COMPUTATION OF GEOMAGNETIC L COORDINATES*

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The study of the distribution of geomagnetically trapped charged particles is aided by the introduction of a parameter L which is a function of the magnitude of the magnetic field and an integral invariant of the particle motion evaluated between the mirror points of the particle trajectory, which are referred to here as absolute conjugate points of the field (McIlwain 1961). The L parameter varies little along any field line and the particle trajectories drift slowly in longitude on surfaces of nearly constant L. This concept has been used by Kilfoyle and Jacka (1968) to establish a latitude–longitude coordinate system for ordering of geophysical data observed near the Earth's surface. Other work includes a satisfactory model of the Earth's magnetic field by Cain *et al.* (1967); it is referred to as the GSFC 12/66 model.

The present paper describes the application of the above contributions to produce more accurate maps of L coordinates computed from the GSFC 12/66 model updated to March 1969.

Tables were prepared, specifying values of L latitude and longitude for 5° geographic latitude and longitude intersections of field lines at a height of 100 km above the Earth's surface.‡ Accurately interpolated contours of constant L latitude at intervals of 10° and L longitude at intervals of 15° were extracted from these values and are presented as polar projections in Figures 1(*a*) and 1(*b*). The accuracy of the tabulated values is believed to be within $\pm 0 \cdot 2°$; the accuracy of the maps is limited by drafting errors to about $0 \cdot 5°$ of a great circle arc.

Computation Procedure

The value λ of L latitude is defined by

$$L \cos^2 \lambda = (R_\phi + h)/6371 \cdot 2 \,,$$

where R_ϕ is the radius of the geoid at geographic latitude ϕ and h is the height in kilometres above the geoid. The parameter L (McIlwain 1961) was calculated, in units of mean Earth radius taken as $6371 \cdot 2$ km, from the magnetic field model through a series of established computer routines.

Computation of L longitude required a modification of these routines. As illustrated by Figure 2, a point P_1, specified by its geographic coordinates at a height of 100 km, defines the field line along which an integration process normally calculates

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‡ Copies of these tables and the maps (23 cm × 26 cm) are available from the Director, The Mawson Institute for Antarctic Research, University of Adelaide, P.O. Box 498D, Adelaide, S.A. 5001.

the L parameter, between P_1 and its absolute conjugate point P_1'. In the present application, the field line was traversed only as far as a point Q, specified as lying within a certain small range of latitude of the geographic equatorial plane. The value of the geographic longitude of Q is ascribed to P_1 as its L longitude. This procedure

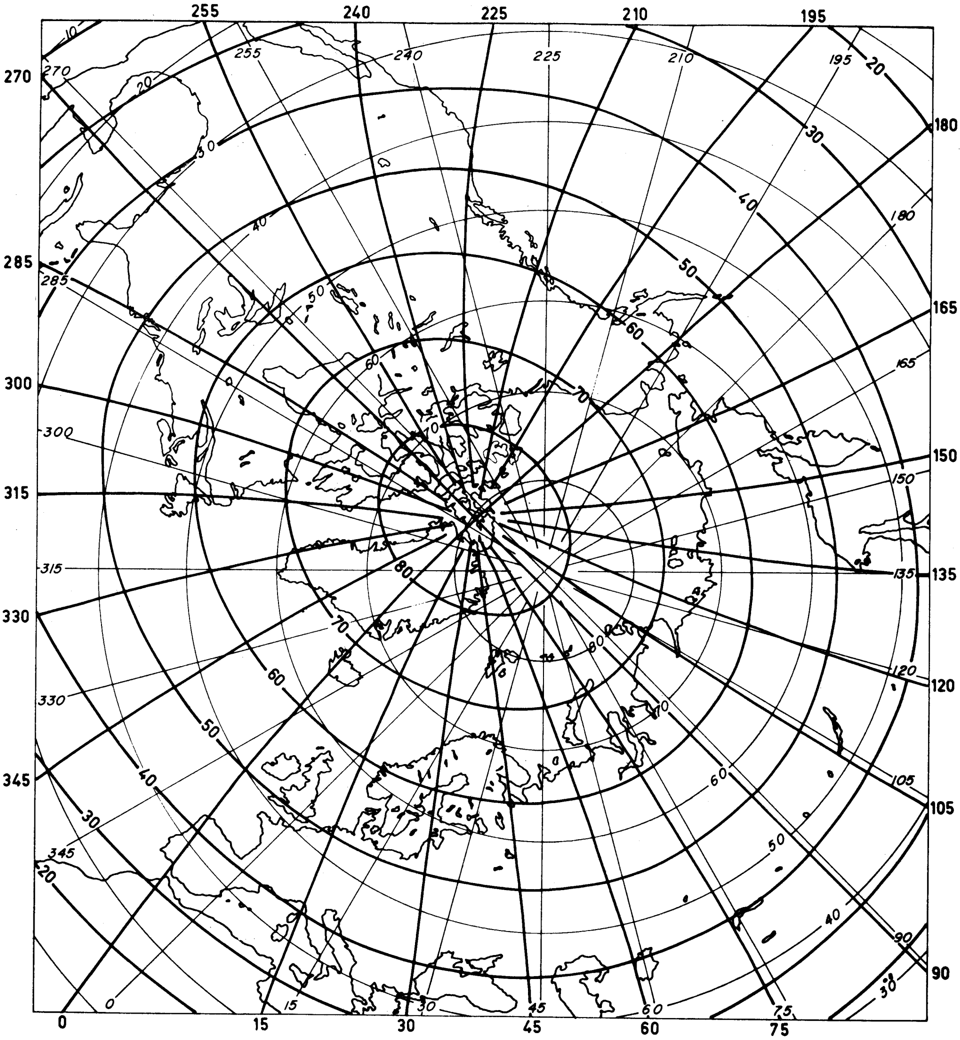

Fig. 1(a).—Polar equidistant geographic projection of the northern hemisphere with the grid of L coordinates superimposed.

introduced some loss of accuracy according to the restriction of latitude range placed on Q and to the declination of the field line at the equator. Also, for a given restriction of range of Q, the routine initially failed for some points P_1, allowing the integration process to proceed beyond the equator towards P_1'. In these cases, the restriction was relaxed sufficiently for a value of longitude at Q to be obtained. The value was then corrected, if necessary, according to the field's declination, to maintain an accuracy of $\pm 0{\cdot}2°$.

It has been pointed out by E. Dunford (personal communication) that the author's use of the FORTRAN program INVAR (written by McIlwain) with the

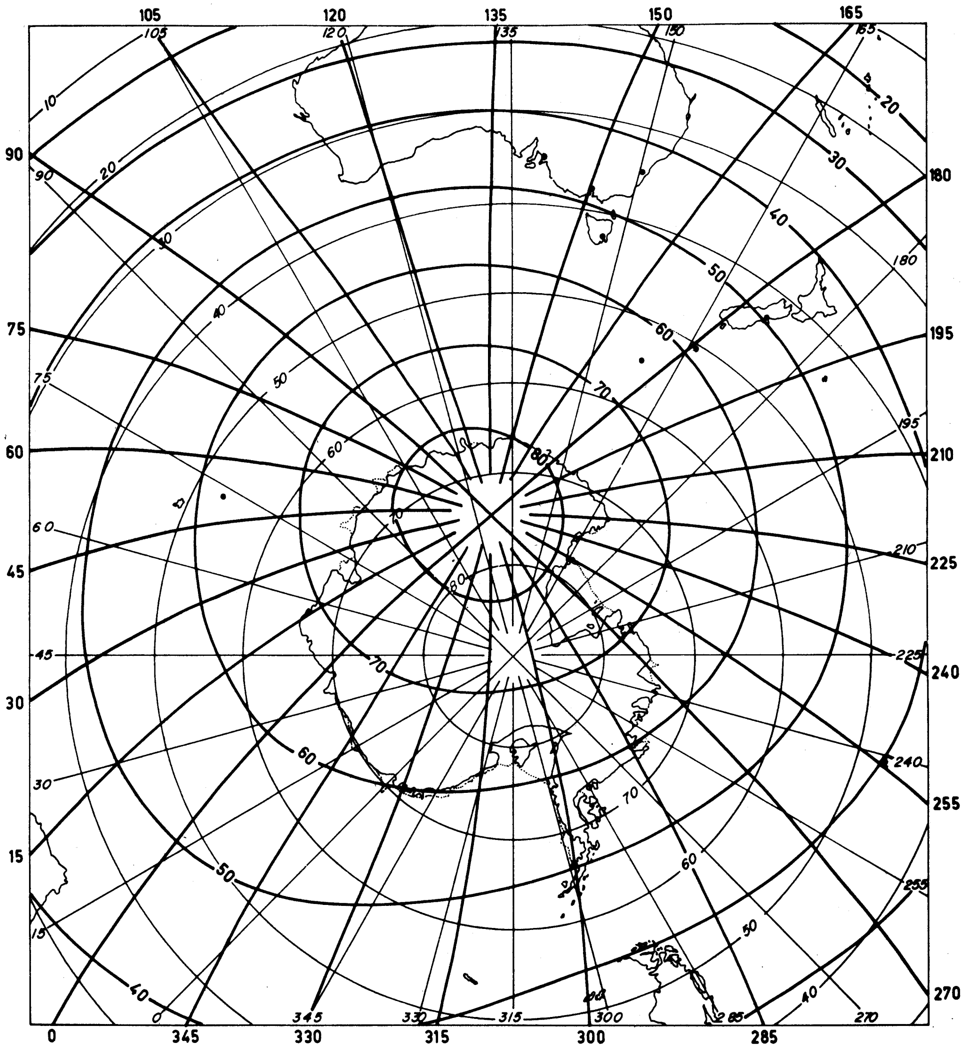

Fig. 1(*b*).—Polar equidistant geographic projection of the southern hemisphere with the grid of L coordinates superimposed.

GSFC 12/66 magnetic field model results in discontinuities in the calculated L values; these have been investigated by Lenhart (1968). A similar investigation reveals comparable discontinuities in L latitude in the present work; however, in middle and high latitudes, no more than about 1% of the tabulated values are in error by more than $\pm 0{\cdot}2°$ as a consequence of this.

Undefined Values

It was found that, for some groups of points at 100 km altitude in certain equatorial regions of the Earth, either the L latitudes or L longitudes (or sometimes both) were not defined. The L latitude of a point was not obtained if the calculated L parameter rendered $\cos^2\lambda$ greater than unity. Although such a value of the L parameter has physical meaning, the corresponding value of L latitude, λ, obviously could not be obtained from the definition given above.

The occurrence of undefined values of L longitude is more significant and reveals a shortcoming of the definition of the longitude coordinate in equatorial regions. It is simply explained by reference to Figure 2. A certain point P_0 specifies a field line which does not intersect the equatorial plane before reaching the absolute conjugate point P_0'. It is obvious that the point Q is without definition. Also, it is evident that the incidence of such cases depends on the displacement of the geomagnetic equator from the geographic equator.

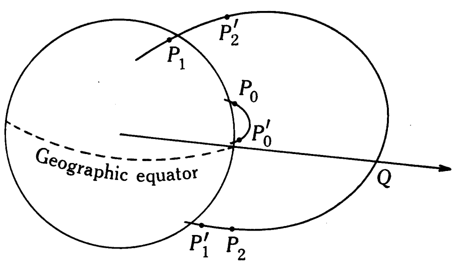

Fig. 2.—Diagram of magnetic field lines of the Earth illustrating the derivation of L latitude and L longitude.

Comparison of L Coordinates at Conjugate 100 km Intersects

The L parameter varies slowly along any given magnetic field line. It would therefore be expected that slightly different values of L latitude would result for the points of conjugate intersect at 100 km for a particular field line.

In order to check the accuracy of the maps of Figure 1 and inspect differences of L latitude for pairs of points at 100 km at each end of a given field line, the following computing routine was used. Referring to Figure 2, the value of L latitude at P_1 was determined as before by calculation of the L parameter by integration between P_1 and its absolute conjugate P_1'. As well, the conjugate intersect P_2 (at 100 km) was obtained; its L latitude was then determined by calculation of the L parameter by integration between P_2 and its absolute conjugate P_2'. The largest differences of L latitude determined for points P_1 and P_2 would be expected where the difference of path lengths P_1P_1' and P_2P_2' is greatest.

Maps of the difference in total geomagnetic field at conjugate intersect points (at a height of 300 km) have been presented by Cole and Thomas (1968). These effectively illustrated the regions where points P_1' and P_2 were furthest separated. In the present work, it was found that the discrepancies in L latitude determined for points P_1 and P_2 in these regions showed a tendency to be greater than in other regions. The maximum discrepancies obtained were less than $\pm 0{\cdot}5°$.

By the nature of its definition, it is apparent that the L longitudes of pairs of conjugate intersects at any given height are precisely equal.

Variation of L Coordinates with Height

In the case of a dipole magnetic field, the value of L latitude is independent of height. For the real field (or that described by the GSFC 12/66 model), variations with height were expected to be small. An indication of the variations over the

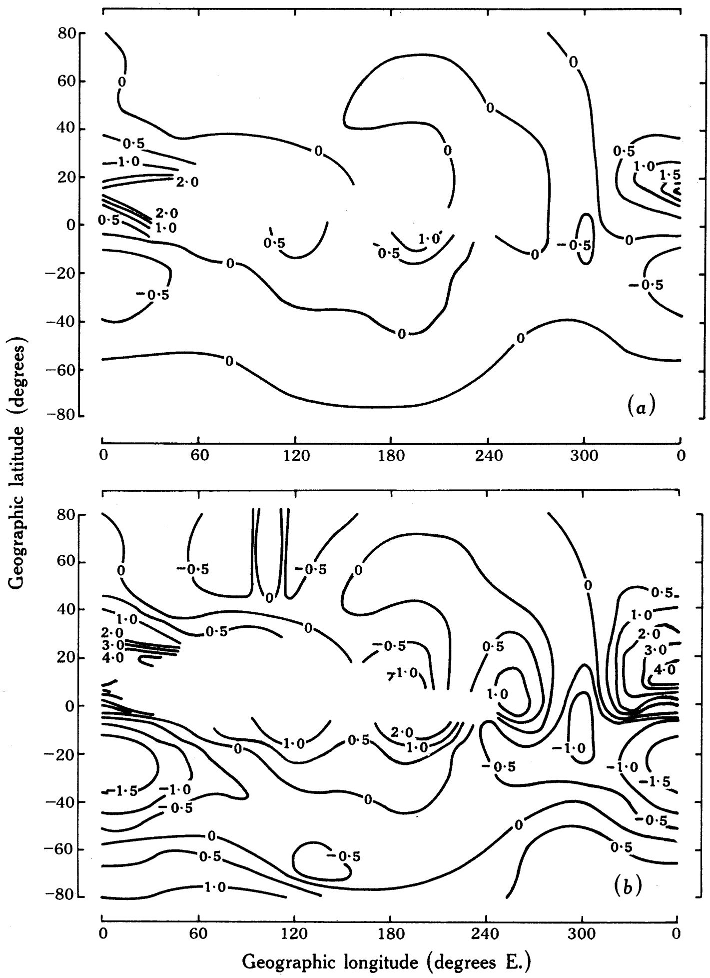

Fig. 3.—World maps in geographic coordinates with contours of the errors (in degrees) resulting at (a) 400 km and (b) 1000 km from adoption of the L latitudes calculated at 100 km.

Earth's surface was obtained through further calculation of L latitude at heights of 400 and 1000 km. Figure 3 shows the results which illustrate the errors incurred in adopting the values calculated at 100 km for heights other than 100 km.

It is noted that over much of the Earth the errors are small; even at 1000 km they lie within $\pm 0 \cdot 5°$ for more than half the total area. Nevertheless, several regions of substantial anomaly are revealed, especially at low latitudes. Although detailed computations have not been made it is apparent from the definition that L longitude will vary only slowly with height.

Acknowledgments

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