N. 57·9	112·6	−22·0
12	Srednikan	N. 62 26	E. 152 19	N. 53·2	210·5	12·6
13	Victoria	N. 48 31	W. 123 25	N. 54·1	293·0	16·1
14	Sverdlovsk	N. 56 44	E. 61 04	N. 48·5	140·7	−13·3
15	Valentia	N. 51 56	W. 10 15	N. 56·7	73·5	−18·1
16	Fredericksburg	N. 38 12	W. 77 23	N. 49·6	349·9	2·6
17	Beloit	N. 39 29	E. 261 52	N. 49·3	324·8	8·6
18	Witteveen	N. 52 49	E. 06 40	N. 54·2	91·2	−19·3
19	Niemegk	N. 52 04	E. 12 41	N. 52·3	96·5	−18·8
20	Tucson	N. 32 15	W. 110 50	N. 40·4	312·1	10·1
21	Vladivostok	N. 43 41	E. 132 10	N. 33·0	198·0	4·9
22	Toledo	N. 39 53	W. 04 03	N. 43·9	74·7	−14·5
23	Tbilisi	N. 42 05	E. 44 42	N. 36·8	122·0	−13·2
24	San Juan	N. 18 23	W. 66 07	N. 29·9	3·2	−0·7
25	Kakioka	N. 36 14	E. 140 11	N. 26·0	206·0	6·2
26	M'Bour	N. 14 24	W. 16 57	N. 21·2	55·1	−9·7
27	Honolulu	N. 21 18	W. 158 06	N. 21·0	266·4	12·3
28	Paramaribo	N. 5 50	W. 55 10	N. 17·0	14·5	−2·9
29	Cha-pa	N. 22 21	E. 103 50	N. 11·0	173·4	−1·4
30	Tatuoca	S. 1 12	W. 48 31	N. 9·5	20·8	−4·1
31	Alibag	N. 18 38	E. 72 52	N. 9·5	143·7	−7·2
32	Addis Ababa	N. 9 2	E. 38 46	N. 5·3	109·2	−11·0
33	Bangui	N. 4 26	E. 18 34	N. 5·0	88·6	−11·5
34	Guam	N. 13 27	E. 144 45	N. 3·9	212·8	6·4
35	Jarvis Is.	S. 00 23	W. 160 2	S. 0·5	269·0	11·5
36	Trivandrum	N. 8 29	E. 76 57	S. 1·1	146·3	−6·4
37	Hollandia	S. 2 34	E. 140 31	S. 12·5	210·3	5·8
38	Apia	S. 13 48	W. 171 46	S. 16·0	260·2	11·7
39	Tananarive	S. 18 55	E. 47 33	S. 23·7	112·6	−11·2
40	Trelew	S. 43 15	W. 65 19	S. 31·7	3·2	−0·9
41	Hermanus	S. 34 26	E. 19 14	S. 33·3	80·3	−13·8
42	Watheroo	S. 30 19	E. 115 53	S. 41·7	185·8	1·3
43	Toolangi	S. 37 32	E. 145 28	S. 46·7	220·9	9·5
44	Amberley	S. 43 09	E. 172 43	S. 47·6	252·6	15·1
45	Port aux Francais	S. 49 21	E. 70 12	S. 57·2	128·0	−14·0
46	Byrd Station	S. 79 59	W. 120 00	S. 70·6	336·0	27·8
47	Mawson	S. 67 35	E. 62 55	S. 73·1	103·0	−30·6
48	Little America	S. 78 16	W. 162 28	S. 74·03	311·98	46·4
49	Wilkes	S. 66 25	E. 110 27	S. 77·13	179·0	−0·5
50	Scott Base	S. 77 51	E. 166 47	S. 78·98	294·37	59·6

II. Arrangement of Data

For the investigation of *SD*, data from 50 magnetic observatories distributed as uniformly as possible over the Earth were taken, except for the region near the auroral zones where *SD* has the greatest variation. Here more observatories were included than in the lower latitudes. The observatories are listed in Table 1 and their locations are shown in Figure 1. The values for the geographic and geomagnetic coordinates of the magnetic observatories in Table 1 were obtained from IGY annals where the position of the geomagnetic north pole was taken to be at geographic colatitude $11\cdot5°$ and geographic longitude $291\cdot0°$ E. The year 1958 was selected for analysis as it was the year nearest sunspot maximum. A subdivision of the data into Lloyds seasons (i.e. southern summer (ss), January, February, November, and December; equinox (e), March, April, September, and October; and northern summer (ns), May, June, July, and August) was carried out for the purpose of determining the seasonal variation of *SD*.

Usually magnetic observatories tabulate hourly mean values of declination D, horizontal component H, and vertical component Z or alternatively the north component X, east component Y, and Z. Therefore it was necessary to convert the hourly mean values or the daily inequalities obtained from the published observatory values to daily inequalities for the geomagnetic north component X_{m} and the geomagnetic east component Y_{m} in order to perform the SHA of Section V. This conversion can be done using the formulae

$$\left.\begin{aligned} \Delta X_{\mathrm{m}} &= \Delta H\cos(D-\psi)-\Delta D\sin(D-\psi),\\ \Delta Y_{\mathrm{m}} &= \Delta H\sin(D-\psi)+\Delta D\cos(D-\psi),\\ \Delta X_{\mathrm{m}} &= \Delta X\cos\psi+\Delta Y\sin\psi,\\ \Delta Y_{\mathrm{m}} &= -\Delta X\sin\psi+\Delta Y\cos\psi, \end{aligned}\right\} \tag{1}$$

where $\Delta D = H\,\Delta D'/34\,380$

and D is the mean declination of the daily sequence (in degrees) taken positive eastward, H the mean horizontal component of the daily sequence (in gammas), $\Delta D'$ the mean hourly inequality for declination taken positive eastward (in tenths of minutes), ΔD the mean hourly inequality for declination taken positive eastward (in gammas), ΔH the mean hourly inequality for the horizontal component (in gammas), ΔX_{m} the mean hourly inequality for the geomagnetic north component taken positive northward (in gammas), ΔY_{m} the mean hourly inequality for the geomagnetic east component taken positive eastward (in gammas), ΔX the mean hourly inequality for the north component (in gammas), and ΔY the mean hourly inequality for the east component (in gammas). The angle ψ (in degrees) was determined for each observatory (see Table 1) using the formula

$$\psi = \sin^{-1}(-\sin\theta_0\sin\Lambda/\sin\theta), \tag{2}$$

where θ is the geographic colatitude (in degrees) of the observatory, θ_0 the geographic colatitude (in degrees) of the north geomagnetic pole ($= 11\cdot5°$), and Λ the geomagnetic longitude (in degrees) of the observatory (as given in Table 1) positive eastward.

From the mean hourly inequalities ΔX_m, ΔY_m, and ΔZ the daily variations $SD(X_m)$, $SD(Y_m)$, and $SD(Z)$ can be calculated by taking the quiet day inequalities from the disturbed day inequalities for each element, season, year, and observatory.

It should be pointed out that equation (2) is strictly valid if the Earth is a sphere and the Earth's magnetic field is that of a centred dipole. In the present work the above assumptions are made which also justify the use of the theory of Sections V and VI.

III. Application of the Method of Least Squares to Deriving SD as a Function of Universal Time

Once $SD(X_m)$, $SD(Y_m)$, and $SD(Z)$ had been obtained from the 50 geomagnetic observatories as shown in Section II for the three seasons and year relative to G.M.T., the method of least squares as applied by Winch (1965) to the noncyclic variation and S_q was used to obtain the noncyclic variation and Fourier coefficients of SD. The mathematical model fitted to the 24 observations y_i was

$$\eta = \alpha_0 + \beta_0 t' + \sum_{i=1}^{4} (\alpha_i \sin mit' + \beta_i \cos mit') \ldots, \tag{3}$$

where $m = 2\pi/24$, t' is G.M.T. in hours, and $24\beta_0$ is the noncyclic variation. It was assumed that the effect of the secular variation, lunar variation, and any residual S_q variation is negligible and can be ignored. The method of least squares was then used to obtain unbiased estimates a_i and b_i ($i = 0, \ldots, 4$) of the parameters α_i and β_i having minimum sampling variance and assuming that the errors have a zero mean, a common variance, and are independent, and that the time t' is measured with negligible error. This was done for each season, year, force component, and observatory. To shorten the computation time the computing algorithm given by Goertzel (1958) was used. The 95% confidence intervals for each of the a_i and b_i were also computed. When doing the computation it was found necessary to work to more than 8-figure accuracy in order to obtain 3-figure accuracy in the values of the 95% confidence limits and 16-figure accuracy was used in the computation. For the computation of the a_i and b_i, 8-figure accuracy was found to be sufficient.

IV. SD Relative to Geomagnetic Time

In the next stage of the work the a_i and b_i (for $i = 1, \ldots, 4$) computed in Section III relative to G.M.T. had to be converted relative to geomagnetic local time in order to perform the SHA given in Section V. To simplify the calculation the non-uniformity of geomagnetic time was ignored. Simonow (1963) pointed out that this may result in at most an error of ± 20 min. To compute the geomagnetic local time t_M (obtained in degrees but converted to hours (1 hr = 15°)) the equation used was

$$t_M = \Lambda + \sin^{-1}\left(\frac{-\cos\delta \sin(\phi_0+\lambda-t)}{[1-\{\cos\theta_0 \sin\delta - \sin\theta_0 \cos\delta \cos(\phi_0+\lambda-t)\}^2]^{\frac{1}{2}}}\right), \tag{4}$$

where ϕ_0 is the geographic longitude (in degrees) positive westward of the north geomagnetic pole (= 69·0°), δ the declination (in degrees) of the Sun, λ the geographic longitude (in degrees) of the observatory, positive eastward from the Greenwich meridian, t the local time (in degrees) of the observatory, and θ_0 and Λ

are as used in equation (2). This equation was given by Hultqvist and Gustafsson (1960).

For G.M.T. (t') $= 0$ (or $t = \lambda$) and for $\delta = 0$ equation (4) becomes

$$(t_{\mathrm{M}})_{\mathrm{G.M.T}=0} = \Lambda - p\,,$$

where p is a positive constant. The value of p varies from the value at equinox by approximately 0·3 hr due to seasonal change of δ. This change in p was considered small enough to be ignored.

Now, making the assumption that each solar hour corresponds to each geomagnetic hour once the initial correspondence between the two times is established, it follows that at a time t' later

$$(t_{\mathrm{M}})_{\mathrm{G.M.T.}=t'} = (t_{\mathrm{M}})_{\mathrm{G.M.T.}=0} + t' = \Lambda - p + t'\,.$$

If we let the geomagnetic time be t_{M} (for G.M.T. $= t'$) then our equation becomes

$$t_{\mathrm{M}} = \Lambda - p + t'\,.$$

Further, let $t_{\mathrm{N}} = t_{\mathrm{M}} + p$ then $t' = t_{\mathrm{N}} - \Lambda$. So a term $a_i \sin it' + b_i \cos it'$ for integer i may be written as

$$a_i \sin i(t_{\mathrm{N}} - \Lambda) + b_i \cos i(t_{\mathrm{N}} - \Lambda) \qquad \text{or} \qquad a_i' \sin it_{\mathrm{N}} + b_i' \cos it_{\mathrm{N}}\,,$$

where

$$a_i' = a_i \cos i\Lambda + b_i \sin i\Lambda\,, \quad b_i' = b_i \cos i\Lambda - a_i \sin i\Lambda\,,$$

and Λ is in degrees. The above formulae were used to compute the a_i' and b_i' ($i = 1, \ldots, 4$) relative to t_{N} from the a_i and b_i ($i = 1, \ldots, 4$) relative to G.M.T. The 95% confidence intervals for the a_i' and b_i' were found using the general statistical theory given in Parratt (1961).

V. Spherical Harmonic Analysis of *SD*

If it is assumed that the Earth is a sphere of radius R with no magnetic matter near its surface and no electric currents passing from the atmosphere to the ground and also that SD depends on local time only but is otherwise independent of longitude λ, then using the theory set out in Chapman and Bartels (1940) the expression for the magnetic potential function V representing the SD field is given by

$$\begin{aligned} V = C + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \Bigg\{ & \left(e_{na}^{m} \frac{r^n}{R^{n-1}} + i_{na}^{m} \frac{R^{n+2}}{r^{n+1}} \right) \cos m\lambda \\ & + \left(e_{nb}^{m} \frac{r^n}{R^{n-1}} + i_{nb}^{m} \frac{R^{n+2}}{r^{n+1}} \right) \sin m\lambda \Bigg\} \mathrm{P}_n^m(\cos\theta) \qquad (5) \\ \equiv C + \sum_{n=1}^{\infty} \sum_{m=0}^{n} V_n^m\,, & \end{aligned}$$

where the longitude λ is a measure of the local time. In equation (5) C is a constant, θ is the colatitude, r is the distance from the Earth's centre, and e and i are the external and internal coefficients respectively of the magnetic potential function. Hence the term V_n^m leads to the following terms in the north X, east Y, and vertically downward Z components of the magnetic field at the surface of the Earth ($r = R$). For X (north),

$$R^{-1}\,\partial V_n^m/\partial\theta = (a_n^m \cos m\lambda + b_n^m \sin m\lambda) X_n^m,$$

for Y (east),

$$-(R\sin\theta)^{-1}\,\partial V_n^m/\partial\lambda = (a_n^m \sin m\lambda - b_n^m \cos m\lambda) Y_n^m,$$

and for Z (downwards)

$$\partial V_n^m/\partial r = (\alpha_n^m \cos m\lambda + \beta_n^m \sin m\lambda) P_n^m,$$

where

$$a_n^m = n(e_{na}^m + i_{na}^m), \qquad b_n^m = n(e_{nb}^m + i_{nb}^m),$$

$$\alpha_n^m = n e_{na}^m - (n+1) i_{na}^m, \qquad \beta_n^m = n e_{nb}^m - (n+1) i_{nb}^m,$$

$$X_n^m = n^{-1}\,\partial P_n^m/\partial\theta, \qquad Y_n^m = m P_n^m / n\sin\theta, \qquad P_n^m \equiv \mathrm{P}_n^m(\cos\theta).$$

Consequently the Fourier series for SD in X, at a station with north polar distance θ, is

$$\sum_{m=1}^{\infty} (x_{ma}\cos m\lambda + x_{mb}\sin m\lambda) = \sum_{m=1}^{\infty} \left\{ \left(\sum_{n=m}^{\infty} a_n^m X_n^m \right) \cos m\lambda + \left(\sum_{n=m}^{\infty} b_n^m X_n^m \right) \sin m\lambda \right\}, \quad (6)$$

which gives

$$x_{ma} = \sum_{n=m}^{\infty} a_n^m X_n^m, \qquad x_{mb} = \sum_{n=m}^{\infty} b_n^m X_n^m. \qquad (7a,\ b)$$

Similarly for Y,

$$y_{ma} = \sum_{n=m}^{\infty} -b_n^m Y_n^m, \qquad y_{mb} = \sum_{n=m}^{\infty} a_n^m Y_n^m, \qquad (8a,\ b)$$

and for Z,

$$z_{ma} = \sum_{n=m}^{\infty} \alpha_n^m P_n^m, \qquad z_{mb} = \sum_{n=m}^{\infty} \beta_n^m P_n^m. \qquad (9a,\ b)$$

The above theory can also be used when $r = R$, $\theta = 0$ is made to be the geomagnetic north pole of a centred dipole field. The coordinates r, θ, λ are now called geomagnetic coordinates. The coordinate which gives the most trouble in this situation is λ as it should represent geomagnetic local time also. As a result of the noncoincidence of the geomagnetic with geographic pole the geomagnetic time will not be uniform. The error which can result in disregarding the non-uniformity of geomagnetic time has already been discussed in Section IV. For simplicity of calculation it seems reasonable to ignore this error although a more accurate treatment is possible if the extra accuracy is thought to be needed. It should also be pointed out that, in using the geomagnetic coordinate system, errors are likely to be created in making the assumption that the Earth is a sphere and that the disturbance phenomena are linked with the centred dipole and not with the observed magnetic field. The errors due to these assumptions, however, are likely to be small. For the analysis of SD the values of θ, the geomagnetic colatitude of the observatory, were taken as given from the geomagnetic latitudes of Table 1. In equations (7)–(9) the Fourier

TABLE 2

EXTERNAL AND INTERNAL COEFFICIENTS OF MAGNETIC POTENTIAL FUNCTION

The coefficients are given (in gammas) for the three seasons and year together with their 95% confidence limits

Season	e^1_{1a}	e^1_{2a}	e^1_{3a}	e^1_{4a}	e^1_{5a}	e^1_{6a}	e^1_{7a}	e^1_{8a}	e^1_{9a}	e^1_{10a}
ss	**1·3**±0·9	**4·5**±1·0	0·8±0·9	−**1·1**±1·0	**1·3**±0·9	−**2·6**±0·9	**1·0**±0·8	−**2·4**±0·7	0·5±0·5	−**0·8**±0·4
e	**1·2**±1·1	**8·6**±1·2	0·9±1·2	0·1±1·1	0·9±1·1	−**3·6**±1·0	0·1±0·8	−**2·2**±0·7	−**0·6**±0·5	−**0·9**±0·5
ns	0·9±0·9	**5·2**±1·0	−**2·1**±1·1	−0·8±1·1	−1·0±1·0	−**3·6**±1·0	−**1·0**±0·8	−**2·6**±0·8	−0·6±0·6	−**1·4**±0·6
y	**1·2**±0·8	**6·3**±0·9	−0·1±1·0	−0·8±0·9	0·9±0·9	−**3·3**±0·8	0·4±0·7	−**2·4**±0·6	−0·2±0·5	−**1·0**±0·4
	e^2_{2a}	e^2_{3a}	e^2_{4a}	e^2_{5a}	e^2_{6a}	e^2_{7a}	e^2_{8a}	e^2_{9a}	e^2_{10a}	e^2_{11a}
ss	−**0·8**±0·3	**0·7**±0·4	−0·3±0·4	0·4±0·5	−0·1±0·5	0·0±0·5	0·1±0·4	−0·2±0·4	0·3±0·3	0·0±0·3
e	0·3±0·4	**0·5**±0·4	−0·2±0·4	**0·7**±0·4	0·0±0·5	0·2±0·4	0·1±0·4	−0·1±0·3	0·0±0·3	0·0±0·3
ns	−**1·1**±0·3	0·0±0·4	0·0±0·4	0·2±0·5	0·0±0·5	−0·3±0·5	0·2±0·4	−0·4±0·4	0·3±0·3	−0·1±0·3
y	−**0·5**±0·3	**0·4**±0·3	0·0±0·4	0·4±0·4	0·0±0·4	0·0±0·4	0·2±0·4	−0·2±0·3	0·3±0·3	0·0±0·2
	e^3_{3a}	e^3_{4a}	e^3_{5a}	e^3_{6a}	e^3_{7a}	e^3_{8a}	e^3_{9a}	e^3_{10a}	e^3_{11a}	e^3_{12a}
ss	0·0±0·1	**0·4**±0·2	−**0·3**±0·2	**0·3**±0·2	−0·2±0·2	0·2±0·2	−0·1±0·2	0·2±0·2	−0·2±0·2	0·0±0·1
e	−0·2±0·2	**0·6**±0·2	−0·2±0·2	0·2±0·2	−0·1±0·2	0·2±0·3	−0·1±0·2	0·2±0·2	−0·1±0·2	0·1±0·2
ns	**0·3**±0·2	**0·3**±0·2	−0·1±0·2	−0·1±0·2	−0·1±0·2	0·0±0·2	−0·1±0·2	0·0±0·2	−0·1±0·2	−0·1±0·2
y	0·0±0·1	**0·5**±0·1	−0·2±0·2	0·2±0·2	−0·1±0·2	0·1±0·2	−0·1±0·2	0·1±0·2	−0·1±0·1	0·0±0·1
	e^1_{1b}	e^1_{2b}	e^1_{3b}	e^1_{4b}	e^1_{5b}	e^1_{6b}	e^1_{7b}	e^1_{8b}	e^1_{9b}	e^1_{10b}
ss	−**1·4**±0·9	0·8±1·0	−0·7±0·9	**2·9**±1·0	−**2·4**±0·9	**3·8**±0·9	−**1·8**±0·8	**2·9**±0·7	−0·1±0·6	**1·2**±0·4
e	−0·2±1·3	−**1·9**±1·3	−**2·5**±1·3	**3·4**±1·3	−**3·0**±1·3	**5·5**±1·1	−**1·8**±1·0	4·2±0·9	0·2±0·8	**2·2**±0·7
ns	1·1±1·1	0·1±1·2	−**1·3**±1·2	**2·7**±1·2	−0·9±1·1	**4·6**±1·0	−0·6±0·9	**4·1**±0·9	0·2±0·7	**2·6**±0·6
y	−0·3±1·0	−0·4±1·1	−**1·7**±1·1	**2·8**±1·1	−**2·2**±1·1	**4·6**±1·0	−**1·5**±0·9	**3·6**±0·8	0·0±0·7	**2·0**±0·6
	e^2_{2b}	e^2_{3b}	e^2_{4b}	e^2_{5b}	e^2_{6b}	e^2_{7b}	e^2_{8b}	e^2_{9b}	e^2_{10b}	e^2_{11b}
ss	0·2±0·3	**1·0**±0·4	−0·3±0·4	**0·8**±0·5	−0·3±0·5	**1·0**±0·5	−**0·5**±0·4	**0·6**±0·4	−0·2±0·3	0·1±0·3
e	−0·2±0·3	**1·3**±0·4	−0·4±0·4	**0·7**±0·4	−0·2±0·4	**1·1**±0·4	0·0±0·4	**0·4**±0·3	0·1±0·3	0·1±0·2
ns	−**0·6**±0·3	**0·8**±0·3	−0·4±0·4	0·4±0·4	0·1±0·4	**0·7**±0·4	0·4±0·4	0·2±0·3	**0·4**±0·3	0·1±0·3
y	−0·2±0·2	**0·9**±0·3	−**0·4**±0·3	**0·6**±0·4	−0·2±0·4	**0·8**±0·4	0·0±0·3	0·3±0·3	0·1±0·2	0·1±0·2
	e^3_{3b}	e^3_{4b}	e^3_{5b}	e^3_{6b}	e^3_{7b}	e^3_{8b}	e^3_{9b}	e^3_{10b}	e^3_{11b}	e^3_{12b}
ss	**0·2**±0·1	−**0·3**±0·1	0·1±0·2	−0·1±0·2	0·0±0·2	−0·2±0·2	0·0±0·2	−0·2±0·2	0·0±0·2	−0·1±0·1
e	−**0·2**±0·1	−0·1±0·1	0·1±0·2	0·0±0·2	0·0±0·2	0·0±0·2	0·0±0·2	0·1±0·2	−0·1±0·1	0·1±0·1
ns	−0·1±0·1	−0·1±0·2	−**0·4**±0·2	−0·2±0·2	−0·1±0·2	0·0±0·2	−0·2±0·2	0·0±0·2	0·0±0·2	0·0±0·2
y	0·0±0·1	−0·1±0·1	−0·1±0·1	−0·1±0·1	0·0±0·1	0·0±0·1	0·0±0·2	0·0±0·1	0·0±0·1	0·1±0·1

	i^{1}_{1a}	i^{1}_{2a}	i^{1}_{3a}	i^{1}_{4a}	i^{1}_{5a}	i^{1}_{6a}	i^{1}_{7a}	i^{1}_{8a}	i^{1}_{9a}	i^{1}_{10a}
ss	**0·7**±0·6	0·5±0·8	**1·0**±0·8	−**1·5**±0·9	**0·9**±0·8	−**0·9**±0·8	0·5±0·7	−**0·7**±0·6	0·4±0·5	−0·3±0·4
e	−0·3±0·8	**1·8**±1·0	−0·8±1·1	−**1·2**±1·0	−0·4±1·0	−**1·3**±0·9	−0·5±0·8	−0·6±0·7	−0·1±0·5	−0·2±0·5
ns	0·4±0·6	−0·2±0·9	−0·5±1·0	−**1·8**±1·0	0·0±1·0	−**1·4**±0·9	−0·4±0·8	−0·4±0·7	−0·3±0·5	−0·4±0·5
y	0·3±0·5	**0·8**±0·7	−0·1±0·9	−**1·7**±0·8	0·2±0·8	−**1·5**±0·7	−0·1±0·6	−**0·7**±0·6	0·0±0·4	−0·4±0·4
	i^{2}_{2a}	i^{2}_{3a}	i^{2}_{4a}	i^{2}_{5a}	i^{2}_{6a}	i^{2}_{7a}	i^{2}_{8a}	i^{2}_{9a}	i^{2}_{10a}	i^{2}_{11a}
ss	−0·2±0·3	0·1±0·4	0·0±0·4	−0·4±0·5	0·2±0·5	−0·3±0·5	0·0±0·4	−0·2±0·4	0·3±0·3	−0·1±0·3
e	0·0±0·3	0·0±0·3	0·0±0·4	−0·2±0·4	0·0±0·4	−**0·5**±0·4	0·3±0·4	−0·1±0·3	0·0±0·3	0·1±0·2
ns	−**0·5**±0·3	0·0±0·3	−0·2±0·4	−0·1±0·4	−0·1±0·4	−0·3±0·4	−0·1±0·4	−0·4±0·4	0·2±0·3	−0·1±0·3
y	−0·2±0·2	0·0±0·3	0·0±0·4	−0·3±0·4	−0·1±0·4	−0·4±0·4	0·1±0·3	−0·3±0·3	0·2±0·2	0·0±0·2
	i^{3}_{3a}	i^{3}_{4a}	i^{3}_{5a}	i^{3}_{6a}	i^{3}_{7a}	i^{3}_{8a}	i^{3}_{9a}	i^{3}_{10a}	i^{3}_{11a}	i^{3}_{12a}
ss	0·1±0·1	0·1±0·1	**0·2**±0·1	−0·1±0·2	**0·3**±0·2	−0·2±0·2	**0·4**±0·2	−0·2±0·2	0·1±0·1	−0·1±0·1
e	−**0·2**±0·1	0·2±0·2	0·0±0·2	0·0±0·2	0·0±0·2	−0·1±0·3	0·0±0·2	−0·1±0·2	−0·1±0·2	0·0±0·1
ns	0·0±0·1	0·1±0·2	0·0±0·2	0·1±0·2	0·0±0·2	0·0±0·2	0·0±0·2	0·0±0·2	0·0±0·2	0·1±0·2
y	0·0±0·1	**0·2**±0·1	0·0±0·1	−0·1±0·2	0·1±0·2	−0·2±0·2	0·2±0·2	−0·1±0·2	0·1±0·1	0·0±0·1
	i^{1}_{1b}	i^{1}_{2b}	i^{1}_{3b}	i^{1}_{4b}	i^{1}_{5b}	i^{1}_{6b}	i^{1}_{7b}	i^{1}_{8b}	i^{1}_{9b}	i^{1}_{10b}
ss	−0·1±0·6	0·3±0·8	0·3±0·8	**1·4**±0·8	−0·5±0·8	**1·6**±0·8	−0·7±0·7	**1·5**±0·7	0·2±0·5	**0·5**±0·4
e	0·8±0·8	0·2±1·0	0·6±1·1	**3·6**±1·1	0·1±1·2	**3·9**±1·0	−0·5±0·9	**2·8**±0·9	−0·1±0·7	**1·5**±0·6
ns	**1·4**±0·7	0·5±1·0	1·0±1·1	**2·7**±1·0	1·0±1·0	**2·2**±0·9	0·3±0·8	**1·6**±0·8	0·6±0·7	**0·9**±0·6
y	**0·8**±0·6	0·3±0·8	0·5±1·0	**2·7**±1·0	−0·1±1·0	**2·9**±0·9	−0·4±0·8	**2·0**±0·8	0·2±0·6	**0·8**±0·5
	i^{2}_{2b}	i^{2}_{3b}	i^{2}_{4b}	i^{2}_{5b}	i^{2}_{6b}	i^{2}_{7b}	i^{2}_{8b}	i^{2}_{9b}	i^{2}_{10b}	i^{2}_{11b}
ss	0·1±0·2	**0·4**±0·3	−**0·5**±0·4	0·2±0·4	−0·3±0·4	0·4±0·4	−0·3±0·4	0·1±0·4	0·0±0·3	−0·1±0·3
e	−**0·3**±0·2	**0·8**±0·3	0·1±0·3	**0·5**±0·4	0·0±0·4	**0·7**±0·4	0·2±0·3	0·3±0·3	0·2±0·2	0·0±0·2
ns	−**0·3**±0·2	**0·6**±0·3	−0·2±0·3	0·4±0·4	0·0±0·4	**0·8**±0·4	0·0±0·3	0·2±0·3	0·2±0·3	0·1±0·3
y	−0·1±0·2	**0·6**±0·2	−0·1±0·3	**0·4**±0·3	−0·1±0·4	**0·6**±0·3	0·0±0·3	0·2±0·3	0·2±0·2	0·0±0·2
	i^{3}_{3b}	i^{3}_{4b}	i^{3}_{5b}	i^{3}_{6b}	i^{3}_{7b}	i^{3}_{8b}	i^{3}_{9b}	i^{3}_{10b}	i^{3}_{11b}	i^{3}_{12b}
ss	0·0±0·1	0·0±0·1	−0·1±0·1	0·2±0·2	−0·2±0·2	0·1±0·2	−0·1±0·2	0·1±0·2	−0·1±0·1	0·0±0·1
e	0·1±0·1	−**0·3**±0·1	0·0±0·1	−**0·3**±0·2	0·2±0·2	−0·2±0·2	0·1±0·2	−0·2±0·2	0·1±0·1	−0·1±0·1
ns	−**0·2**±0·1	0·0±0·1	0·0±0·2	−0·2±0·2	0·2±0·2	0·0±0·2	**0·3**±0·2	0·0±0·2	**0·2**±0·1	0·1±0·1
y	0·0±0·1	−0·1±0·1	0·0±0·1	−0·1±0·1	0·1±0·1	0·0±0·2	0·0±0·1	0·0±0·1	0·0±0·1	0·0±0·1

coefficients x_{ma}, x_{mb}, y_{ma}, y_{mb}, z_{ma}, and z_{mb} and their respective standard deviations were derived from the a'_i and b'_i ($i = 1, \ldots, 4$) of Section IV.

The values for m of 1, 2, 3 and for n of m to $m+9$ were chosen. The reason for such a large range for n is so that the sharp changes in the *SD* field in the vicinity of the auroral zones would be approximated as closely as possible. Equations (7a), (8b), and (9a) for $m = 1$ and $n = m$ to $m+9$ give rise from the 50 observatories to 150 equations of condition in the 20 unknowns (i.e. 10 e^1_{na} and 10 i^1_{na}) whose values were estimated using the weighted method of least squares and their 95% confidence limits determined. The weights were taken to be the inverse of the variance of the respective Fourier coefficient. Equations (7b), (8a), and (9b) were similarly used to obtain e^1_{nb} and i^1_{nb} and their respective 95% confidence limits. A similar procedure was carried out for $m = 2$ and $m = 3$. Thus the e and i coefficients were obtained for the three seasons and year and are given in Table 2 together with their 95% confidence limits.

Once the e and i values are obtained the internal and external current systems can be computed. Equation (5) may be written in the form

$$V = C + \sum_{m=1}^{3} \sum_{n=m}^{m+9} \{(e^m_{na} \cos m\lambda + e^m_{nb} \sin m\lambda) r^n / R^{n-1}$$

$$+ (i^m_{na} \cos m\lambda + i^m_{nb} \sin m\lambda) R^{n+2} / r^{n+1}\} \mathrm{P}^m_n(\cos\theta)$$

$$= C + \sum_{m=1}^{3} \sum_{n=m}^{m+9} (V^{me}_n + V^{mi}_n),$$

where the ranges of m and n are those used for *SD*, and V^{me}_n and V^{mi}_n now represent the external and internal parts of the magnetic potential function. Now following the theory given in Chapman and Bartels (1940) for current flowing in a thin spherical shell, the current functions (in amperes) are given by

$$J^{me}_n = -\frac{10}{4\pi}\frac{2n+1}{n+1}\left(\frac{r}{R}\right)^n V^{me}_n \qquad \text{and} \qquad J^{mi}_n = \frac{10}{4\pi}\frac{2n+1}{n}\left(\frac{R}{r}\right)^{n+1} V^{mi}_n,$$

where R, the Earth's radius, is taken as $6 \cdot 37 \times 10^8$ cm and r is the radius of the sphere at which the currents are assumed to be flowing. However, $r = R$ is assumed because $|r - R| \ll R$. The current systems for the three seasons and year are given in Figures 2(a)–2(h). In computing the current functions from V for these figures, θ represents the geomagnetic colatitude and λ the time t_{N}.

VI. Results and Discussions

Table 2 lists the external and internal coefficients of the magnetic potential function of *SD* for the three seasons and year. Those terms significant at the 95% confidence level have been shown in bold. From the table the following may be observed.

Figs 2(a)–2(h).—External and internal current systems corresponding to the *SD* field for the three seasons and year. Between consecutive lines 20 000 A flow anticlockwise around maxima and clockwise around minima of the current function. The time scale is for t_{N} time and the arrows indicate midnight geomagnetic local time.

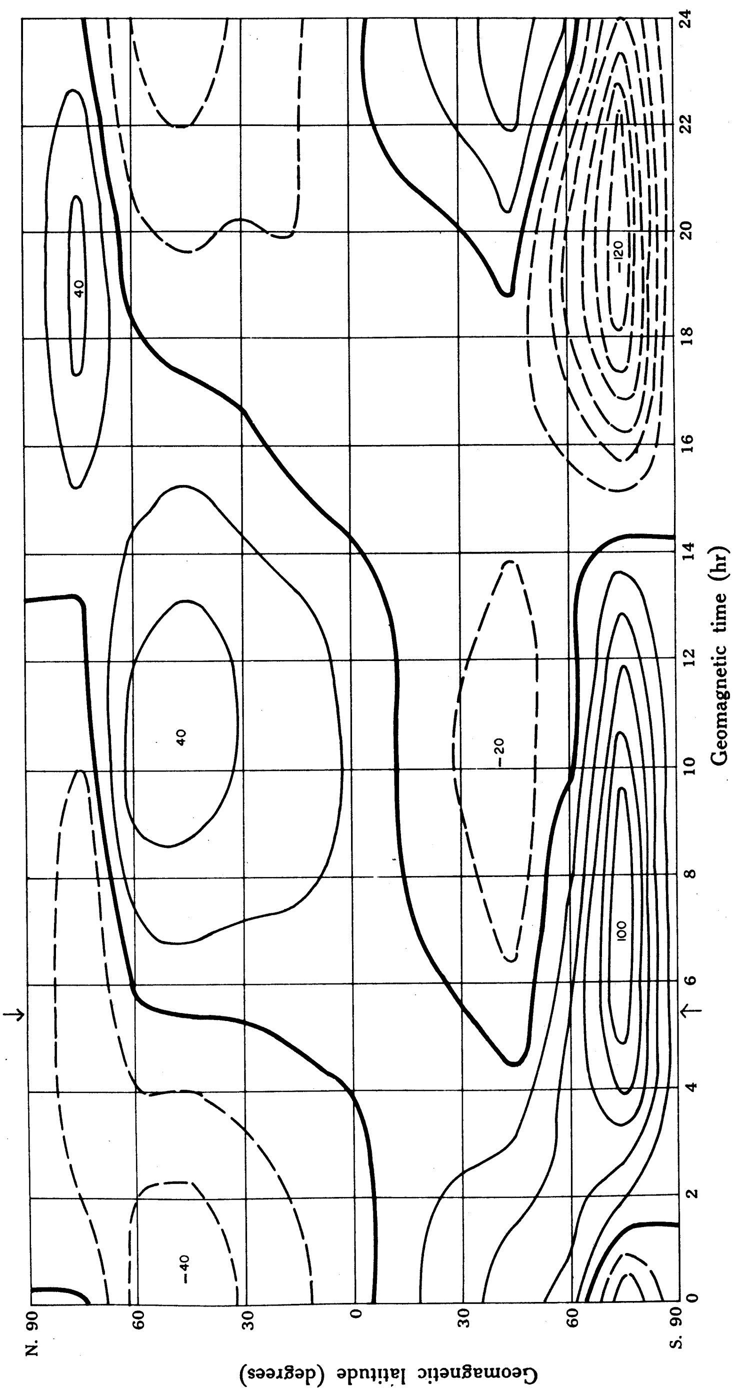

Fig. 2(*a*). External current, southern summer

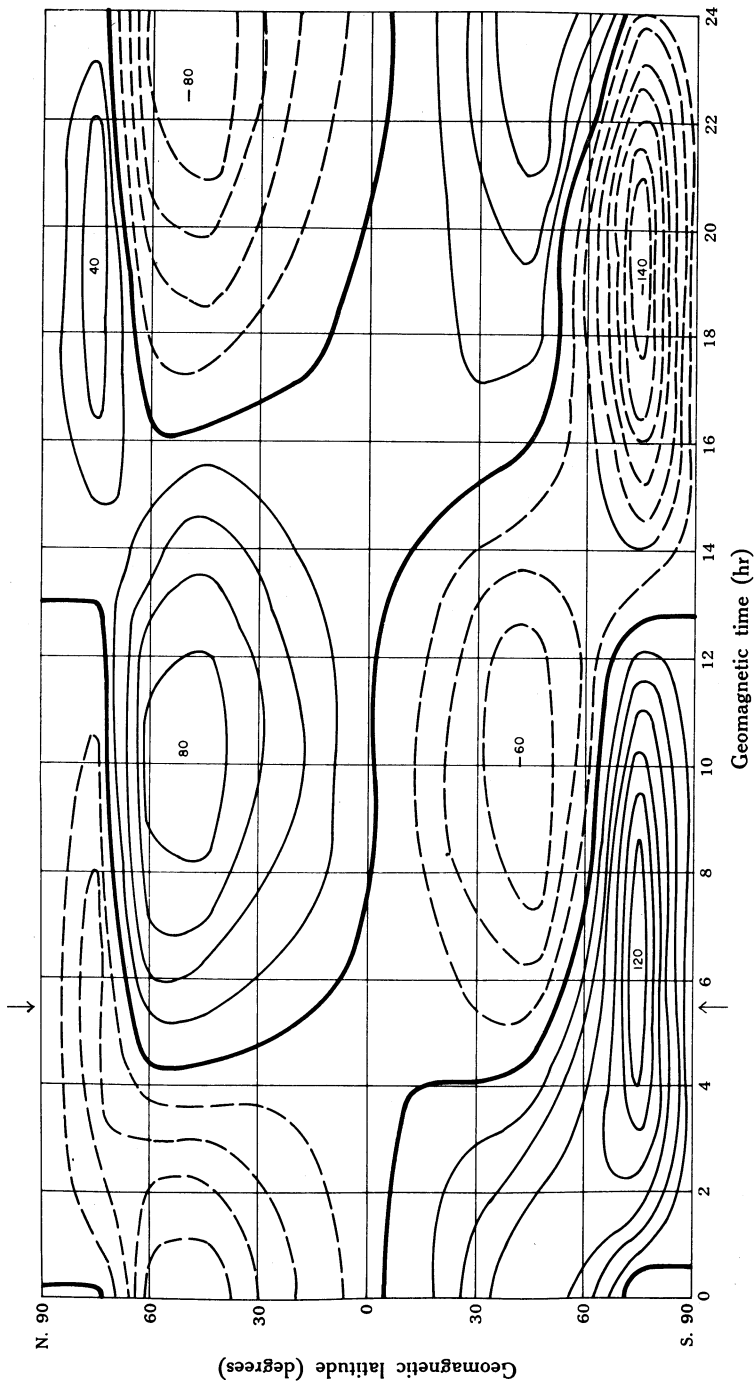

Fig. 2(b). External current, equinox

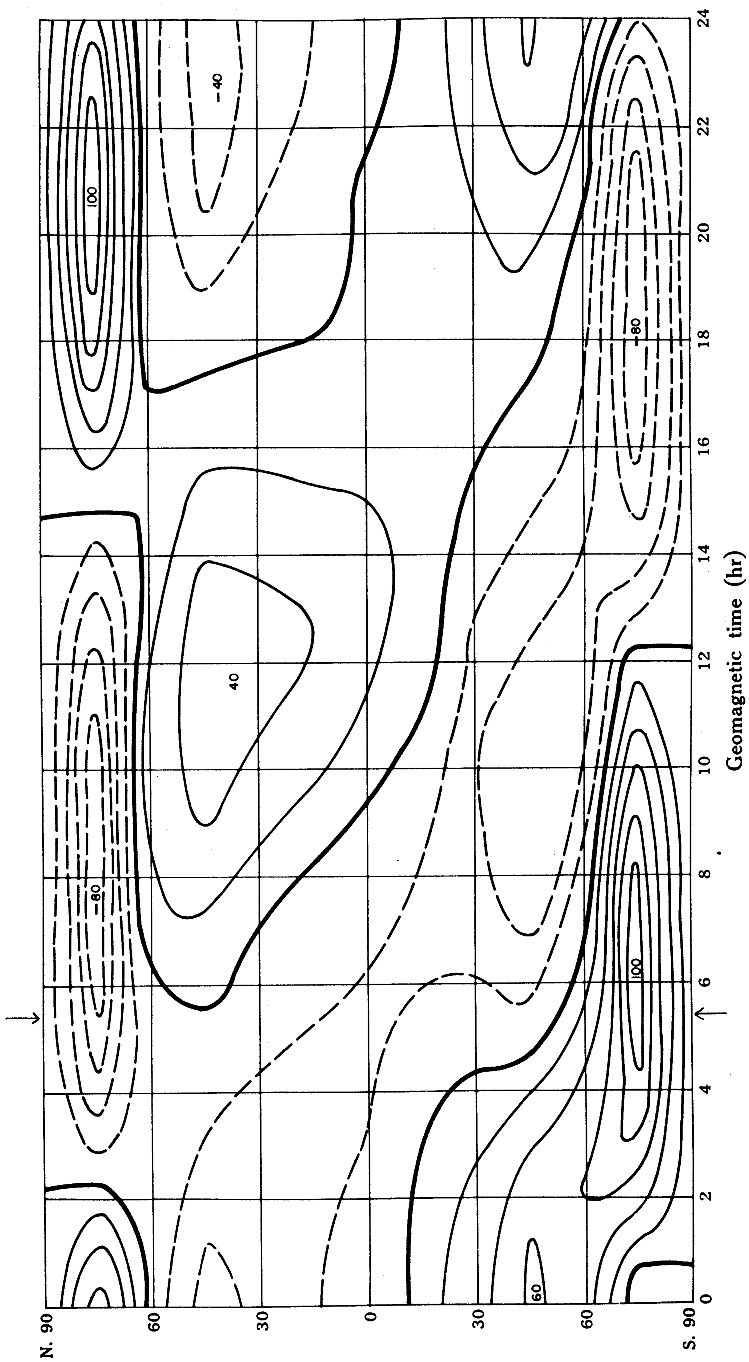

Fig. 2(c). External current, northern summer

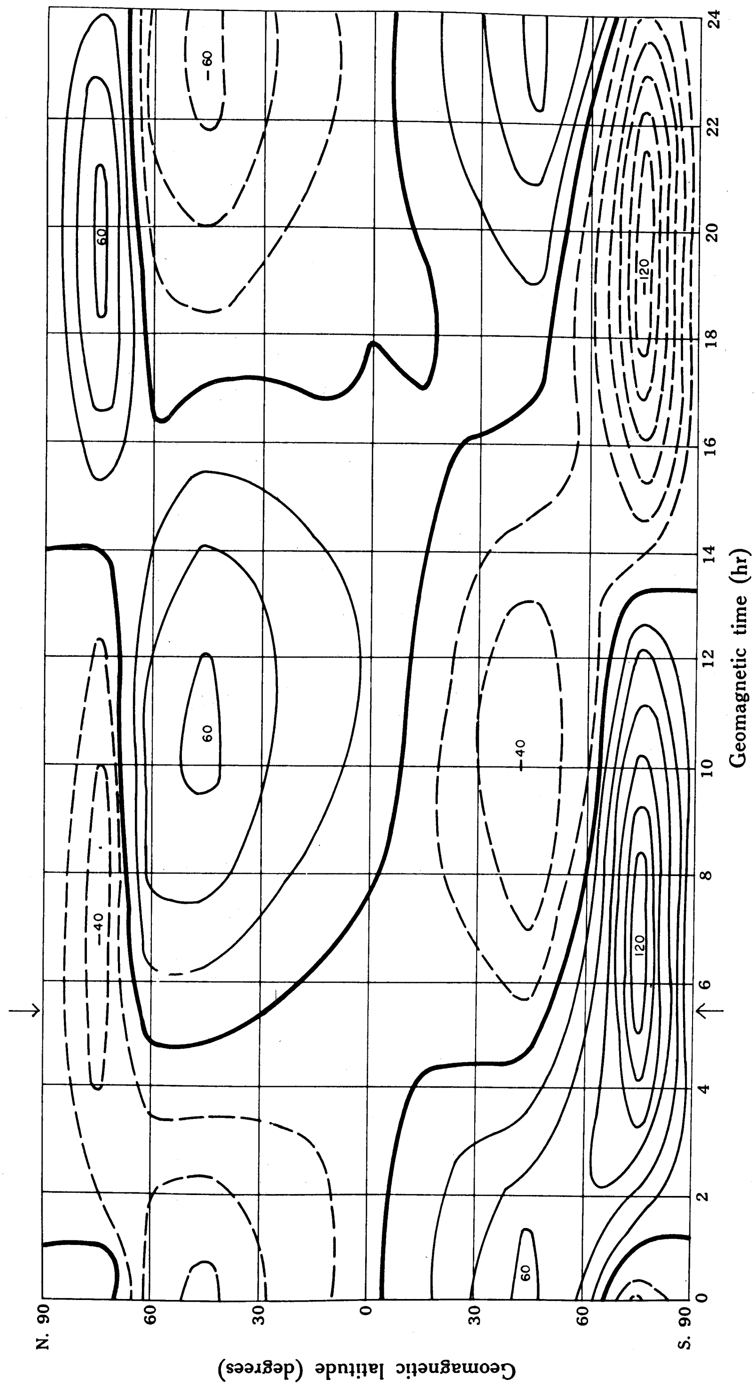

Fig. 2(*d*). External current, year

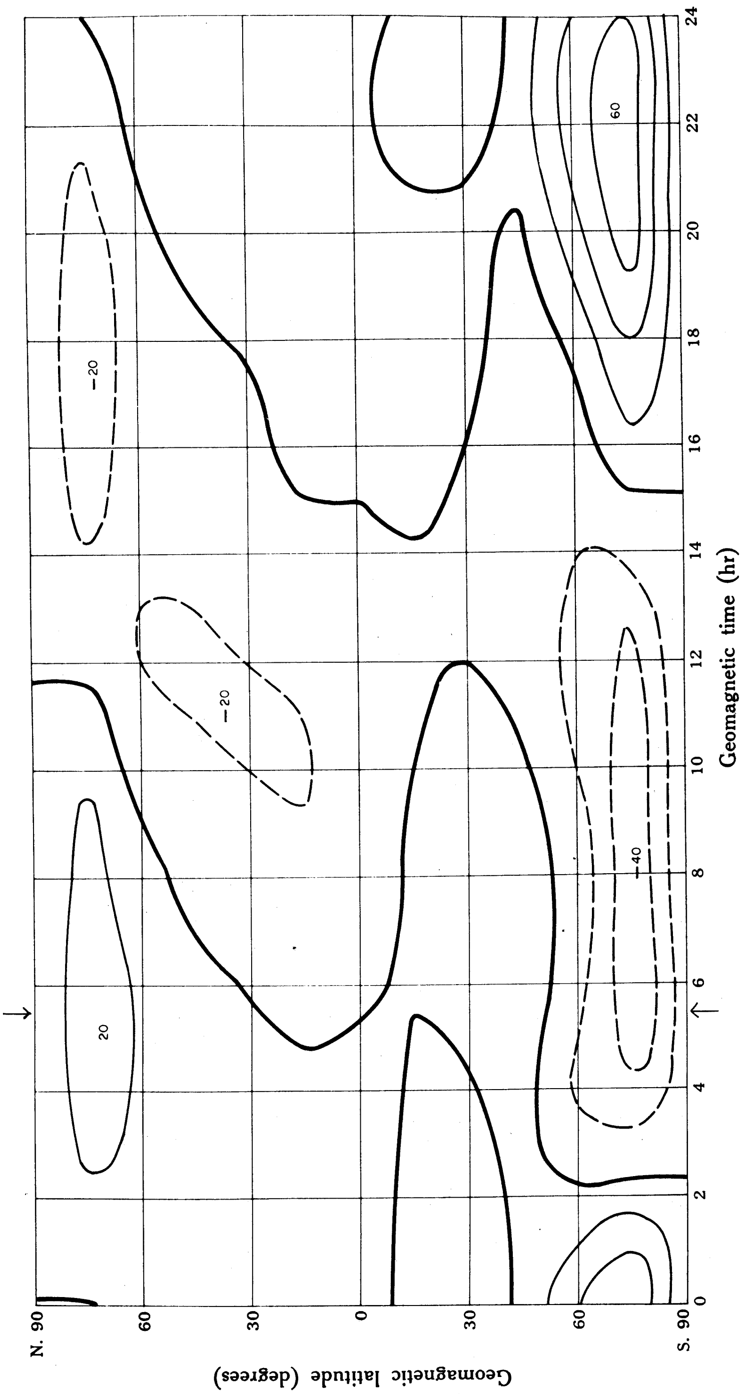

Fig. 2(*e*). Internal current, southern summer

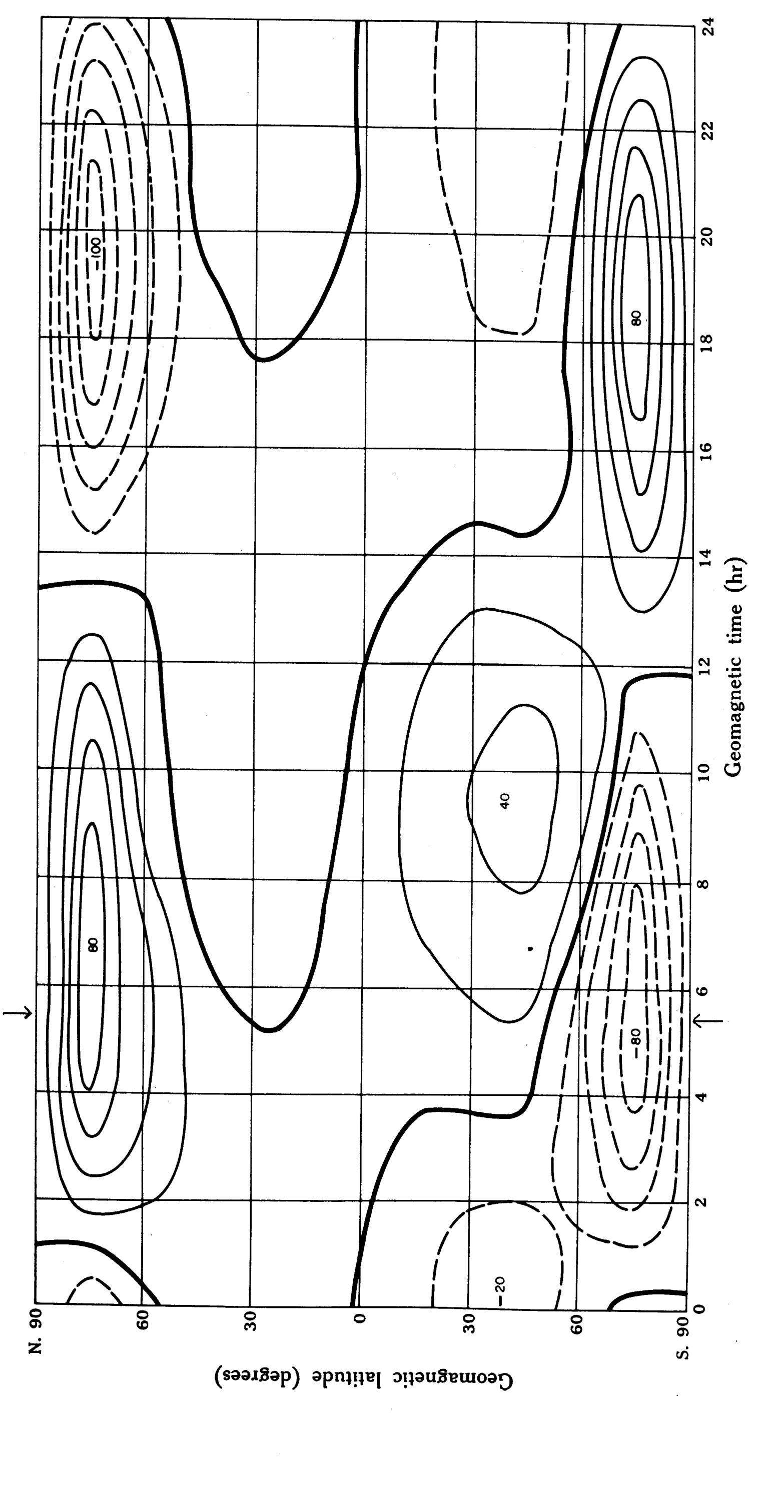

Fig. 2(f). Internal current, equinox

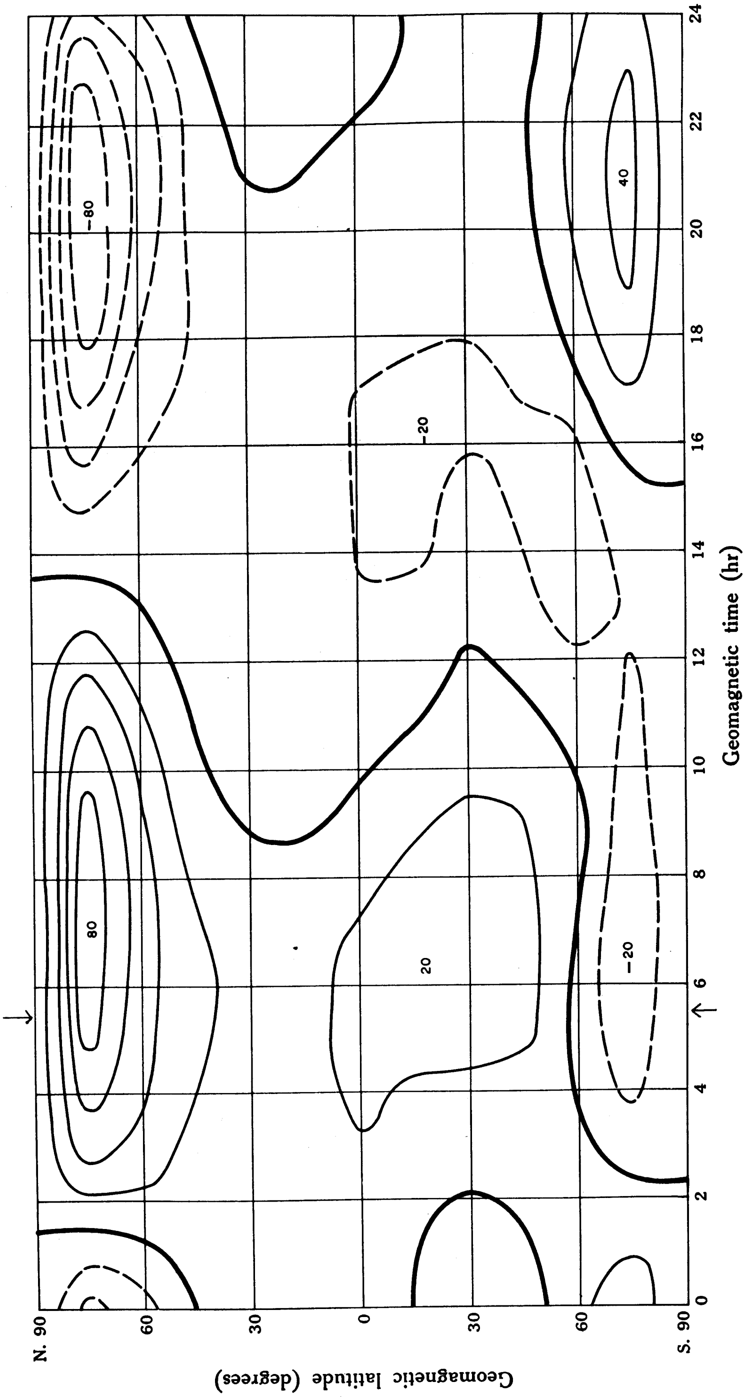

Fig. 2(*g*). Internal current, northern summer

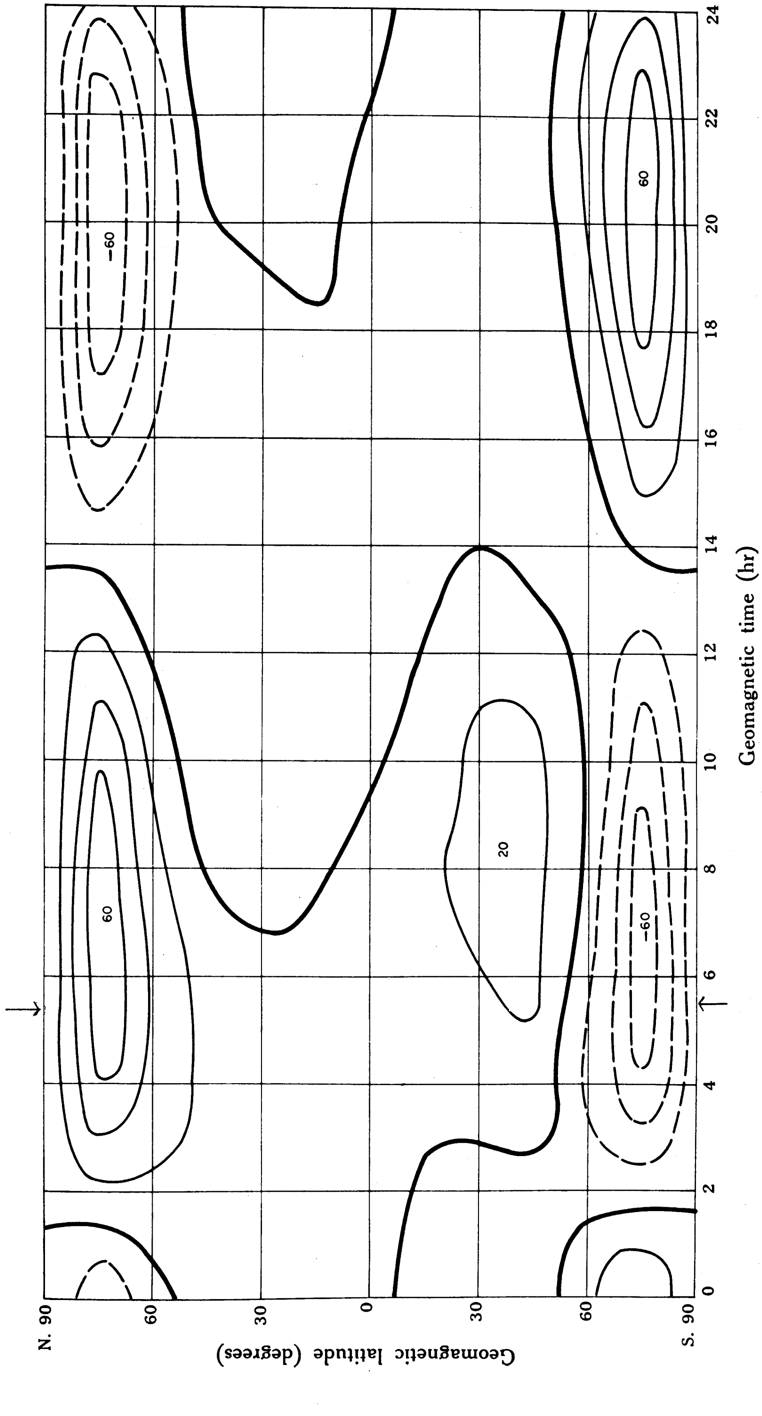

Fig. 2(h). Internal current, year

(1) Fewer coefficients with $m = 2$ and $m = 3$ are significant than for $m = 1$. This applies to both the internal and external coefficients.
(2) The absolute magnitudes of the coefficients for $m = 1$ are generally larger than for $m = 2$ and the latter are generally larger than for $m = 3$. This also applies to both the external and internal coefficients.
(3) Terms where $n-m$ is odd are in general larger than where $n-m$ is even both for the internal and external coefficients. It appears that it would be more important to include more terms with $m = 1$ than to go to higher values of m judging by the size of the terms involved, especially for $m = 1$ and $n-m$ odd.
(4) Fewer i_{na}^{m} are significant compared with the i_{nb}^{m}.
(5) The absolute magnitude of the internal coefficients are generally smaller than the absolute magnitudes of the corresponding external coefficients.
(6) Observations (1)–(5) apply to the coefficients for each of the three seasons and year.

TABLE 3

ABSOLUTE RATIO OF MAXIMUM NEGATIVE INTERNAL VALUE TO MAXIMUM POSITIVE EXTERNAL VALUE OF ELECTRIC CURRENT FUNCTION

Season	Geomagnetic Latitude (°N.)											Mean
	75	60	45	30	15	0	−15	−30	−45	−60	−75	
ss	0·72	0·44	0·41	0·53	0·62	0·65	0·19	0·06	0·26	0·63	0·50	0·46
e	1·81	0·53	0·20	0·32	0·29	1·08	1·13	0·65	0·46	0·49	0·70	0·70
ns	0·90	2·00	0·35	0·32	0·44	0·75	1·71	0·62	0·35	0·53	0·35	0·76
y	1·27	0·64	0·26	0·41	0·49	0·80	0·92	0·49	0·26	0·42	0·56	0·59

Figures 2(*a*)–2(*h*) were calculated for every 15° of geomagnetic latitude and longitude. Table 3 gives the absolute value of the ratio of the maximum negative value of the internal current function to the maximum positive value of the external current function for a given geomagnetic latitude, season, and year. The values for 75° N. are considerably larger than the values for 75° S. Table 4 shows that this is mainly due to the considerable increase of the value of the external current function from its value at 75° N. for southern summer, equinox, and year, while for northern summer it is due mainly to the decrease in the value of the absolute value of the internal current function from its value at 75° N. Table 3 shows that the mean value of the ratio for equinox and year is 0·70 and 0·59 in reasonable agreement with that observed for S_q. The mean ratio is a little larger for northern summer and smaller for southern summer than the value for equinox and year.

From Figures 2(*a*)–2(*h*), which show the external and internal current systems for the centred dipole system for the three seasons and year, the following may be observed.

(1) The external and internal current systems for *SD* are different from those of S_q as obtained by Matsushita and Maeda (1965).
(2) At the poles two current vortices are observed for the external current system in close agreement with the theoretical current systems of Chapman (1935) and different from the current systems of Fel'dshteyn and Zaytsev (1965).

(3) The external current systems for the three seasons and year are only approximately symmetrical about the geomagnetic equator. Best approach to symmetry in the case of the external current systems is observed for equinox and year. This is true if the form and not the intensity is considered. In the case of the internal current systems one finds reasonable symmetry for equinox and year in both the form and intensity if one looks only at the high latitude current function distributions.

TABLE 4

MAXIMUM NEGATIVE INTERNAL AND MAXIMUM POSITIVE EXTERNAL VALUES OF ELECTRIC CURRENT FUNCTION AND RATIO OF THEIR ABSOLUTE MAGNITUDES FOR HIGH LATITUDES

Season	Geomagnetic Latitude	Max. Negative Internal	Max. Positive External	Magnitude Ratio
ss	75° N.	−34	47	0·72
	75° S.	−62	125	0·50
e	75° N.	−112	62	1·81
	75° S.	−102	145	0·70
ns	75° N.	−106	118	0·90
	75° S.	−42	119	0·35
y	75° N.	−85	67	1·27
	75° S.	−75	134	0·56

(4) For the external current system the equatorial vortices are not always independent of the polar vortices, i.e. in some cases the current from the poles flows down to the low latitudes.

(5) The external current vortices are stronger for ss at the south pole than at the north pole. For ns the current vortices are much the same for the two hemispheres.

(6) For equinox and year the external current vortices for the south pole are stronger than for the north pole.

(7) The external and internal current systems for equinox are more intense than the corresponding current systems for the year.

(8) The external current systems are more intense than the corresponding internal current systems for each season and year.

(9) The zero line for the external current systems near the north and south poles for the three seasons and year occurs at approximately 2 and 14 hr t_N time.

(10) The internal current systems have a similar pattern to the corresponding external current systems suggesting that the internal current systems are induced by the corresponding external current systems.

(11) The internal current systems are much less consistent at lower latitudes.

(12) The internal current systems at the south pole are less intense than for the north pole in the case of ss and vice versa for ns. For equinox and year the intensity near the two poles is about the same.

(13) Well-defined current vortices were observed for the internal current systems near the poles for all seasons and year except near the north pole for ss.

(14) The ss internal current system is weaker than the other current systems.

VII. Acknowledgments

The author wishes to acknowledge the encouragement and assistance given to him by Professor L. Slaucitajs and Dr. D. E. Winch in their capacity as supervisors of his Ph.D. thesis, part of which is represented by the work of this paper.

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