SUB–COULOMB STRIPPING IN THE ^{124}Sn(d, p)^{125}Sn REACTION

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Abstract

Angular distributions for the reaction ^{124}Sn(d, p)^{125}Sn leading to the unresolved ground and 0·026 MeV levels and to the 0·210, 0·936, 1·257, 1·362, and 1·540 MeV levels of ^{125}Sn have been measured with deuteron energies of 5·1 and 12·0 MeV. Spectroscopic factors have been extracted using the DWBA theory. The dependence of the calculations on various assumptions of the theory has been studied by comparing the results at both energies. The sub-Coulomb data have been found to yield more reliable spectroscopic factors because of the lack of sensitivity to the optical model parameters used in the distorting potentials.

I. Introduction

The extraction of spectroscopic factors from the DWBA analysis of sub-Coulomb stripping data has been investigated by a number of authors (e.g. Dost and Hering 1965; Brient *et al.* 1966; Dally, Nelson, and Smith 1966; Dost, Hering, and Smith 1967; Posner 1967; Jeans *et al.* 1969; van der Merwe and Heymann 1969; Yamaya *et al.* 1969). The conclusion from this work has been that, although the dependence on optical model parameters is removed, the sensitivity of the cross section to the geometry of the bound state potential prevents accurate absolute spectroscopic factors from being obtained. More generally, the cross section is found to depend on factors which affect the wavefunctions outside the nuclear radius, which is the region where most of the overlap occurs in sub-Coulomb stripping reactions. However, stripping above the Coulomb barrier is also sensitive to these factors so that sub-Coulomb stripping may still give more accurate spectroscopic factors, even though the cross section is sensitive to surface effects. The work reported here is a study of the ^{124}Sn(d, p)^{125}Sn reaction at energies above and below the Coulomb barrier so as to compare the spectroscopic factors obtained by a DWBA analysis in both energy regions.

The Coulomb barrier for deuterons and protons incident on tin is approximately 9 MeV, providing a suitable reaction for studying stripping above and below the Coulomb barrier using a deuteron beam from an EN tandem accelerator. The Q value for the reaction is 3·506 MeV (Nealy and Sheline 1964) and, apart from the ground state and first excited state (0·026 MeV), the low lying levels are sufficiently resolved to provide a test of sub-Coulomb stripping for a range of Q values.

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Spectroscopic factors from ^{124}Sn(d, p)^{125}Sn have been reported previously by Schneid, Prakash, and Cohen (1967) with a deuteron energy of 15 MeV. Their results provide a further check on the values extracted from the sub-Coulomb data.

II. Experimental Details

The deuteron beams were provided by the A.N.U. tandem accelerator. The target was prepared by evaporation of isotopic (94·74%) ^{124}Sn onto a thin (20 μg) carbon backing. The reaction products were detected in an array of solid state counters.

Absolute cross sections were obtained in the following manner. The product of the target thickness and solid angle was measured for each detector by comparing the cross section for elastic scattering of 3 MeV deuterons with that predicted by the Rutherford scattering formula. The measurements were taken at eight forward angles and the average values and standard deviations were then used in estimating the cross sections and the errors in the (d, p) reactions.

The target thickness was estimated to be approximately 220 μg cm^{-2} and the errors in the absolute cross sections were $\pm 5\%$.

III. Experimental Results

Data were obtained at 5·1 and 12·0 MeV incident deuteron energies. Typical spectra are shown in Figure 1. Angular distributions were obtained for transitions to the following final states of ^{125}Sn: g.s.+0·026, 0·210, 0·936, 1·257, 1·362, and 1·540 MeV. The energy level positions have been taken from the work of Nealy and Sheline (1964). The angular distributions are shown in Figures 2 and 3, where the errors on the points are statistical only. The ground state and first excited state were not resolved but, since the transition to the ground state is considerably weaker than to the first excited state (Nealy and Sheline 1964), the angular distribution of the sum of the two transitions was extracted and analysed, initially ignoring the ground-state contribution. At 5·1 MeV, data forward of 90° were difficult to obtain, but sufficient points were measured for the groups of the ground-state doublet and the 0·210 MeV state to show the backward peaking typical of sub-Coulomb stripping. The "two-point" angular distribution for the protons leading to the 0·936 MeV level is a result of overlap with unidentified contaminant groups at many angles.

IV. DWBA Analysis

The data were analysed using the DWBA theory. Contributions to the cross sections from the compound nucleus process were considered to be insignificant because of the large number of open exit channels.

The DWBA computer code DWUCK (Kunz 1966) used in the analysis employs the zero-range approximation and has options to include the local energy approximation (LEA) (Bencze and Zimanyi 1964; Buttle and Goldfarb 1964; Perey and Saxon 1964) for finite range of the deuteron and non-locality in the potentials generating the distorted waves as well as the bound state potential. A spin–orbit term of the Thomas form,

$$V_{\mathrm{so}}(r) = -V_{\mathrm{R}}\frac{VSOR}{45\cdot 2}\frac{1}{r}\frac{\mathrm{d}f(X_{\mathrm{R}})}{\mathrm{d}r},$$

where

$$f(X_i) = 1 + \exp\{(r - R_{0i}A^{\frac{1}{3}})/a_i\}^{-1}$$

and *VSOR* is the usual λ factor, was included in the deuteron, proton, and neutron potentials.

The optical model parameters used by Schneid, Prakash, and Cohen (1967) in fitting (d, p) data for the tin isotopes at $E_d = 15$ MeV were taken as a starting point in the present analysis. The 12 MeV data were fitted first in an attempt to verify their tentative l_n and J^π assignments.

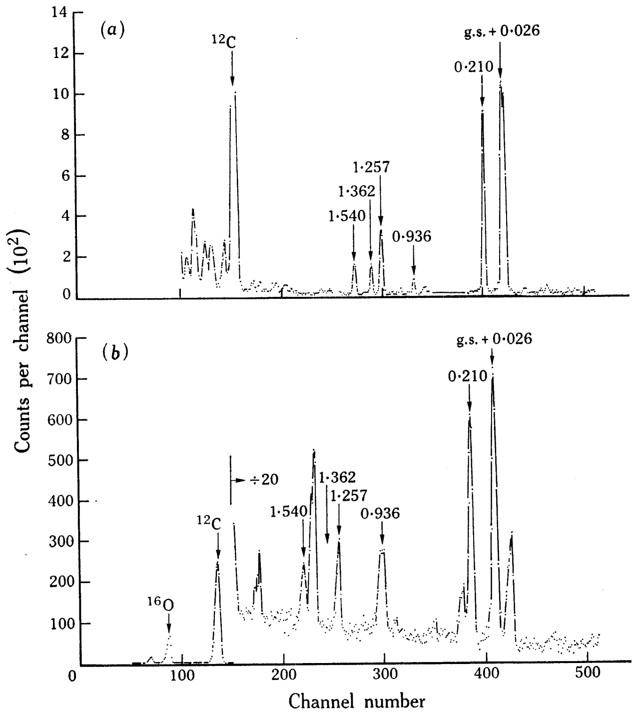

Fig. 1.—Spectra of ^{125}Sn from the reaction ^{124}Sn(d, p)^{125}Sn for (*a*) E_d = 12·0 MeV at a laboratory angle of 110° and (*b*) E_d = 5·1 MeV at 105°.

The spins and parities of the ground and first two excited states are known from β- and γ-ray studies and from inelastic proton scattering to be $11/2^-$, $3/2^+$, and $1/2^+$ respectively (Goldhaber and Hill 1952; Dubey, Mandeville, and Rothman 1956; Bursom, Blanc, and Martin 1957; Yuta and Morigana 1960; Nealy and Sheline 1964). The procedure adopted was to fit the angular distribution for the 0·210 MeV, $3s_{1/2}$ state and use the same parameters to fit the other data. The fits were obtained by varying the potential depths only and maintaining the same geometry as used by Schneid, Prakash, and Cohen (1967).

The angular distribution for the g.s.–0·026 MeV doublet was fitted using a program to optimize the fit of a sum of $2d_{3/2}$ and $1h_{11/2}$ calculations to the experimental points by varying the two spectroscopic factors. The final fits were obtained with the potentials D1 for the deuteron and P1 for the proton (see Table 1) and are shown in Figure 3, where the DWBA calculations include a non-local range of $\beta = 0 \cdot 8$ and the LEA correction for the finite range of the deuteron.

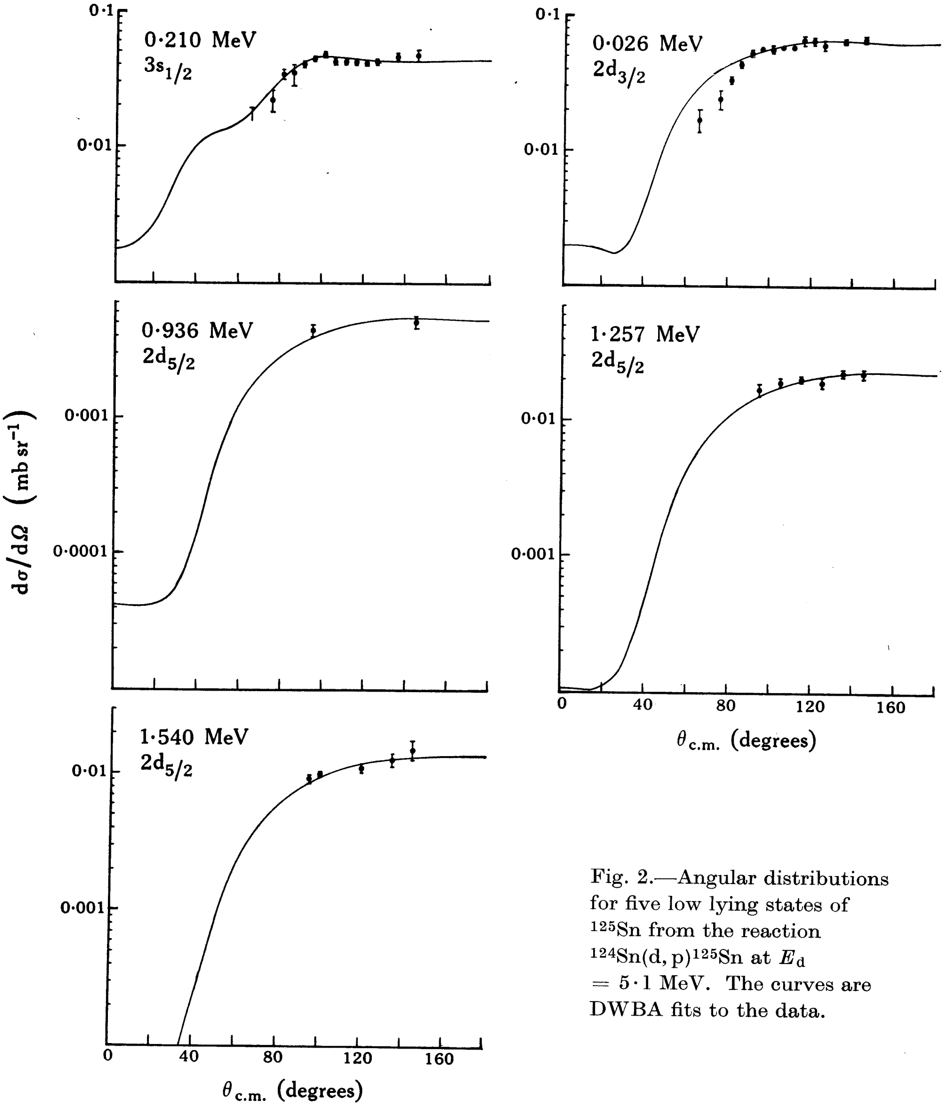

Fig. 2.—Angular distributions for five low lying states of ^{125}Sn from the reaction ^{124}Sn(d, p)^{125}Sn at $E_d = 5 \cdot 1$ MeV. The curves are DWBA fits to the data.

The angular momentum assignments are shown in Table 2. The DWBA fits to the ground-state doublet and 0·210 MeV state verify the assignments mentioned above. Cohen and Price (1961) have observed a state at approximately 0·9 MeV in essentially all of the odd tin isotopes. The regularity of occurrence, low excitation, and relatively high cross section are very suggestive of a single-particle state. However, the angular distributions of protons from these states in each of the nuclei are not consistently typical of any one l value and hence the state has been described as a non-single-particle state. Schneid, Prakash, and Cohen (1967) arrived at the same conclusion from more accurate data. Furthermore, a state at 0·90 MeV in ^{119}Sn has been reported from Coulomb excitation work (Alkhazov *et al.* 1958), suggesting a collective mode of excitation. For the present work, the cross section for the transition to the 0·936 MeV level has been observed to be comparable with that of the 1·54 MeV level,

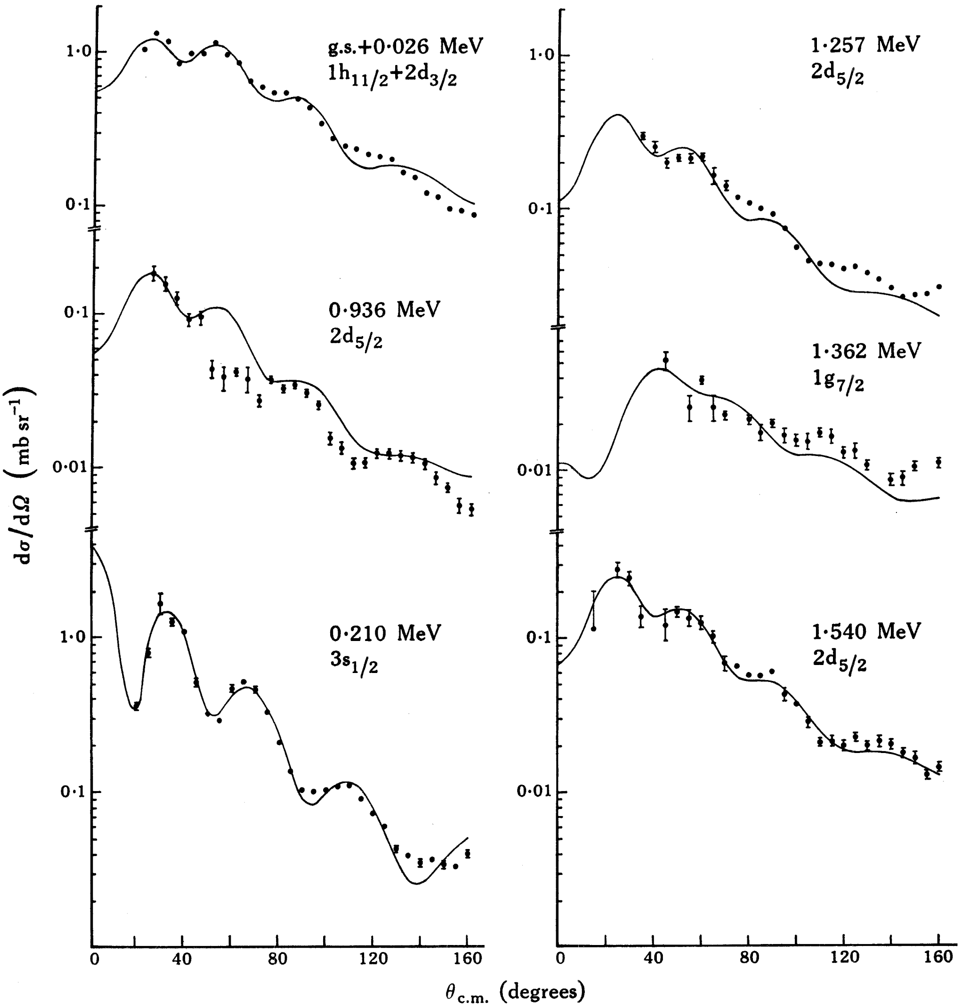

Fig. 3.—Angular distributions for the first seven states of ^{125}Sn from the reaction $^{124}Sn(d, p)^{125}Sn$ at $E_d = 12 \cdot 0$ MeV. The curves are DWBA fits to the data.

which has been successfully described by an $l = 2$ DWBA calculation, as shown in Figure 3. The calculation for the 0·936 MeV level shown in Figure 3 is for an $l = 2$ transition which gave the best fit of all possible l values. However, the fit obtained for this state was not as good as for the known single-particle states, especially around 50°. The accumulated evidence suggests that this state is not a single-particle state, but involves some other mode of excitation, perhaps collective.

The level at 1·362 MeV has been previously observed by Nealy and Sheline (1964) but was not resolved in the study by Schneid, Prakash, and Cohen (1967). The fit shown to the data for this state was calculated for an $l = 4$ ($1g_{7/2}$) transition which gave the best comparison with the experimental angular distribution. However, the assignment of $l = 4$ to the 1·362 MeV level is to be regarded as tentative because of the uncertainty in the forward angle data due to the low cross section for this transition.

TABLE 1

PARAMETER VALUES FOR FITS TO DATA IN DWBA ANALYSIS

Particle	Potential	V (MeV)	R_V (fm)	a_V (fm)	W_D (MeV)	r_W (fm)	a_W (fm)	r_C (fm)	*VSOR*
Deuteron	D1	120	1·15	0·81	54	1·34	0·68	1·115	10
	D2	120	1·15	0·81	64	1·34	0·68	1·115	10
	D3	80	1·15	0·81	64	1·34	0·68	1·115	10
	D4	100	1·15	0·81	64	1·34	0·68	1·115	10
	D5	50	1·15	0·81	54	1·34	0·68	1·115	10
Proton	P1	49	1·25	0·65	54	1·25	0·47	1·25	10
	P2	39	1·25	0·65	54	1·25	0·47	1·25	10
	P3	69	1·25	0·65	54	1·25	0·47	1·25	10
Neutron	N1	Adjusted for BE	1·25	0·65					20

The 1·257 and 1·540 MeV levels have been tentatively assigned J^π values of $5/2^+$ by Schneid, Prakash, and Cohen (1967), and this value has been used in both calculations shown in Figure 3. The theoretical predictions for $d_{3/2}$ transitions to both states differ from those for the $d_{5/2}$ in magnitude only, allowing no discrimination between either of the possible values.

TABLE 2

SPECTROSCOPIC FACTORS

Level Energies (MeV)	l_n	J^π	Spectroscopic Factor With LEA (5·1 MeV)	With LEA (12·0 MeV)	Zero Range (5·1 MeV)	Zero Range (12·0 MeV)	SPC*
0+0·026	{5, 2}	{$11/2^-$, $3/2^+$}	0·267	0·262	0·336	0·300	0·340
0	5	$11/2^-$		0·272		0·315	
0·026	2	$3/2^+$		0·164		0·212	
0·210	0	$1/2^+$	0·148	0·144	0·186	0·168	0·250
0·936	2	($5/2^+$)	0·007	0·012	0·008	0·014	
1·257	2	($5/2^+$)	0·026	0·028	0·032	0·034	0·039
1·362	4	($7/2^+$)		0·024		0·032	
1·540	2	($5/2^+$)	0·013	0·016	0·016	0·021	0·023

* From Schneid, Prakash, and Cohen (1967).

The potentials were then used in the analysis of the 5·1 MeV data together with angular momentum assignments obtained from the 12·0 MeV data. The results of the analysis are shown in Figure 2 and Table 2. The table includes the spectroscopic factors obtained with and without the LEA corrections so as to provide a comparison with the values of Schneid, Prakash, and Cohen (1967), which were obtained with the zero-range Julie code employing a lower cutoff of 6·7 fm in the radial integrals.

The spectroscopic factors derived from the higher energy data were obtained by fitting the DWBA curve to the experimental points and finding a minimum in χ^2. The values obtained were not significantly different from those derived from fitting forward angle points up to 90°.

The results for both energies are in close agreement and compare well with the work of Schneid, Prakash, and Cohen (1967).

It was not possible to obtain separate spectroscopic factors for the ground state and first excited state at 5·1 MeV because of the lack of structure in the angular distribution. Schneid, Prakash, and Cohen (1967) assume the ground-state contribution to be much weaker than that of the 0·026 MeV level and extract a spectroscopic factor for the 0·026 MeV level by comparing the DWBA calculation for the transition to that level with the experimental angular distribution of the doublet. The present values of 0·336 measured at 5·1 MeV and 0·300 at 12 MeV agree with their value of 0·34 at 15·0 MeV using the same method.

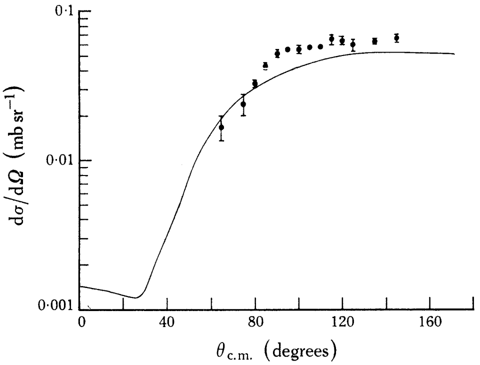

Fig. 4.—DWBA calculation for the ground state ($1h_{11/2}$) and 0·026 MeV state ($2d_{3/2}$) at 5·1 MeV using the spectroscopic factors obtained at E_d = 12·0 MeV.

In order to check the consistency of the spectroscopic factors determined by fitting two DWBA calculations to the g.s.–0·026 MeV doublet at 12 MeV, calculations at 5·1 MeV were carried out using the spectroscopic factors obtained at the higher energy. The result is shown in Figure 4 where the agreement is well within the uncertainties and supports the values obtained at 12 MeV.

V. Comparison of Stripping Above and Below the Coulomb Barrier

(a) *Effect of Optical Model Parameters*

It is well known that ambiguities exist in the deuteron optical potentials so that several discrete sets of parameters give equally good fits to the same elastic scattering data (Bassel *et al.* 1964; Halbert 1964). Each of these potentials generates a wavefunction which can be used in the DWBA analysis of appropriate stripping reactions. As well as the uncertainty due to ambiguities in the deuteron optical potential there is the problem of knowing which proton potential to use in the exit channel. Quite often elastic scattering data are not available or are unobtainable for elastic scattering from the final nucleus involved.

No search for ambiguities was attempted here, but the parameters were varied over a wide range to investigate the sensitivity of the cross section for sub-Coulomb stripping to the nuclear distortion. Figure 5 demonstrates the lack of sensitivity to the optical potentials of Table 1 for ^{124}Sn(d, p)^{125}Sn leading to the $0 \cdot 210$ MeV, $l = 0$ state which has a Q value of $3 \cdot 296$ MeV and should have the largest dependence on the optical model parameters of all the states studied. The curves are shown for up to 100% variations in well depths so as to include the possibility of ambiguous elastic scattering potentials. The same calculations were made for the transitions to the $0 \cdot 026$ and $1 \cdot 540$ MeV levels. The variation in cross section with the parameters was largest for the $0 \cdot 210$ MeV state, being always less than 10% for angles larger than 110°. Changes in the real well depth for the proton had the largest effect on the shape of the curve, emphasizing the need to ensure that the outgoing protons are

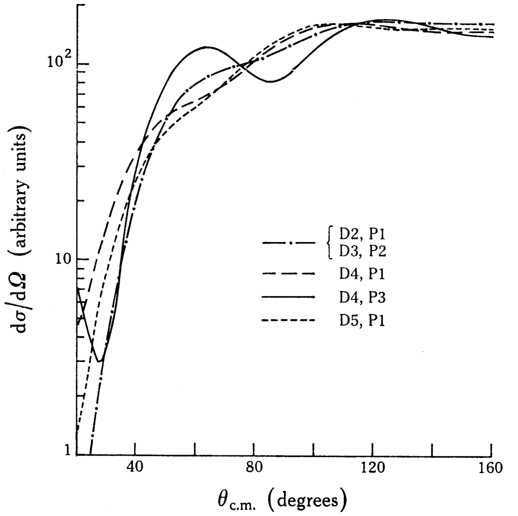

Fig. 5.—DWBA calculations for the $0 \cdot 210$ MeV state ($3s_{1/2}$) using the optical model potentials of Table 1.

sufficiently below the Coulomb barrier in a sub-Coulomb stripping experiment. The recent work of van der Merwe and Heymann (1969) on ^{208}Pb(d, d)^{208}Pb and ^{208}Pb(d, p)^{209}Pb verifies that spectroscopic factors obtained from DWBA analysis of the (d, p) reaction above the Coulomb barrier can vary by up to 100% using the different optical potentials which fit the elastic scattering data, whereas, below the Coulomb barrier, the variation is less than 5%.

(*b*) *Geometry of Bound State Potential*

The effect on the spectroscopic factor of varying the bound state potential geometry for stripping above and below the Coulomb barrier was investigated. The results are shown in Figure 6, where the cross section has been plotted for three values of r_n and with $a_n = 0 \cdot 65$ fm for the $0 \cdot 210$ MeV, $l = 0$, $1 \cdot 540$ MeV, $l = 2$, and the ground state, $l = 5$ transitions. The open circles are for $5 \cdot 1$ MeV incident deuterons and the full circles are for $12 \cdot 0$ MeV. The $r_n = 1 \cdot 25$ fm point at $12 \cdot 0$ MeV has been normalized to the same value as the $5 \cdot 1$ MeV cross section to allow easy comparison. The conclusion is that the variation of the spectroscopic factor with the geometry of the bound state potential is the same for incident deuteron energies above and below the Coulomb barrier.

(c) *Spin–Orbit Coupling in Bound State Potential*

It has been suggested that spin–orbit coupling in the bound state potential affects the cross section and therefore the spectroscopic factor, thereby preventing the extraction of absolute spectroscopic factors from sub-Coulomb stripping data (Dost, Hering, and Smith 1967).

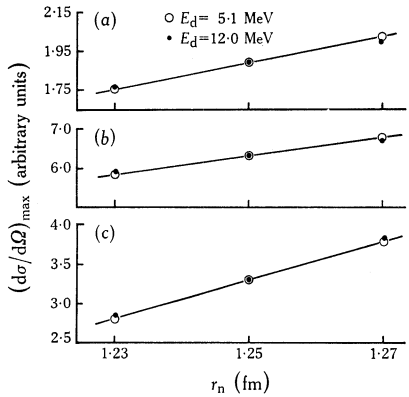

Fig. 6.—Variation of the peak cross section with the bound state radius r_n for a diffuseness $a_n = 0 \cdot 65$ fm for the transitions:
(*a*) 0·210 MeV, $l_n = 0$
(*b*) 1·540 MeV, $l_n = 2$
(*c*) g.s., $l_n = 5$

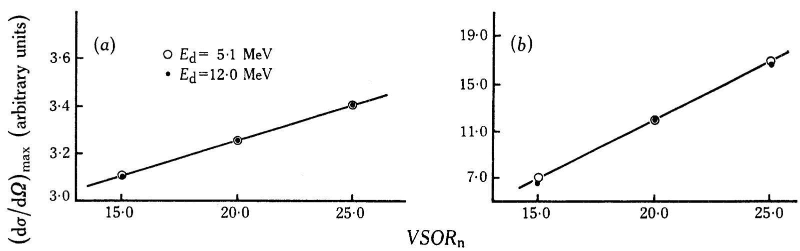

Fig. 7.—Variation of the peak cross section with the spin–orbit strength $VSOR_n$ in the bound state potential for (*a*) the ground state, $l_n = 5$ and (*b*) the 1·54 MeV, $l_n = 2$ transitions.

The effect of varying the spin–orbit strength for both energies was investigated and the results are shown graphically in Figure 7. The variations in the cross section for the $l = 5$ transition to the ground state and the $l = 2$ transition to the 1·54 MeV level for three values of the spin–orbit strength are shown. Once again the effect is the same for energies above and below the Coulomb barrier.

(d) *Inclusion of LEA*

The local energy approximation appears to be good for finite range of the deuteron and non-locality in the potentials generating the wavefunctions for a DWBA calculation (Halbert 1964). The effect of non-locality in the bound state potential is expected to be more important in reactions which occur at the nuclear surface. This is because the correction factor for non-locality dampens the wavefunctions in the nuclear interior, and to maintain the overall normalization the wavefunction in the exterior is increased. The correction should be particularly important in sub-Coulomb stripping because the major contribution to the overlap integral comes from a region outside the nucleus. An investigation of the effect of including the LEA in calculations for stripping reactions above and below the Coulomb barrier is shown in Figure 8. The percentage changes in the cross section for both 12 and $5 \cdot 1$ MeV were found to be approximately the same when:

(1) the non-locality of the bound state was corrected for using the LEA with $\beta = 0 \cdot 8$;

(2) LEA corrections for non-locality in the bound state potential and for finite range were included;

(3) LEA corrections for non-locality in all potentials and for finite range were included.

Figure 8 shows the results of calculations for the ground state, $l = 5$ and $1 \cdot 54$ MeV, $l = 2$ transitions.

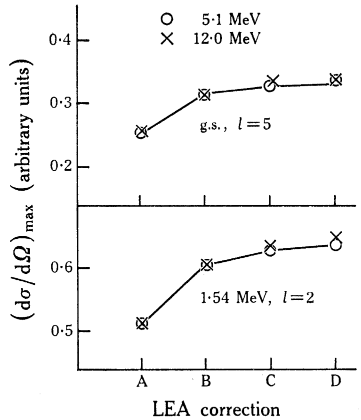

Fig. 8.—Variation of the peak cross section with the inclusion of different LEA corrections:

A, zero range;

B, non-locality in the bound state ($\beta = 0 \cdot 8$);

C, non-locality in the bound state and LEA correction for finite range;

D, non-locality in all potentials and LEA correction for finite range.

A similar result was obtained for the $0 \cdot 210$ MeV, $l = 0$ transition.

VI. Conclusions

The study of the $^{124}Sn(d, p)^{125}Sn$ reaction with 12 MeV deuterons has confirmed orbital angular momentum values for five low lying states of ^{125}Sn and suggested values for two other states. These values have been used in the analysis of data taken with $5 \cdot 1$ MeV deuterons in order to compare the extraction of spectroscopic factors from deuteron stripping reactions with incident energies above and below the Coulomb barrier. The spectroscopic factors obtained at both energies are in close agreement and compare well with measurements from a previous work at $15 \cdot 0$ MeV.

From the comparison of the dependence of the spectroscopic factors on various assumptions in the calculations at both energies that has been carried out, it has been seen that wide variations in the optical model parameters used in calculating the distorted waves have negligible effect on the spectroscopic factors derived from the sub-Coulomb stripping data, whereas at the higher energy such variations result in large discrepancies in the spectroscopic factors. The dependence of the spectroscopic factor on the geometry of the bound state potential, the inclusion of spin–orbit coupling in the bound state potential, and the inclusion of the local energy approximation for non-locality in the optical and bound state potentials and for the finite range of the deuteron has been found to be the same for both 5·1 and 12·0 MeV incident deuterons for $l = 0$, $l = 2$, and $l = 5$ transitions.

The conclusion is that more reliable absolute and relative spectroscopic factors can be obtained from stripping reactions when the incident deuteron energy is below the Coulomb barrier because of the lack of sensitivity to the parameters of the distorting potentials, even for Q values up to 3·5 MeV. The low cross sections for sub-Coulomb stripping reactions make the collection of data slow, but one reliable value of the cross section beyond 120° should be sufficient to extract a spectroscopic factor, compared with the need for a complete angular distribution for energies above the Coulomb barrier.

VII. Acknowledgments

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