CALCULATION OF THE $^{12}C+\alpha$ CAPTURE CROSS SECTION AT STELLAR ENERGIES*

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Abstract

The $^{12}C(\alpha,\gamma)^{16}O$ cross section is calculated at stellar energies, using R-matrix parameters obtained by fitting consistently the $^{12}C+\alpha$ scattering phase shifts and the α-spectrum from ^{16}N β-decay. This limits the $^{12}C+\alpha$ channel radius to the range 5–7 fm. The S-factor at $E_\alpha = 400$ keV is calculated to lie in the range $0\cdot05$–$0\cdot33$ MeV . b.

I. Introduction

The cross section of the $^{12}C(\alpha,\gamma)^{16}O$ reaction at α-particle energies of the order of a few hundred keV is of interest for theories of nucleosynthesis in stars. Direct experimental measurement of the cross section is not feasible below a few MeV, so that an indirect determination is necessary. The assumption usually made is that the main contributions are from the known 1^- levels of ^{16}O at $7\cdot12$ MeV (46 keV below the $^{12}C+\alpha$ threshold) and $9\cdot59$ MeV, and the greatest uncertainty comes from the value of the α-particle dimensionless reduced width θ_α^2 of the $7\cdot12$ MeV level.

Previous estimates of θ_α^2 have differed widely. A theoretical estimate by Stephenson (1966) based on models for the 1^- and 3^- levels of ^{16}O gave $\theta_\alpha^2 = 0\cdot08\pm0\cdot04$.‡ Experimental estimates have been made by various methods:

(1) Stripping reactions. From the $^6Li(^{12}C,d)^{16}O$ reaction Loebenstein *et al.* (1967) obtained $\theta_\alpha^2 = 0\cdot06$–$0\cdot14$,‡ while from $^{12}C(^7Li,t)^{16}O$, Pühlhofer *et al.* (1970) gave $\theta_\alpha^2 = 0\cdot025$‡ with the suggestion that this may be too low by a factor of $1\cdot5$ or 2. Dolinsky, Turovtsev, and Yarmukhamedov (1970) have pointed out some difficulties in obtaining reliable estimates in this way.

(2) Analysis of the $^{12}C+\alpha$ elastic scattering p-wave phase shift. With the energy dependence of the phase shift represented by a many-level R-matrix formalism, Clark (1969) obtained $\theta_\alpha^2 = 0\cdot71^{+0\cdot37}_{-0\cdot18}$, while a Caltech group (Weisser, Morgan, and Thompson 1970) found for θ_α^2 a very small value (Thompson, personal communication) with a large uncertainty. This is because θ_α^2 is involved in both the incoming and outgoing channels so that the $7\cdot12$ MeV level contributes little to the phase shift.

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‡ These estimates are all appropriate to a $^{12}C+\alpha$ channel radius of $1\cdot40(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ fm $= 5\cdot43$ fm, as they were normalized by comparing reduced widths for higher ^{16}O levels with those obtained by Hill (1953) and Bittner and Moffat (1954).

(3) Analysis of the $^{12}C(\alpha,\gamma)^{16}O$ cross section near the 9·59 MeV peak, used in conjunction with (2) above. The Caltech group obtained $\theta_\alpha^2 \approx 0\cdot4$ with a large uncertainty (Thompson, personal communication). In principle, this reaction is more favourable for determining θ_α^2 than is elastic scattering, as the γ-ray reduced width of the 7·12 MeV level is large compared with that of the 9·59 MeV level (this is consistent with the 7·12 MeV level being mainly $1p$–$1h$ relative to the ^{16}O ground state and the 9·59 MeV level being mainly $3p$–$3h$). There are, however, great experimental difficulties in measuring the $^{12}C(\alpha,\gamma)^{16}O$ cross section, and recent measurements at Oak Ridge (Jaszczak, Gibbons, and Macklin 1970; Jaszczak and Macklin 1970) and at Caltech differ considerably (Weisser, personal communication).

In the present paper therefore a different method is proposed in which θ_α^2 is estimated from a many-level R-matrix analysis of the spectrum of delayed α-particles following ^{16}N β-decay, used in conjunction with $^{12}C+\alpha$ phase shift analysis.† The advantages of this method are that the spectrum depends sensitively on θ_α^2 because the β-decay matrix element to the 7·12 MeV level is large compared with that to the 9·59 MeV level (since the ^{16}N ground state also belongs mainly to $1p$–$1h$ configurations), and that the spectrum has been measured accurately down to relatively low excitation energies in ^{16}O (Hättig, Hünchen, and Wäffler 1970). A disadvantage is that there is also a contribution to the spectrum from 3^- levels of ^{16}O, so that the $^{12}C+\alpha$ f-wave phase shift must also be fitted.

In principle, information about θ_α^2 could also be obtained by using a particle reaction to populate 1^- states of ^{16}O, e.g. $^{15}N(^3He,d)^{16}O$, but it would be difficult to obtain an accuracy comparable with that from ^{16}N β-decay. There are several reasons for this: the particle reaction is less selective in the J^π values of the ^{16}O states it can populate (unless the contribution from $l = 0$ transfer can be separated out), a background contribution to the deuteron spectrum in the region of interest can come from the competing mode of decay $^{15}N(^3He,\alpha)^{14}N^*(d)^{12}C$, and a very clean ^3He beam profile is needed as one is essentially looking for the ghost of the 7·12 MeV level (Barker and Treacy 1962).

The next section considers the experimental data that are fitted. Section III gives the R-matrix formulae; those used in the analyses of the $^{12}C+\alpha$ phase shifts and the α-spectrum from ^{16}N β-decay are similar to formulae used previously in analyses of $\alpha+\alpha$ scattering and reactions involving states of ^8Be (Barker, Hay, and Treacy 1968; Barker 1969). The results are contained in Section IV. In each of these sections, the $^{12}C+\alpha$ scattering, the α-spectrum from ^{16}N β-decay, and the $^{12}C(\alpha,\gamma)^{16}O$ cross section are treated separately.

II. Experimental Data

(a) $^{12}C+\alpha$ *Scattering*

From the experimental $^{12}C+\alpha$ phase shifts $\delta_l^{\exp}(l = 1,\ 3)$ we wish to obtain information about the broad 1^-, 9·59 MeV and 3^-, 11·63 MeV levels in addition to the bound 1^-, 7·12 MeV and 3^-, 6·13 MeV levels. Thus we use values of $\delta_1^{\exp}$ and

† After this work was essentially completed, it was learnt that the same approach has been used by Werntz (personal communication).

$\delta_3^{\exp}$ obtained by Jones *et al.* (1962) for the energy range $E_\alpha = 2\cdot5$–$4\cdot8$ MeV, and by Clark, Sullivan, and Treacy (1968) for $E_\alpha = 2\cdot8$–$6\cdot6$ MeV. As the $11\cdot63$ MeV level occurs near the highest energy of these measurements, we also include some of the values of $\delta_3^{\exp}$ obtained by Morris, Kerr, and Ophel (1968) for $E_\alpha = 6\cdot6$–$8\cdot5$ MeV. The phase shifts may be complex for $E_\alpha > 5\cdot92$ MeV, where the inelastic channel is open, but the results of Clark, Sullivan, and Treacy and of Morris, Kerr, and Ophel suggest that the imaginary parts of the phase shifts are negligible for $E_\alpha \lesssim 7\cdot5$ MeV. Thus we include the $\delta_3^{\exp}$ values of Morris, Kerr, and Ophel for $E_\alpha = 6\cdot6$–$7\cdot5$ MeV.

The phase shift errors ϵ_l for the data of Jones *et al.* (1962) are taken as $\epsilon_1 = 2°$ and $\epsilon_3 = 1°$; these are compromise values as Jones *et al.* have used smoothed phase shifts but their values have to be read from figures rather than tables. The $\delta_3^{\exp}$ values of Clark, Sullivan, and Treacy (1968) at their three highest energies and of Morris, Kerr, and Ophel (1968) do not show a smooth dependence on energy, and their values differ by 10° at the common energy of $E_\alpha = 6\cdot6$ MeV, so we take $\epsilon_3 = 5°$ for all of these points but use the smaller tabulated errors given by Clark, Sullivan, and Treacy for ϵ_3 at other energies and for all ϵ_1.

In the analysis it is important that the energy calibrations of the different experimental data should be consistent. We assume that the ^{16}O excitation energies obtained by Browne and Michael (1964) from the $^{14}N(^3He, p)^{16}O$ reaction are correct,* at any rate for narrow levels, and adjust other energy scales to agree with these. Thus Browne and Michael give the energy of a 2^+ level as $9\cdot847 \pm 0\cdot003$ MeV and of a 4^+ level as $10\cdot353 \pm 0\cdot004$ MeV. These levels are observed by Jones *et al.* (1962) and their energies are given as $9\cdot815 \pm 0\cdot020$ MeV and $10\cdot329 \pm 0\cdot025$ MeV. These values are increased by 13 keV when the $^{12}C+\alpha-^{16}O$ mass value of $7\cdot148$ MeV used by Jones *et al.* is replaced by the present value of $7\cdot161$ MeV. We therefore increase the energy scale of Jones *et al.* by 15 keV in excitation energy or 20 keV in α-particle energy. Jones *et al.* had noted that the α-particle energy scales in earlier experiments at Wisconsin (Hill 1953; Bittner and Moffat 1954) were about 30 keV higher than their own. Clark, Sullivan, and Treacy (1968) did not make measurements near the narrow resonances and their energy scale can be compared with others only by looking at the increase in $\delta_1^{\exp}$ for $E_\alpha \approx 3\cdot2$ MeV. To obtain agreement with the adjusted data of Jones *et al.*, the α-particle energy scale of Clark, Sullivan, and Treacy is reduced by 20 keV. The energy scale of Morris, Kerr, and Ophel (1968) is not changed, as δ_3 appears to be approximately independent of energy over the part of their energy range used here.

The experimental phase shifts are shown later in Figure 2, together with calculated fits described in Section IV.

(*b*) *α-spectrum from ^{16}N β-decay*

The spectrum of delayed α-particles following ^{16}N β-decay has been measured by Hättig, Hünchen, and Wäffler (1970) with very good statistics, the total number of α-particles being $1\cdot3 \times 10^8$. From the published work it is not possible to obtain

* A check on the accuracy of Browne and Michael's (1964) values is that their energies for the bound $6\cdot13$ and $7\cdot12$ MeV levels differ by only 0 and 4 keV respectively from those obtained by precision measurements of γ-ray energies (Chasman *et al.* 1967).

the spectrum with sufficient accuracy; however, detailed values of the pulse height distribution for one run containing $3\cdot 2 \times 10^7$ α-particles spread over 98 channels and of the corresponding background due to electrons have been made available (Wäffler, personal communication).

In order to obtain an experimental spectrum to compare with the calculated spectrum, the measured pulse height distribution is corrected for the experimental energy resolution and for the background. The calculated energy resolution (FWHM) is $32\cdot 5$ keV for an α-energy* $E_\alpha = 2$ MeV and 36 keV for $E_\alpha = 1\cdot 28$ MeV (Hättig *et al.* 1969). The measured resolution at $E_\alpha = 2$ MeV is 37 keV (Hättig *et al.* 1969), while at $E_\alpha = 1\cdot 28$ MeV Hättig, Hünchen, and Wäffler (1970) used a value of 53 keV although a smaller value would seem to give a better fit. We therefore assume that the resolution is a linear function of E_α, equal to 37 keV at $E_\alpha = 2$ MeV and $(36/32\cdot 5)$ times this at $E_\alpha = 1\cdot 28$ MeV. The effect of the experimental resolution is treated as a perturbation, so that the corrected spectrum $f_c(i)$ for channel i is given by first-order perturbation theory as

$$f_c(i) = 2f_a(i) - f_b(i)\,, \tag{1}$$

where $f_a(i)$ is the raw spectrum and $f_b(i)$ is the result of smearing the raw spectrum with a normalized Gaussian of width equal to the resolution appropriate to the channel i. The fractional corrections $(f_c - f_a)/f_a$ obtained in this way are at most of order 2%. The accuracy of the procedure may be estimated by smearing f_c and comparing the result with f_a: the differences are at most of the order of the statistical errors, so that use of higher order perturbation relations does not give substantially better accuracy. The values of f_c for the four or five channels at each end of the range are unreliable and are omitted from the fits. The background is fitted by a function that is the sum of two terms, one being constant and the other having an exponential energy dependence, and this function, suitably normalized, is subtracted from the corrected spectrum f_c to obtain the experimental α-particle spectrum $W^{\exp}(i)$. Because $W^{\exp}$ varies so smoothly with energy, data from every fourth channel only are included in the fits.

The corresponding uncertainty $\eta(i)$ in the α-particle spectrum includes contributions from the statistical errors in the measured pulse height distribution and in the background, the uncertainty in the normalization of the background, and the uncertainty in the α-particle energy. This last contribution is needed as the spectrum is in some regions a very rapidly varying function of energy. From their external calibration, Hättig *et al.* (1969) estimated the uncertainty in the α-particle energy scale to be 10 keV. This calibration gave excitation energies of $8\cdot 867 \pm 0\cdot 013$ MeV (Hättig, Hünchen, and Wäffler 1970) and $9\cdot 851 \pm 0\cdot 013$ MeV (Hättig *et al.* 1969) for weak groups of α-particles attributed to the 2^- and 2^+ levels of ^{16}O located by Browne and Michael (1964) at $8\cdot 870 \pm 0\cdot 003$ and $9\cdot 847 \pm 0\cdot 003$ MeV respectively. We calibrate the energy scale of the α-particle spectrum by assuming Browne and

* It should be noted that the quantity E_α used in discussing the α-spectrum from ^{16}N β-decay, and plotted as abscissa in Figures 3 and 5, corresponds to 9/16 times the quantity E_α used in discussing $^{12}C + \alpha$ scattering and the $^{12}C(\alpha,\gamma)^{16}O$ cross section, and used as abscissa in Figures 2 and 6.

Michael's energies for these weak groups, and then take an uncertainty in the energy scale of 5 keV in calculating η. Except at low energies ($E_\alpha \lesssim 1\cdot 2$ MeV) and near the peak of the spectrum, the dominant contribution to η comes from this uncertainty in energy.

The experimental α-particle spectrum W^{exp} is shown later in Figure 5 as a function of α-particle energy, together with the background subtracted and a calculated fit to the spectrum described in Section IV. It may be noted that a similar experimental spectrum with poorer statistics (with a total of 10^6 α-particles) obtained by Sprenkel-Segel, Segel, and Siemssen (1970) differs appreciably from the spectrum shown in Figure 5.

In the analysis it is necessary to use experimental values of the branching ratios of the ^{16}N β-decay to the $9\cdot 59$ MeV level (which contributes the bulk of the observed α-spectrum) and to the bound 1^- and 3^- levels at $7\cdot 12$ and $6\cdot 13$ MeV. These are taken respectively as $1\cdot 19\times 10^{-5}$ (Hättig *et al.* 1969) and $0\cdot 049$ and $0\cdot 68$ (Alburger, Gallmann, and Wilkinson 1959). The Q-value for the ^{16}N β-decay to the ^{16}O ground state is taken as $10\cdot 422$ MeV ($\pm 3\cdot 5$ keV) from the mass tables of Mattauch, Thiele, and Wapstra (1965). The value $10\cdot 402$ MeV given by Hättig *et al.* (1969) appears to be a misprint; early fits to the data using this value were very poor for the high-energy end of the α-particle spectrum.

(c) $^{12}C(\alpha,\gamma)^{16}O$ *Cross Section*

In calculating the $^{12}C(\alpha,\gamma)^{16}O$ cross section, we use the measured values of 62 ± 5 meV for the radiation width of the $7\cdot 12$ MeV 1^- level of ^{16}O (Swann 1970) and 50 nb for the maximum of the cross section at the $9\cdot 59$ MeV peak (Jaszczak and Macklin 1970).

III. R-Matrix Formulae and Fitting Procedures

As mentioned in the Introduction, the formulae and notation used in this section are similar to those used previously in analyses of $\alpha+\alpha$ scattering and reactions involving states of ^8Be (Barker, Hay, and Treacy 1968; Barker 1969), and the justification for their use is similar to that given there.

(a) $^{12}C+\alpha$ *Scattering*

The best fits to the experimental $^{12}C+\alpha$ phase shifts δ_l^{exp} ($l = 1, 3$) are obtained by minimizing

$$X_l = \frac{1}{N_l}\sum_{i=1}^{N_l}\left|\frac{\delta_l^{\text{exp}}(E_i)-\delta_l(E_i)}{\epsilon_l(E_i)}\right|^2, \tag{2}$$

where the E_i are the energies of the N_l data points and $\delta_l(E_i)$ is the calculated phase shift at E_i:

$$\delta_l(E) = -\phi_l+\arctan\left[P_l\div\left\{\left(\sum_{\lambda=1}^{q_l}\gamma_{\lambda l}^2/(E_{\lambda l}-E)\right)^{-1}-S_l+B_l\right\}\right]. \tag{3}$$

Here P_l, S_l, and $-\phi_l$ are respectively the energy-dependent penetration factor, shift factor, and hard-sphere phase shift calculated at the channel radius a_l, and B_l is the constant boundary condition parameter. It is assumed that q_l levels of the compound nucleus ^{16}O are contributing, with eigenenergies $E_{\lambda l}$ and α-particle reduced width amplitudes $\gamma_{\lambda l}$ $(\lambda = 1, ..., q_l)$. The energies E_i and $E_{\lambda l}$ are taken as c.m. energies measured from the ^{12}C$+\alpha$ threshold.

In order to fit the p-wave phase shift over the energy range $E_\alpha = 2\cdot5$–$6\cdot6$ MeV $(N_1 = 45)$, we take $q_1 = 3$, corresponding to the $7\cdot12$ and $9\cdot59$ MeV levels and a background term representing all other 1^- levels. Because the lowest level is bound and its energy accurately known, we restrict the level parameters so that this energy is fitted exactly. Also, since the effects on X_1 of variations of the two background parameters are strongly correlated, we restrict the parameters to fix the energy of the background level at 10 MeV. Thus for fixed values of a_1 and B_1, there remain four free parameters.

Similarly the f-wave phase shift is fitted over the range $E_\alpha = 2\cdot5$–$7\cdot5$ MeV $(N_3 = 54)$ with $q_3 = 3$, corresponding to the $6\cdot13$ and $11\cdot63$ MeV 3^- levels and a background term, and restrictions are imposed in order to fit the energy of the bound level and to fix the background level at 15 MeV.

For given a_l, identical fits to the data can be obtained for any value of B_l, the corresponding values of the level parameters $E_{\lambda l}$ and $\gamma^2_{\lambda l}$ depending on the choice of B_l according to relations given in Appendix II of Barker, Hay, and Treacy (1968). The dependence on the choice of B_l is more significant here than in the previous ^{8}Be analyses because a greater range of B_l values is of interest. In the next subsection, arguments based on shell model descriptions of the ^{16}O states are used. The R-matrix eigenstate in the internal region should most nearly resemble a shell model state for B_l equal to S_l calculated at the energy of the state appropriate to that value of B_l, that is, for the eigenstate λ

$$B_l = S_l(E_{\lambda l}(B_l)) \,. \tag{4}$$

We use the notation $B_l^{(\lambda)}$ for this value of B_l and label the corresponding level parameters by a similar index (λ), e.g.

$$E^{(\lambda)}_{\lambda' l} = E_{\lambda' l}(B_l^{(\lambda)}) \,. \tag{5}$$

Then the restrictions obtained from fitting the energies of the bound and background levels can be expressed as

$$\left.\begin{aligned} E^{(1)}_{11} &= -0\cdot046\text{ MeV}\,, & E^{(3)}_{31} &= 10\cdot0\text{ MeV}\,,\\ E^{(1)}_{13} &= -1\cdot030\text{ MeV}\,, & E^{(3)}_{33} &= 15\cdot0\text{ MeV}\,. \end{aligned}\right\} \tag{6}$$

The quantity θ^2_α of prime interest here is given by

$$\theta^2_\alpha = \gamma^{(1)2}_{11} \div (\tfrac{3}{2}\hbar^2/\mu a_1^2) \,, \tag{7}$$

where μ is the reduced mass of the ^{12}C$+\alpha$ system.

(b) α-spectrum from ^{16}N β-decay

The quality of fit to the experimental α-spectrum from ^{16}N β-decay is measured by

$$X_\beta = \frac{1}{N_\beta}\sum_{i=1}^{N_\beta}\left|\frac{W^{\exp}(c_i)-W(c_i)}{\eta(c_i)}\right|^2, \tag{8}$$

where c_i is the channel label for the N_β channels included in the fit ($N_\beta = 23$). Since allowed β-decays from the 2^- ground state of ^{16}N can populate 1^-, 2^-, and 3^- states of ^{16}O and decay of 2^- states to $^{12}C+\alpha$ is parity forbidden, the calculated α-particle spectrum $W(c_i)$ is assumed to contain contributions from 1^- and 3^- states of ^{16}O, not interfering with each other. Because $\Delta J = \pm 1$, Fermi contributions are zero. Thus the form assumed for W is

$$W(c_i) = f_\beta \sum_{l=1,3} P_l \left(\frac{\left|\sum_{\lambda=1}^{q_l}\{A_{\lambda l}/(E_{\lambda l}-E_i)\}\right|^2}{\left|1-(S_l-B_l+\mathrm{i}P_l)\sum_{\lambda=1}^{q_l}\{\gamma_{\lambda l}^2/(E_{\lambda l}-E_i)\}\right|^2}\right), \tag{9}$$

where E_i is the c.m. energy corresponding to the channel c_i, $f_\beta \equiv f(8, 3\cdot772\text{ MeV} - E_i)$ is the integrated Fermi function, and the energy-independent amplitudes $A_{\lambda l}$ can be expressed in terms of the Gamow–Teller matrix elements $(\int\sigma)_{\lambda l}$ by

$$A_{\lambda l} = \left(\frac{mc^2}{2\pi^3\hbar}\frac{Nt_{\frac{1}{2}}}{(\ln 2)\,Y(9\cdot59)}\right)^{\frac{1}{2}}\frac{G_{\mathrm{G}}(\int\sigma)_{\lambda l}}{I_{\lambda l}^{\frac{1}{2}}}. \tag{10}$$

Here

$$N = \int_0^\infty W(E)\,\mathrm{d}E \approx \int_0^\infty W^{\exp}(E)\,\mathrm{d}E \tag{11}$$

(the main contribution coming from the $9\cdot59$ MeV level), $t_{\frac{1}{2}}$ is the half-life of the decay, $Y(9\cdot59)$ is the branching ratio to the $9\cdot59$ MeV level, G_{G} is the dimensionless Gamow–Teller coupling constant (the value of which is not needed), and

$$I_{\lambda l} = \int_0^\infty \frac{P_l\,\mathrm{d}E}{|E_{\lambda l}-E-(S_l-B_l+\mathrm{i}P_l)\gamma_{\lambda l}^2|^2}. \tag{12}$$

For the bound levels this gives*

$$I_{1l} = \pi\gamma_{1l}^{-2}\left(1+\gamma_{1l}^2\frac{\mathrm{d}S_l}{\mathrm{d}E}\right)_{E_{1l}}^{-1}. \tag{13}$$

Also, since

$$G_{\mathrm{G}}^2(\int\sigma)_{\lambda l}^2 = 2\pi^3(\hbar/mc^2)\{(\ln 2)/(ft)_{\lambda l}\}, \tag{14}$$

equation (10) can be written

$$A_{\lambda l}^2 = NY_{\lambda l}/Y(9\cdot59)\,I_{\lambda l}\,f_{\lambda l}, \tag{15}$$

* The formula (A19) in Barker (1969) corresponding to equation (13) contains an incorrect sign.

where

$$Y_{\lambda l} = t_{\frac{1}{2}}/t_{\lambda l} \tag{16}$$

is the branching ratio to the level λl. With $B_l = B_l^{(1)}$, equation (15) is used with equations (11) and (13) and the experimental values of $W^{\exp}$ and of the branching ratios to determine the $A_{1l}^{(1)2}$ for the bound 1^- and 3^- levels as functions of the reduced widths $\gamma_{1l}^{(1)2}$.

Values of the amplitudes A_{2l} and A_{3l} are limited by the use of model-dependent arguments about the natures of the states involved. Only levels with large α-particle reduced widths will contribute appreciably to the background terms, and in a shell model description these will be states of higher configurations, i.e. many-particle, many-hole states. On the other hand, the ^{16}N ground state is expected to be mainly $1p$–$1h$, so that Gamow–Teller matrix elements will be zero except for ^{16}O states containing $1p$–$1h$ configurations. Because the matrix elements of the Hamiltonian coupling such $1p$–$1h$ states with the many-particle, many-hole states should be small, there will be little mixing of them unless they are almost degenerate, in which case the mixed states will also be almost degenerate and will not contribute to the α-spectrum. Thus we assume that the background levels are not fed in the β-decay. Similarly the 9·59 MeV 1^- and 11·63 MeV 3^- states are believed to be mainly $3p$–$3h$ states, as compared with the 7·12 MeV 1^- and 6·13 MeV 3^- states, which are mainly $1p$–$1h$. Thus the Gamow–Teller matrix elements to the 9·59 and 11·63 MeV states should be small compared with those to the 7·12 and 6·13 MeV states.

Since the values of $A_{\lambda l}$ or $(\int \sigma)_{\lambda l}$ are dependent on the choice of B_l, we have to choose the most appropriate value of B_l at which to apply these restrictions. Thus we take for the background levels

$$A_{3l}^{(3)} = 0\,, \tag{17}$$

and expect to be small the ratios of the reduced matrix elements

$$\mathscr{R}_l \equiv (\int \sigma)_{2l}^{(2)}/(\int \sigma)_{1l}^{(1)}\,. \tag{18}$$

The fits obtained all give

$$\mathscr{R}_1 \approx 0\cdot 3 \tag{19}$$

(see Table 2), and model calculations (Brown and Green 1965; Kelson 1965) suggest a smaller ratio for the corresponding $l = 3$ states, i.e.

$$|\mathscr{R}_3| \lesssim 0\cdot 3\,. \tag{20}$$

(c) ^{12}C$(\alpha,\gamma)^{16}$O *Cross Section*

The contribution to the ^{12}C$(\alpha,\gamma)^{16}$O cross section from 1^- intermediate states of ^{16}O is calculated using the many-level R-matrix formula (Lane and Thomas 1958)

$$\sigma^{(1^-)}(\alpha\gamma) = \frac{6\pi}{k_\alpha^2} P_1 \left| \frac{\sum_{\lambda=1}^{q_1} \{\gamma_{\lambda 1} \Gamma_{\lambda\gamma}^{\frac{1}{2}}/(E_{\lambda 1} - E)\}}{1-(S_1 - B_1 + \mathrm{i}P_1) \sum_{\lambda=1}^{q_1} \{\gamma_{\lambda 1}^2/(E_{\lambda 1} - E)\}} \right|^2, \tag{21}$$

where contributions to the denominator from the γ-ray channel are neglected and $\Gamma_{\lambda\gamma}$ is the formal $E1$ γ-ray width of the 1^- level λ, say

$$\Gamma_{\lambda\gamma} = 2E_\lambda^3\gamma_{\lambda\gamma}^2 . \tag{22}$$

The measured radiation width of the $7\cdot12$ MeV level is a value of the observed width

$$\Gamma_{1\gamma}^{\mathrm{obs}} = \Gamma_{1\gamma}^{(1)}\left(1+\gamma_{11}^{(1)2}\frac{\mathrm{d}S_1}{\mathrm{d}E}\right)_{E_{11}^{(1)}}^{-1} \tag{23}$$

rather than of the formal width $\Gamma_{1\gamma}^{(1)}$, the difference being due to the energy dependence of the level shift in the α-particle channel. Fitting the measured width gives $\gamma_{1\gamma}^{(1)2}$ as a function of $\gamma_{11}^{(1)2}$.

The value of $\gamma_{3\gamma}$ is restricted by model-dependent arguments similar to those given above for the A_{3l}. The only components of the background states that will contribute to $\gamma_{3\gamma}$ will be $T = 1$, $1p$–$1h$ states, and the amount of these in the mainly $T = 0$ many-particle, many-hole background states that give the main contributions to γ_{31} is expected to be small. Thus we assume

$$\gamma_{3\gamma}^{(3)} = 0 . \tag{24}$$

Then only one free parameter remains, say $\gamma_{2\gamma}$, and fitting the measured peak cross section gives two solutions for $\gamma_{2\gamma}$ with opposite signs of $\gamma_{2\gamma}/\gamma_{1\gamma}$. Model-dependent arguments similar to those above suggest

$$\mathscr{R}_\gamma^2 \ll 1 , \tag{25}$$

where

$$\mathscr{R}_\gamma \equiv \gamma_{2\gamma}^{(2)}/\gamma_{1\gamma}^{(1)} . \tag{26}$$

For these two cases, corresponding to constructive or destructive interference in the region between the $7\cdot12$ and $9\cdot59$ MeV levels, the $^{12}C(\alpha,\gamma)^{16}O$ cross section is calculated as a function of energy down to the region of astrophysical interest.

For definiteness and without loss of generality, we assume that all $\gamma_{\lambda l}^{(1)} \geqslant 0$, $A_{1l}^{(1)} \geqslant 0$, and $\gamma_{1\gamma}^{(1)} \geqslant 0$.

IV. Results

(a) $^{12}C+\alpha$ *Scattering*

Acceptable fits to the experimental p- and f-wave phase shifts can be obtained for a wide range of channel radii. Figure 1 shows minimum values of X_l as functions of $\gamma_{1l}^{(1)2}$ for various channel radii a_l from $4\cdot5$ to $6\cdot5$ fm, for $l = 1$ and 3. The rather abrupt worsening of the fits for the smaller a_l values ($a_1 \lesssim 4\cdot75$ fm, $a_3 \lesssim 5\cdot0$ fm) is due to the necessity in these cases of imposing an extra restriction that all reduced widths should be non-negative, as there is a tendency for the background reduced width to become negative especially for larger values of $\gamma_{1l}^{(1)2}$.

For both l values it is seen that small values of $\gamma_{1l}^{(1)2}$ are favoured. For $l = 1$, this is in agreement with the Caltech phase shift analysis and with the theoretical and stripping estimates of θ_α^2 referred to in Section I, and, for $l = 3$, it agrees with estimates of $\gamma_{13}^{(1)2}$ (for $a_3 = 5\cdot43$ fm) of 0·07–0·14 MeV (Stephenson 1966), 0–0·014 MeV (Loebenstein *et al.* 1967), and 0·02–0·04 MeV (Pühlhofer *et al.* 1970). The large value of $\theta_\alpha^2 = 0\cdot71$ obtained by Clark (1969) for $a_1 = 5\cdot3$ fm appears to be due partly

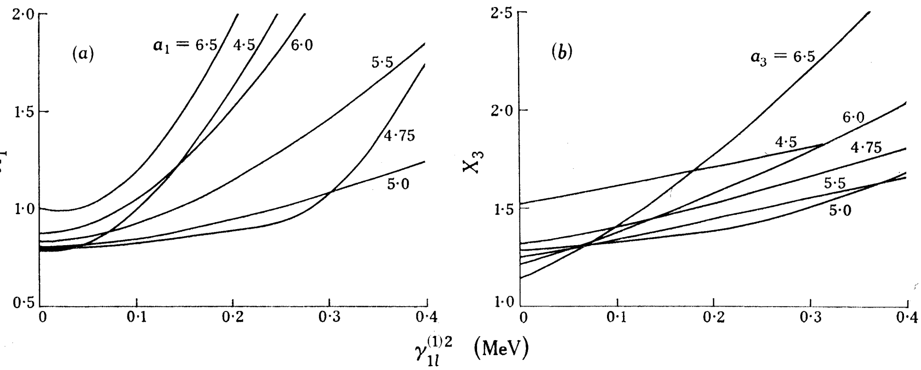

Fig. 1.—Minimum values of X_l as functions of $\gamma_{1l}^{(1)2}$ for the indicated values of a_l (in fm) for (*a*) $l = 1$ and (*b*) $l = 3$.

to his use of a one-level approximation in order to fix the energy of the 7·12 MeV level, which is not consistent with the three-level formula used in the remainder of his analysis, but mainly to the fact that his value of θ_α^2 is appropriate to $B_1 = 0$; with $B_1 = B_1^{(1)}$, the corresponding value of θ_α^2 is 0·11, giving $\gamma_{11}^{(1)2} = 0\cdot08$ MeV.

Table 1

DIMENSIONLESS α-PARTICLE REDUCED WIDTHS FOR 9·59 AND 11·63 MeV LEVELS OF ^{16}O

The values of θ^2 are from best fits to ^{12}C+α scattering phase shifts

a_l (fm)	$3\hbar^2/2\mu a_l^2$ (MeV)	α-particle Reduced Widths $\theta^2(9\cdot59)$	$\theta^2(11\cdot63)$
4·5	1·025	3·28	7·02
4·75	0·920	1·94	2·79
5·0	0·830	1·35	1·68
5·5	0·686	0·82	0·91
6·0	0·576	0·60	0·64
6·5	0·491	0·48	0·51

The best-fit value of the reduced width $\gamma_{2l}^{(2)2}$ for the 9·59 or 11·63 MeV level depends strongly on the value of a_l, and the ratio θ^2 to the sum rule limit $(3\hbar^2/2\mu a_l^2)$ increases rapidly as a_l decreases, as shown in Table 1. Since values of θ^2 much larger than unity are unlikely, we should probably exclude fits unless

$$a_l \gtrsim 5\ \mathrm{fm}. \tag{27}$$

An example of the calculated fits to the p- and f-wave phase shifts is shown in Figure 2 for the parameter values that give the best overall fit to the phase shifts and the β-decay data for $a_l = 5\cdot 5$ fm.

(b) α-spectrum from ^{16}N β-decay

Consistent fits to the $^{12}C+\alpha$ phase shifts and to the α-spectrum from ^{16}N β-decay may be obtained by using sets of parameters a_l, B_l, $E_{\lambda l}$, and $\gamma^2_{\lambda l}$ that minimize X_l, and then minimizing X_β by varying only the amplitudes $A_{\lambda l}$ in (9), subject to the restrictions (17) and (20). Since, however, some of the $l = 1$ parameters may be determined as well from the α-spectrum as from the phase shift, the following fitting procedure is used.

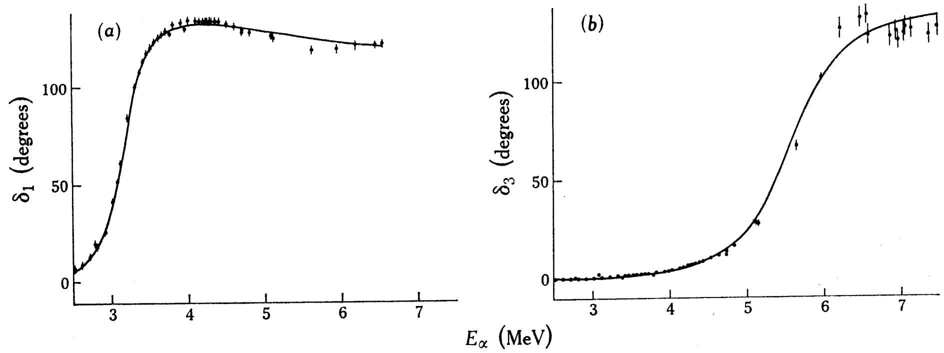

Fig. 2.—$^{12}C+\alpha$ phase shifts δ_l as functions of α-particle energy for (a) $l = 1$ and (b) $l = 3$. The points are experimental values, with energies adjusted as described in the text, and the curves are fits for $a_l = 5\cdot 5$ fm and other parameters as given in Table 2.

For given a_l (for simplicity we assume $a_1 = a_3 = a$) and given $\gamma^{(1)2}_{1l}$, an overall best fit to the phase shifts and the α-spectrum is obtained by minimizing

$$X = X_1+X_3+X_\beta, \tag{28}$$

by using as starting values sets of parameters that minimize the X_l, and varying the $E_{\lambda 1}$ and $\gamma^2_{\lambda 1}$ subject to the restrictions (6) and the $A_{\lambda l}$ subject to (17). The condition (20) is used to limit the acceptable range of values of $\gamma^{(1)2}_{13}$ for given values of a and $\gamma^{(1)2}_{11}$.

The character of the fits can be described qualitatively. For definiteness we consider parameter values for $B_l = B^{(1)}_l$. For given a, the magnitude of $A^{(1)}_{21}$ is determined by the size of the peak in the α-particle spectrum and its sign by the asymmetry in the shape of $W^{\exp}/f_\beta P_1$, which is plotted in Figure 3 for two "extreme" values of a_1. The more rapid decrease on the low-energy side indicates destructive interference in the region below the $9\cdot 59$ MeV level, that is, $A^{(1)}_{21} > 0$ (since $A^{(1)}_{11} > 0$). The magnitude of the asymmetry near the peak gives a lower limit to the value of $\gamma^{(1)2}_{11}$; an upper limit is obtained by requiring the energy at which the 1^- contribution is zero to fall below the region of observation. The best fits are obtained with the 1^- contribution small near the lower end of the observed range, so that the 3^- contribution must account for what is observed there. This leads to a relation between the values of $\gamma_{13}^{(1)2}$ and $\mathscr{R}_3$; $\mathscr{R}_3$ is negative for small $\gamma_{13}^{(1)2}$ and becomes positive as $\gamma_{13}^{1(2)}$ increases. For each value of a, the best fit is a compromise between good fits to the p-wave phase shift and the low-energy end of the α-spectrum, which favour

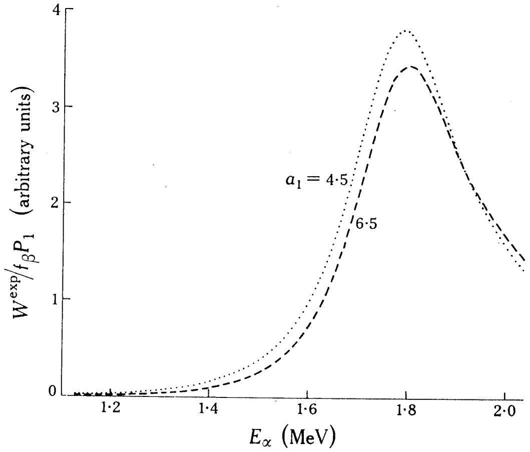

Fig. 3.—α-particle spectrum from ^{16}N β-decay, after division by the Fermi function and p-wave penetration factor, as a function of α-particle energy for $a_1 = 4{\cdot}5$ and $6{\cdot}5$ fm.

smaller values of $\gamma_{11}^{(1)2}$, and a good fit to the high-energy end of the α-spectrum, which favours larger $\gamma_{11}^{(1)2}$. The difficulty in fitting this high-energy end of the spectrum increases as a increases.

Quantitative results are given in Figure 4, which shows minimum values of X as functions of $\gamma_{11}^{(1)2}$, for various values of a and allowed values of $\mathscr{R}_3$. The best fits

Table 2

PARAMETER VALUES FOR BEST FITS TO ^{12}C+α PHASE SHIFTS AND α-SPECTRUM FROM ^{16}N β-DECAY

a (fm)	l	$B_l^{(1)}$	$E_{1l}^{(1)}$ (MeV)	$\gamma_{1l}^{(1)2}$ (MeV)	$E_{2l}^{(1)}$ (MeV)	$\gamma_{2l}^{(1)2}$ (MeV)	$E_{3l}^{(1)}$ (MeV)	$\gamma_{3l}^{(1)2}$ (MeV)	$\mathscr{R}_l$	X_l	X_β	X
4·5	1	−3·391	−0·046	0·165	4·70	0·595	1553	653	0·31	1·42	1·11	4·10
	3	−4·806	−1·030	0·041	9·24	0·531	4143	1251	−0·30	1·57		
4·75	1	−3·478	−0·046	0·146	4·18	0·489	70·4	23·3	0·31	0·98	0·42	2·79
	3	−4·905	−1·030	0·085	6·96	0·340	3244	812	−0·30	1·39		
5·0	1	−3·564	−0·046	0·111	3·80	0·413	30·8	7·45	0·31	1·08	0·44	2·84
	3	−5·004	−1·030	0·062	6·16	0·249	109·7	22·0	−0·30	1·32		
5·5	1	−3·729	−0·046	0·065	3·32	0·278	18·1	2·56	0·31	1·06	0·63	2·96
	3	−5·204	−1·030	0·022	5·56	0·198	32·5	3·75	−0·30	1·27		
6·0	1	−3·888	−0·046	0·041	3·03	0·170	16·5	1·87	0·30	1·17	0·90	3·30
	3	−5·397	−1·030	0·008	5·12	0·127	27·8	2·54	−0·30	1·23		
6·5	1	−4·042	−0·046	0·027	2·84	0·104	16·8	1·80	0·29	1·33	1·32	3·80
	3	−5·597	−1·030	0·003	4·84	0·081	27·4	2·32	−0·30	1·15		

satisfying (20) are obtained for $\mathscr{R}_3 = -0{\cdot}3$. Only slightly better fits would be obtained if more negative values of $\mathscr{R}_3$ were allowed, as the restriction $\gamma_{13}^{(1)2} \geqslant 0$ prevents $\mathscr{R}_3$ from becoming too negative. Complete sets of parameter values giving the best fits for each a value are listed in Table 2.

The best fits are obtained for a near 5 fm and give $X = X_{\min} \approx 2{\cdot}8$. The minimum value of X increases rapidly for smaller a and slowly for larger a. If the condition for an acceptable fit is taken as $X \lesssim 1\cdot5\,X_{\min} \approx 4\cdot2$, then the channel radii are restricted to

$$4\cdot5\ \text{fm} \lesssim a \lesssim 6\cdot8\ \text{fm}. \tag{29}$$

By taking account of the previous restriction (27), we get

$$5\cdot0\ \text{fm} \lesssim a \lesssim 6\cdot8\ \text{fm}. \tag{30}$$

For each value of a within this range, there is a range of acceptable values of $\gamma_{11}^{(1)2}$, which can be obtained from Figure 4 and is given in Table 3. It seems reasonable to take $a = 5\cdot5$ fm as the value giving the most satisfactory overall fit (including consideration of the parameter values). These fits for $a = 5\cdot5$ fm and the parameter values in Table 2 are shown in Figure 2 for the phase shifts and Figure 5 for the α-spectrum.

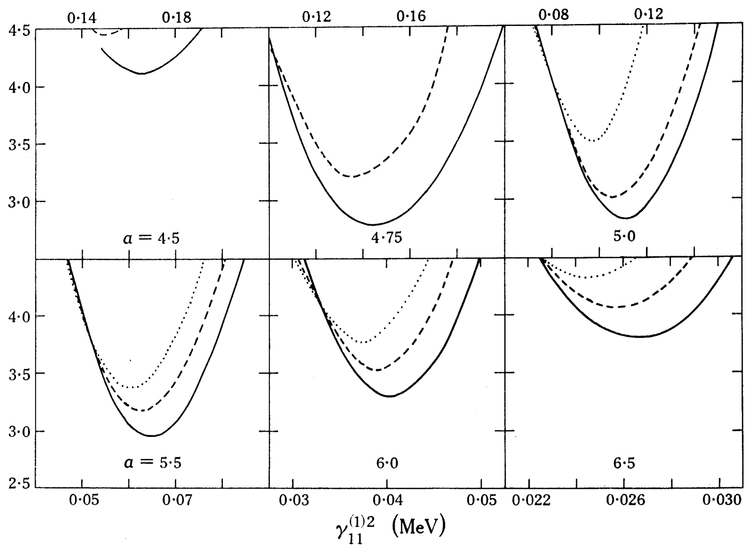

Fig. 4.—Minimum values of X as functions of $\gamma_{11}^{(1)2}$ for the indicated values of a (in fm) and $\mathscr{R}_3 = 0\cdot3$ (dotted curves), 0 (dashed curves), and $-0\cdot3$ (solid curves).

(c) $^{12}C(\alpha,\gamma)^{16}O$ *Cross Section*

The cross section is calculated as a function of energy, for the two cases of constructive and destructive interference, for various channel radii within the acceptable range (30), and for parameter values corresponding to the optimum values of $\gamma_{11}^{(1)2}$, as given in Table 2, and to the upper and lower limits of the acceptable $\gamma_{11}^{(1)2}$.

The low-energy behaviour of the cross section is most conveniently expressed in terms of the S-factor defined by

$$S = E\exp(2\pi\eta)\,\sigma^{(1^-)}(\alpha\gamma)\,, \tag{31}$$

where $\eta = 3\cdot273\,E^{-\frac{1}{2}}\ \text{MeV}^{\frac{1}{2}}$. The quantity of interest in astrophysics is the value $S(400)$ of S at $E_\alpha = 400$ keV ($E = 0\cdot3$ MeV). It is also of interest to compare the calculated cross section or S values near the $9\cdot59$ MeV peak with the measured values.

Some of the results are given in Figure 6 and some in Table 3. Figure 6 shows S as a function of E_α. The calculated curves and hatched area show the effect of constructive or destructive interference, of changes in the value of a, and of changes in the value of $\gamma_{11}^{(1)2}$. Table 3 lists values of θ_α^2 (for comparison with previous work), $\mathscr{R}_\gamma$ (defined by equation (26)), and $S(400)$, for constructive and destructive interference, for various acceptable channel radii, and for the optimum value and the upper and lower limits of the acceptable values of $\gamma_{11}^{(1)2}$.

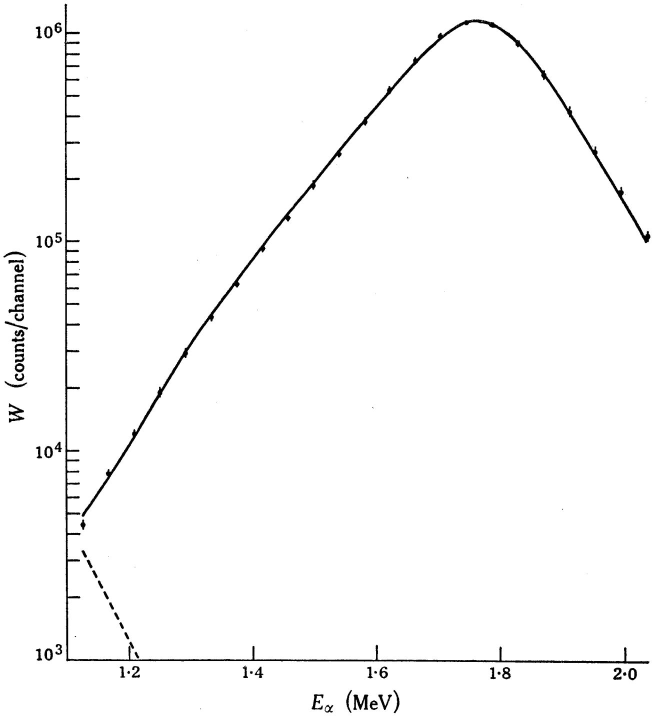

Fig. 5.—α-particle spectrum from ^{16}N β-decay as a function of α-particle energy. The points represent experimental measurements (Wäffler, personal communication) corrected for background (shown by the dashed curve) and experimental resolution, plotted for every fourth channel from channel 5 ($E_\alpha = 1\cdot125$ MeV) to channel 93 ($E_\alpha = 2\cdot040$ MeV). The error bars include a contribution from uncertainty in the α-particle energy. The solid curve is a fit for $a = 5\cdot5$ fm and other parameters as given in Table 2.

Uncertainties in the experimental radiation width of the 7·12 MeV level and maximum cross section of the 9·59 MeV peak (allowing, for example, for possible E2 contributions to the measured value; Werntz 1970) cause uncertainties in the values of $\mathscr{R}_\gamma$ and $S(400)$. The 8% uncertainty in the radiation width leads to about 4% uncertainty in $\mathscr{R}_\gamma$ and 7% in $S(400)$; a 20% uncertainty in the peak cross section gives 11% uncertainty in $\mathscr{R}_\gamma$ but less than 3% in $S(400)$. The $\mathscr{R}_\gamma$ values in Table 3 satisfy (25) moderately well.

The range of acceptable values of $S(400)$, from Table 3, is

$$0\cdot 05\ \mathrm{MeV\,.\,b} \lesssim S(400) \lesssim 0\cdot 33\ \mathrm{MeV\,.\,b}. \tag{32}$$

If $^{12}C(\alpha,\gamma)^{16}O$ cross section measurements can exclude the possibility of destructive

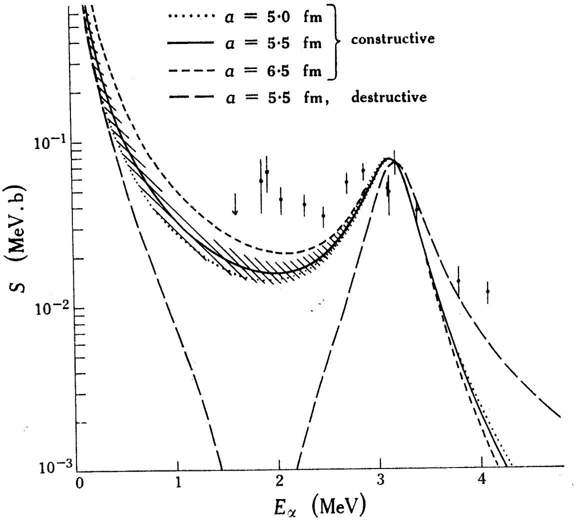

Fig. 6.—S-factor for the $^{12}C(\alpha,\gamma)^{16}O$ cross section as a function of α-particle energy. The experimental points with error bars are from Jaszczak, Gibbons, and Macklin (1970) and Jaszczak and Macklin (1970). The curves are calculated using the parameters of Table 2 for the indicated channel radii with constructive and destructive interference. The hatched area shows the effect of varying $\gamma_{11}^{(1)2}$ within its acceptable range for $a = 5\cdot 5$ fm and constructive interference.

Table 3

CALCULATED VALUES OF S-FACTOR FOR $^{12}C(\alpha,\gamma)^{16}O$ AT $E_\alpha = 400$ keV

The values of $S(400)$ are for various sets of parameter values giving acceptable fits to $^{12}C+\alpha$ phase shifts and α-spectrum from ^{16}N β-decay

a (fm)	$\gamma_{11}^{(1)2}$ (MeV)	θ_α^2	$\mathscr{R}_\gamma$ Constructive	$\mathscr{R}_\gamma$ Destructive	$S(400)$ (MeV . b) Constructive	$S(400)$ (MeV . b) Destructive
5·0	0·111	0·134	−0·47	0·49	0·121	0·073
	0·146	0·176	−0·48	0·50	0·153	0·097
	0·077	0·093	−0·45	0·47	0·088	0·050
5·5	0·065	0·095	−0·41	0·42	0·154	0·095
	0·082	0·120	−0·41	0·44	0·189	0·120
	0·049	0·071	−0·40	0·41	0·120	0·070
6·0	0·041	0·071	−0·38	0·39	0·200	0·125
	0·049	0·085	−0·38	0·39	0·235	0·151
	0·032	0·056	−0·37	0·38	0·160	0·096
6·5	0·027	0·055	−0·36	0·36	0·261	0·166
	0·030	0·061	−0·36	0·37	0·288	0·185
	0·023	0·047	−0·35	0·36	0·225	0·140

interference below the $9\cdot 59$ MeV peak, as is suggested by Figure 6, then the range is smaller,

$$0\cdot 09\ \mathrm{MeV\,.\,b} \lesssim S(400) \lesssim 0\cdot 33\ \mathrm{MeV\,.\,b}, \tag{33}$$

and the parameter values for the most satisfactory fit to the phase shifts and α-spectrum give $S(400) \approx 0\cdot 15$ MeV . b.

In astrophysical calculations the cross section has been represented by a one-level approximation corresponding to the $7\cdot12$ MeV level alone, with channel radius

$$a_1 = 1\cdot44\,(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})\ \mathrm{fm} = 5\cdot58\ \mathrm{fm}\,,$$

giving for small energies (Fowler, Caughlan, and Zimmerman 1967)

$$S = 2\cdot67\times10^6\,\theta_\alpha^2\,\Gamma_{1\gamma}^{\mathrm{obs}}(E-E_{11}^{(1)})^{-2} \quad \mathrm{MeV}^2\,.\,\mathrm{b}\,. \tag{34}$$

The value of θ_α^2 to be used in such calculations is therefore given by

$$\theta_\alpha^2 = 0\cdot72\,S(400) \quad \mathrm{MeV}^{-1}\,\mathrm{b}^{-1}\,. \tag{35}$$

V. Acknowledgments

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