SHORT COMMUNICATION

THE EFFECTS OF CYLINDRICAL GEOMETRY ON NORMAL IONIZING SHOCK WAVES*

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Most of the theoretical work on the characteristics of normal ionizing shock waves, as exemplified by the work of Taussig (1967), has been restricted to a one-dimensional model. Only Woods (1965) has attempted to include the effects of cylindrical geometry. However, the complexity of the problem in cylindrical geometry forced Woods to ignore the radial dependence of the azimuthal drive field b_θ, which he took to be given by the relation (his equation (3))

$$b_\theta = \mu_0 I/2\pi r_0 \,,$$

where I is the total current passing through the shock front and r_0 is an "effective outer radius" which Woods took to be $\sim 90\%$ of the shock tube radius.

Because of these limitations in theory, many experimental investigations have been performed in coaxial shock tubes with electrodes of large radii and small separation in an effort to minimize the effects of the radial dependence of the various parameters and thus more closely to approach the conditions of the one-dimensional theoretical calculations. Experiments of this type include those by Patrick and Pugh (1965), Heywood (1966), and Miller (1967).

Another common type of shock tube, used in studies of ionizing shock waves by Brennan *et al.* (1963), Blackburn, Brennan, and Fletcher (1969), and Cross, James, and Watson-Munro (1969), has a short centre electrode and, generally, a large electrode separation. Blackburn, Brennan, and Fletcher (1969) refer to this electrode arrangement as the "hybrid configuration". These authors also investigated the performance of the "short electrode configuration", in which both electrodes extend only a few centimetres into the tube, and found that the performance of this type of configuration was identical to that of the hybrid type.

In comparing the predictions of the one-dimensional theories with the results of experiments in the various types of cylindrical shock tubes it has been assumed that the effective drive field can be taken to be the *average* azimuthal magnetic field defined by the relation

$$\bar{b}_\theta = \frac{1}{b-a}\int_a^b \frac{\mu_0 I \,\mathrm{d}r}{2\pi r}\,,$$

where I is the total current driving the shock and a and b are the inner and outer electrode radii respectively (see e.g. Cross, James, and Watson-Munro 1969). The

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present work was undertaken to test the validity of this assumption by observing the shock velocity for a number of electrode radii ratios and separations for both the coaxial and the hybrid electrode configurations.

The experiments were performed in the plasma source FPS I (Flinders Plasma Source I) described by Blackburn, Brennan, and Fletcher (1969). In this device the ionizing shock is produced by an approximately square current pulse of up to 40 kA flowing between the two electrodes in the presence of an axial magnetic field of up to 8 kG. The present experiments were performed in hydrogen at an initial neutral gas pressure of 150 mtorr.

The ratio of the electrode radii and the separation were varied by using a number of centre electrodes. For the coaxial configuration, centre electrode radii of 0·25, 0·50, 0·95, 1·90, and 3·15 cm were used, while centre electrode radii of 1·0, 2·0, and 3·0 cm were used for the hybrid configuration. The outer electrode radius was 3·8 cm in all cases. The shock tube was operated with the centre electrode positive.

The arrival time of the shock front at a particular axial position and the magnitude of the azimuthal magnetic drive field were determined by using a small pickup coil, oriented so as to be sensitive only to the b_θ component of the field. The coil was housed in a glass tube of 3 mm diameter and inserted along a line parallel to the axis of the machine and at a radial position approximately midway between the inner and outer electrodes. Earlier experiments by Sharp and Watson-Munro (1964) and Blackburn, Brennan, and Fletcher (1969) have shown that, at the moderate drive currents used in the present experiments, the drive current flows into the shock within a radius very close to the inner electrode radius, thus producing an r^{-1} dependence for the azimuthal field. It was only at high drive currents, in the vicinity of 100 kA, that Cross *et al.* (1968) found a more complex current flow pattern.

Some information on the curvature of the front was obtained, for three of the electrode systems, by inserting a second probe into the plasma at a different radial position. Both probes were calibrated *in situ* by passing a known current through a central electrode in the absence of any plasma.

The shock speed was determined for a variety of shock drive currents and axial magnetic fields. For each condition the arrival time of the shock at five different axial positions was determined.

Results

Before a detailed investigation of the dependence of shock velocity on the average drive field could be made it was necessary to determine whether the shock front was indeed in an equilibrium condition with all points on the front propagating at the same axial velocity. Physical limitations imposed by the construction of the shock tube and the probe assemblies precluded a complete investigation of this question. Instead, for three electrode configurations, a comparison was made of the velocity of two points on the front separated by a radial distance of 1–2 cm. In each case the velocities of the two points on the front were the same within experimental error.

The present measurements also yield some information on the inclination of the shock front to the tube axis. For the two coaxial cases the front was observed to arrive first at the probe closer to the centre of the tube. For example, for the coaxial configuration with an inner electrode radius of 0·5 cm and a shock velocity of 8·4 cm μs^{-1} there was a 0·35 μs difference in the arrival times at two probes separated by a radial distance of 1·5 cm. For the hybrid configuration and a centre electrode radius a of 1·0 cm, the time delay for a shock velocity of 3·0 cm μs^{-1} and a probe separation of 1·0 cm was 0·40 μs with the front arriving at the outer probe first.

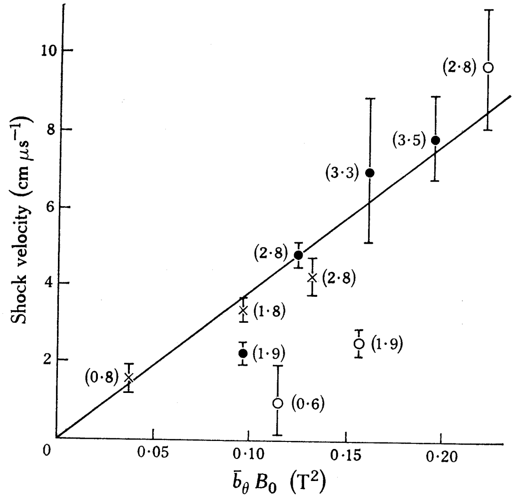

Fig. 1.—Variation of the shock velocity with the product of the axial magnetic field B_0 and the average azimuthal drive field $\bar{b}_\theta$ for the configurations:

●, coaxial at 6 kG;

o, coaxial at 8 kG;

x, hybrid at 8 kG.

The results are for hydrogen at 150 mtorr and the values in parentheses at the points indicate the electrode separations in centimetres.

The dependence of shock velocity on the average field $\bar{b}_\theta$ was investigated for the hybrid configuration at an axial field B_0 of 8 kG and for the coaxial configuration at fields of 6 and 8 kG. Previous measurements, such as those by Brennan *et al.* (1963), have shown that, for the rather limited range of B_0 and $\bar{b}_\theta$ used in the present experiment, the shock velocity is approximately proportional to the product $\bar{b}_\theta B_0$. The observed shock velocities are shown as a function of this product in Figure 1.

Discussion

The observations of the front arrival times at two radially separated probes show that the ionizing shock front is not, in general, entirely normal to the axial magnetic field. As perhaps might be expected, this inclination of the shock front is not observed in tubes of larger diameter, where the variation of b_θ over the shock front is much less pronounced (see e.g. Cross, James, and Watson-Munro 1969).

Fishman and Petschek (1962) and Pert (1968) have calculated the shock curvature arising from the radial nonuniformity in b_θ for a coaxial shock tube in the absence of an axial magnetic field. Although the interaction of the axial field with the current in the curved front may render these calculations invalid for the present experiments, it is interesting to note that the time delays predicted for the shock arrival times at the two probe positions are close to the observed values. Clearly, a more detailed theory of shock curvature including the effects of the axial field is required before this detail can be included in the description of ionizing shock waves in cylindrical geometry.

Two facts emerge from the results presented in Figure 1. Firstly, except for very small electrode separations, the shock speed is proportional to the product of the average azimuthal drive field and the axial magnetic field; the average azimuthal drive field is indeed the appropriate drive field to use for cylindrical shock tubes. Secondly, for coaxial configurations with an electrode spacing of less than 2 cm, the measured shock velocity is much less than the predicted value. This latter effect does not occur for small spacings in the hybrid geometry. The decrease in shock velocity for small electrode separations in the coaxial geometry is presumably due to the increasing influence of boundary layer effects for small separations. Electrode boundary layer effects have not been considered in the theoretical treatments of normal ionizing shocks. It is therefore not possible, at this stage, to identify the mechanisms primarily responsible for the effects observed in the present work.

Finally, it is noted that Sharp and Watson-Munro (1964) measured the radial variation in the azimuthal flow velocity following the passage of a normal ionizing shock in a tube with the hybrid electrode configuration. The experimental conditions, namely gas pressure, axial magnetic field, and azimuthal drive field, were similar to those used in the present work. Their measurements showed a sharp decrease in azimuthal flow velocity for points within about 2 cm of the outer electrode. No corresponding anomaly was observed near the centre electrode. Apparently, the boundary layer effects observed in the present work result from processes occurring at the outer, negative electrode.

Acknowledgments

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