# PROPAGATION OF ELECTROMAGNETIC WAVES IN A WEAKLY IONIZED WARM MAGNETOPLASMA

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*Abstract*

Modified wave equations for a weakly ionized warm magnetoplasma are obtained by linearizing Maxwell's equations with the help of the equation of state and the conservation equations for mass, momentum, and energy. The expressions derived for the refractive index for oblique and normal wave propagation from solutions of the equation of motion and from the modified wave equations are found to be equivalent. The results are used to study wave propagation in the ionosphere and the magnetosphere.

## I. Introduction

During the past decade a number of papers dealing with the propagation of electromagnetic waves in warm stationary plasmas have been published (e.g. Allis, Buchsbaum, and Bers 1963; Seshadri 1964; Unz 1965*a*, 1965*b*, 1967). To generalize the magnetoionic theory for such media, these authors have obtained an expression for the refractive index and have explored the possibility of propagation of electromagnetic waves. In their papers they have assumed that the hot plasmas may be described from a macroscopic point of view in terms of the fluid continuum theory of plasma dynamics and have discussed propagation of the electromagnetic waves for oblique and normal incidence. Mishra, Shukla, and Misra (1972) have obtained coupled wave equations describing propagation in a weakly ionized warm magnetoplasma for the case of normal incidence. Expressions for refractive index have been obtained from the solution of modified wave equations.

In the present paper, we consider oblique and normal propagations of electromagnetic and electroacoustic waves in a warm magnetoplasma. In order to describe the propagation of these waves we obtain an expression for the refractive index from the solution of a cubic equation as well as from the solution of a modified wave equation. A comparative study between the two approaches shows that the results obtained are in agreement. The Booker quartic equation is also derived with the help of the refractive index equation thus obtained.

## II. Theoretical Considerations

Let us consider an electron plasma permeated by an external magnetic field $\boldsymbol{H}_0(r,t)$. We assume that in this medium the macroscopic equations of plasma dynamics are valid and that the plasma is unbounded, neutral with no net static electric field,

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and obeys adiabatic conditions. The basic equations can then be written as follows. Maxwell's equations:

$$\nabla \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0 \boldsymbol{H}, \qquad \nabla \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0 \boldsymbol{E} - eN_0 \boldsymbol{v}; \tag{1a, b}$$

the force equation:

$$mN_0(\nu+\mathrm{i}\omega)\boldsymbol{v} = -\nabla p - eN_0 \boldsymbol{E} - eN_0 \mu_0(\boldsymbol{v}\times \boldsymbol{H}_0); \tag{2}$$

the conservation equations of mass and energy and the equation of state:

$$\frac{N}{N_0} = \frac{p}{\gamma p_0} = \frac{T}{(\gamma-1)T_0} = -\frac{\nabla \cdot \boldsymbol{v}}{\mathrm{i}\omega} = -\frac{\epsilon_0 \nabla \cdot \boldsymbol{E}}{eN_0}; \tag{3}$$

where $\mu_0$ and $\epsilon_0$ are the permeability and dielectric constant of free space, $\boldsymbol{v}$ and $N_0$ are the velocity and average number density of the electrons respectively, $\nu$ is the collision frequency, and $\gamma$ is the ratio of the specific heat at constant pressure to the specific heat at constant volume of the electron gas.

Let us define

$$k_1^2 = \omega^2/a^2 = \omega^2 mN_0/\gamma p_0, \tag{4}$$

where $a$ is the acoustic velocity in the electron gas. One may obtain from equations (3) and (4)

$$p = -(\gamma p_0/\mathrm{i}\omega)\nabla \cdot \boldsymbol{v} = \mathrm{i}\omega mN_0 k_1^{-2} \nabla \cdot \boldsymbol{v}. \tag{5}$$

Substituting (5) into (2) then gives

$$U\boldsymbol{v} = -k_1^{-2} \nabla(\nabla \cdot \boldsymbol{v}) + \mathrm{i}e\boldsymbol{E}/\omega m + \boldsymbol{v}\times \mathrm{i}\boldsymbol{Y}, \tag{6}$$

where we have defined

$$U = 1-\mathrm{i}Z = 1-\mathrm{i}\nu/\omega, \qquad \boldsymbol{Y} = e\mu_0 \boldsymbol{H}_0/m\omega.$$

Taking the curl of equation (1a) and using the value of curl $\boldsymbol{H}$ from (1b), we obtain

$$\nabla \times \nabla \times \boldsymbol{E} - k_0^2 \boldsymbol{E} = \mathrm{i}\omega\mu_0 eN_0 \boldsymbol{v}, \tag{7}$$

where $k_0 = \omega/c$ is the electromagnetic wave number. Substituting the value of $\boldsymbol{v}$ from (7) into (6) and defining $X = \omega_\mathrm{p}^2/\omega^2$, we then have

$$-\frac{\nabla\times\nabla\times\boldsymbol{E}}{k_0^2} + \frac{\nabla(\nabla \cdot \boldsymbol{E})}{Uk_1^2} + \frac{(U-X)\boldsymbol{E}}{U} + \frac{(\nabla\times\nabla\times\boldsymbol{E} - k_0^2\boldsymbol{E})\times\mathrm{i}\boldsymbol{Y}}{Uk_0^2} = 0. \tag{8}$$

Equation (8) represents the three-dimensional wave equation for a homogeneous compressible warm plasma in the presence of an external magnetic field. If we consider propagation when the external magnetic field is applied in the direction of the magnetic field of the electromagnetic waves, equation (8) after some rearrangement may be written as

$$-\frac{\nabla\times\nabla\times\boldsymbol{E}}{k_0^2}+\frac{\nabla(\nabla\,.\,\boldsymbol{E})}{U'k_1^2}+\left(1-\frac{X}{U'}\right)\boldsymbol{E}=0\,, \tag{9}$$

where

$$U'=1-\mathrm{i}Z-\mathrm{i}Z'=1-\mathrm{i}(Z+Z')\,,\qquad Z'=\omega_H/\omega\,.$$

## III. Oblique Wave Propagation

### (a) *Equation of Motion*

To consider the propagation of oblique waves in a warm magnetoplasma, we define the field variations in a rectangular coordinate system as

$$\boldsymbol{E}=\boldsymbol{E}_0\exp(-\mathrm{i}k_0\,\boldsymbol{n}\,.\,\boldsymbol{r})\,, \tag{10}$$

with similar variations for the other parameters $\boldsymbol{H}$, $p$, $N$, $T$, and $\boldsymbol{v}$. In equation (10) the term $\boldsymbol{n}\,.\,\boldsymbol{r}$ is defined as

$$\boldsymbol{n}\,.\,\boldsymbol{r}=s_1x+s_2y+qz \tag{11}$$

and thus the expression for the refractive index $n$ can be written as

$$|\,n\,|^2=n^2=c^2/v_{\mathrm{p}}^2=s_1^2+s_2^2+q^2\,, \tag{12}$$

where $v_{\mathrm{p}}$ is the phase velocity of the plasma waves. Further, from equation (10),

$$\nabla\,.\,\boldsymbol{E}=-\mathrm{i}k_0\,\boldsymbol{n}\,.\,\boldsymbol{E}\,,\qquad \nabla\times\boldsymbol{E}=-\mathrm{i}k_0\,\boldsymbol{n}\times\boldsymbol{E}\,. \tag{13}$$

Hence, combining equations (9), (10), and (13) and using the vector identity

$$\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{E})=(\boldsymbol{n}\,.\,\boldsymbol{E})\boldsymbol{n}-n^2\boldsymbol{E}\,,$$

we have

$$U'(1-n^2-X/U')\boldsymbol{E}+(U'-a^2/c^2)(\boldsymbol{n}\,.\,\boldsymbol{E})\boldsymbol{n}=0\,. \tag{14}$$

After some rearrangement equation (14) may be written in matrix form as

$$\begin{pmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & \alpha_{22} & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & \alpha_{33}\end{pmatrix}\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}=0\,, \tag{15}$$

where

$$\alpha_{11}=U'(1-n^2-X/U')+(U'-a^2/c^2)s_1^2\,, \tag{16a}$$

$$\alpha_{21}=\alpha_{12}=(U'-a^2/c^2)s_1\,s_2\,, \tag{16b}$$

$$\alpha_{31}=\alpha_{13}=(U'-a^2/c)^2s_1\,q\,, \tag{16c}$$

$$\alpha_{22}=U'(1-n^2-X/U')+U'(U'-a^2/c^2)s_2^2\,, \tag{16d}$$

$$\alpha_{32}=\alpha_{23}=(U'-a^2/c^2)s_2\,q\,, \tag{16e}$$

$$\alpha_{33}=U'(1-n^2-X/U')+(U'-a^2/c^2)q^2\,. \tag{16f}$$

### (*b*) *Refractive Index*

In order to find an expression for the refractive index when the wave is propagated obliquely, we equate the determinant of the coefficients of $\boldsymbol{E}$ in equation (15) to zero:

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix} = 0\,. \tag{17}$$

Solving equation (17) with the help of the boundary conditions, we obtain the cubic equation in $q$

$$\alpha_1 q^3 + \alpha_2 q^2 + \alpha_3 q + \alpha_0 = 0\,, \tag{18}$$

where

$$\alpha_1 = -\delta U'^2\,, \qquad \delta = a^2/c^2\,, \tag{19a}$$

$$\alpha_2 = \{U'^3 - XU'^2\} + \delta\{2c'^2U'^2 - 2XU' - U'^2(1-c'^2)\}\,, \tag{19b}$$

$$\alpha_3 = 2\{c'^2U' - X\}\{U'(X-U')\} + \delta\{c'^2U' - X\}\{2U'(1-c'^2) - (c'^2U' - X)\}\,, \tag{19c}$$

$$\alpha_0 = \{c'^2U' - X\}\{(c'^2U' - X)(U' - X)\} - \delta(1-c'^2)(c'^2U' - X)^2\,, \tag{19d}$$

with

$$c'^2 = 1 - s_1^2 - s_2^2\,. \tag{20}$$

From equations (12) and (20) the refractive index may be written as

$$n^2 = 1 - c'^2 + q^2\,. \tag{21}$$

Simplifying equation (18) with the help of (21),

$$\{U'q^2 - (U'c'^2 - X)\}^2\{\delta(q^2 + 1 - c'^2) - (U' - X)\} = 0\,. \tag{22}$$

The solution of equation (22) for the complex refractive index can then be expressed as

$$n_{1,2}^2 = 1 - X/U'\,, \qquad n_3^2 = (U' - X)c^2/a^2\,. \tag{23a, b}$$

Since it is well known that the refractive index for electroacoustic waves is not affected by an external magnetic field, equation (23b) becomes

$$n_3^2 = (U - X)c^2/a^2\,. \tag{24}$$

Thus an expression for the refractive index for electromagnetic waves in the presence of an external magnetic field has been obtained in the form

$$n^2 = 1 - X/\{1 - \mathrm{i}(Z + Z')\}\,. \tag{25}$$

Since $n = \mu - \mathrm{i}\psi$, where $\mu$ is the real part of the refractive index, which exhibits the propagation characteristics of the electromagnetic waves, and $\psi$ is the absorption coefficient, equation (25) can be written as

$$\mu^2-\psi^2-2\mathrm{i}\mu\psi = 1-X/\{1-\mathrm{i}(Z+Z')\}. \tag{26}$$

Separating real and imaginary parts we have

$$\mu^2-\psi^2 = 1-\frac{X}{1+(Z+Z')^2}, \qquad 2\mu\psi = \frac{X(Z+Z')}{1+(Z+Z')^2}, \tag{27}$$

Solution of equations (27) for the refractive index $\mu$ yields the expression

$$\mu^2 = \frac{1}{2}\left[1-\frac{X}{1+\bar{Z}^2}\pm\left\{\left(1-\frac{X}{1+\bar{Z}^2}\right)^2+\left(\frac{X\bar{Z}}{1+\bar{Z}^2}\right)^2\right\}^{\frac{1}{2}}\right], \tag{28}$$

where $\bar{Z} = Z+Z'$.

There are two particular cases to consider:

(i) When there is no steady magnetic field $\omega_H = 0$ and thus $Z' = \omega_H/\omega = 0$ and equation (28) reduces to

$$\mu^2 = \frac{1}{2}\left[1-\frac{X}{1+Z^2}\pm\left\{\left(1-\frac{X}{1+Z^2}\right)^2+\frac{X^2Z^2}{(1+Z^2)^2}\right\}^{\frac{1}{2}}\right]. \tag{29}$$

(ii) At high frequencies, collisions may be neglected and, for $\nu = 0$, $Z = \nu/\omega = 0$. Equation (28) then becomes

$$\mu^2 = \frac{1}{2}\left[1-\frac{X}{1+Z'^2}\pm\left\{\left(1-\frac{X}{1+Z'^2}\right)^2+\left(\frac{XZ'}{1+Z'^2}\right)^2\right\}^{\frac{1}{2}}\right]. \tag{30}$$

## IV. Solution of Modified Wave Equations

Let the electric field be separated into two modes as

$$\boldsymbol{E} = \boldsymbol{E}_{\mathrm{T}}+\boldsymbol{E}_{\mathrm{L}}, \tag{31}$$

with

$$\nabla\cdot\boldsymbol{E}_{\mathrm{T}} = 0, \qquad \nabla\times\boldsymbol{E}_{\mathrm{L}} = 0,$$

$$\nabla\times\nabla\times\boldsymbol{E}_{\mathrm{T}} = -\nabla^2\boldsymbol{E}_{\mathrm{T}}, \qquad \nabla(\nabla\cdot\boldsymbol{E}_{\mathrm{L}}) = \nabla^2\boldsymbol{E}_{\mathrm{L}},$$

where $\boldsymbol{E}_{\mathrm{T}}$ and $\boldsymbol{E}_{\mathrm{L}}$ are the electric field components of the electromagnetic wave and the longitudinal acoustic wave respectively. Substituting equation (31) into (9) we can write the coupled wave equations

$$\nabla^2\boldsymbol{E}_{\mathrm{T}}+k_0^2(1-X/U')\boldsymbol{E}_{\mathrm{T}} = 0, \qquad \nabla^2\boldsymbol{E}_{\mathrm{L}}+k_1^2(U'-X)\boldsymbol{E}_{\mathrm{L}} = 0. \tag{32a, b}$$

In order to study the refractive index of a warm magnetoplasma, we rewrite equations (32) in the form

$$\frac{\partial^2\boldsymbol{E}_{\mathrm{T}}}{\partial x^2}+\frac{\partial^2\boldsymbol{E}_{\mathrm{T}}}{\partial y^2}+k_0^2(1-X/U')\boldsymbol{E}_{\mathrm{T}} = 0, \qquad \frac{\partial^2\boldsymbol{E}_{\mathrm{L}}}{\partial z^2}+k_1^2(U'-X)\boldsymbol{E}_{\mathrm{L}} = 0. \tag{33a, b}$$

These modified coupled wave equations can be solved to obtain an expression for the refractive index; their solutions are of the form

$$\boldsymbol{E} = \boldsymbol{E}_0 \exp\{\mathrm{i}(\omega t - \boldsymbol{k} \cdot \boldsymbol{r})\}\,, \tag{34}$$

and this is the characteristic of propagation of electromagnetic and electroacoustic waves in the medium. The wave number $k$ is a complex quantity,

$$k = k_{\mathrm{r}} - \mathrm{i}k_{\mathrm{i}}\,. \tag{35}$$

*(a) Refractive Index of Electromagnetic Waves*

An expression for the refractive index for electromagnetic waves in the case of normal wave propagation can be obtained by noting the form of the solution (34) and substituting

$$\boldsymbol{E}_{\mathrm{T}} = \boldsymbol{E}_{\mathrm{T}0} \exp[\mathrm{i}\{\omega t - (k_x x + k_y y)\}]$$

into equation (33a). This gives the result

$$k^2 = k_0^2(1 - X/U_{\mathrm{m}})\,, \tag{36}$$

where we have defined $k_x = k\cos\theta$ and $k_y = k\sin\theta$. From the definition (35) of $k$, equation (36) can be written as

$$k_{\mathrm{r}}^2 - k_{\mathrm{i}}^2 - 2k_{\mathrm{r}}\,k_{\mathrm{i}} = k_0^2\left(1 - \frac{X}{1+\bar{Z}^2} - \frac{\mathrm{i}X\bar{Z}}{1+\bar{Z}^2}\right). \tag{37}$$

Separating real and imaginary parts we have

$$k_{\mathrm{r}}^2 - k_{\mathrm{i}}^2 = k_0^2\left(1 - \frac{X}{1+\bar{Z}^2}\right), \qquad 2k_{\mathrm{r}}\,k_{\mathrm{i}} = k_0^2\left(\frac{XZ}{1+\bar{Z}^2}\right). \tag{38}$$

Solution of equations (38) gives the real part of $k$ as

$$k_{\mathrm{r}} = \frac{k_0}{\sqrt{2}}\left[1 - \frac{X}{1+\bar{Z}^2} \pm \left\{\left(1 - \frac{X}{1+\bar{Z}^2}\right)^2 + \frac{X^2\bar{Z}^2}{(1+\bar{Z}^2)^2}\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}}. \tag{39}$$

The phase velocity of the electromagnetic wave is

$$v_{\mathrm{g}} = \frac{\omega}{k_{\mathrm{r}}} = \frac{\omega\sqrt{2}}{k_0}\left[1 - \frac{X}{1+\bar{Z}^2} \pm \left\{\left(1 - \frac{X}{1+\bar{Z}^2}\right)^2 + \frac{X^2\bar{Z}^2}{(1+\bar{Z}^2)^2}\right\}^{\frac{1}{2}}\right]^{-\frac{1}{2}}, \tag{40}$$

while the refractive index of the magnetoplasma is, by definition,

$$\mu_{\mathrm{em}} = \frac{c}{v_{\mathrm{g}}} = \frac{ck_{\mathrm{r}}}{\omega} = \frac{1}{\sqrt{2}}\left[1 - \frac{X}{1+\bar{Z}^2} \pm \left\{\left(1 - \frac{X}{1+\bar{Z}^2}\right)^2 + \frac{X^2\bar{Z}^2}{(1+\bar{Z}^2)^2}\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}}. \tag{41}$$

Thus the expression obtained is the same as equation (28), which was derived from solution of the equation of motion for oblique propagation. This result is utilized in the study of wave propagation in the ionosphere and the magnetosphere.

From (41) it follows that similar results to (29) and (30) hold for $\mu_{\mathrm{em}}$ in the particular cases of no steady magnetic field and at high frequency, considered at the end of Section III.

*(b) Refractive Index of Electroacoustic Waves*

Consider now the propagation of electroacoustic waves of the form

$$\boldsymbol{E}_{\mathrm{L}} = \boldsymbol{E}_{\mathrm{L0}}\exp\{\mathrm{i}(\omega t - k_l z)\}\,. \tag{42}$$

Substituting in equation (33b) we obtain

$$k_l^2 = k_1^2(U_{\mathrm{m}} - X)\,. \tag{43}$$

The real part of (43) is

$$k_r = (k_1/\sqrt{2})[1-X \pm \{(1-X)^2+\bar{Z}^2\}^{\frac{1}{2}}]^{\frac{1}{2}}\,, \tag{44}$$

so that the phase velocity of the longitudinal wave is

$$v_{\mathrm{g}} = \omega/k_{\mathrm{r}} = (\omega\sqrt{2}/k_1)[1-X \pm \{(1-X)^2+\bar{Z}^2\}^{\frac{1}{2}}]^{-\frac{1}{2}} \tag{45}$$

and the refractive index of the magnetoplasma is given by

$$\mu_{\mathrm{ea}} = (c/\sqrt{2}\,a)[1-X \pm \{(1-X)^2+\bar{Z}^2\}^{\frac{1}{2}}]^{\frac{1}{2}}\,. \tag{46}$$

Since electroacoustic waves are not affected by the presence of an external magnetic field, $\bar{Z} = Z = \nu/\omega$ and equation (46) can be replaced by

$$\mu_{\mathrm{ea}} = (c/\sqrt{2}\,a)[1-X \pm \{(1-X)^2+Z^2\}^{\frac{1}{2}}]^{\frac{1}{2}}\,. \tag{47}$$

At high frequencies when $\nu = 0$ equation (47) reduces to

$$\mu_{\mathrm{ea}} = (c/\sqrt{2}\,a)\{1-X \pm (1-X)\}^{\frac{1}{2}}\,, \tag{48}$$

which agrees with the result (24).

Thus the same results for the refractive index for both oblique and normal wave propagation have been obtained from the two methods of solution.

## V. Booker Quartic Equation

The Booker quartic equation describing propagation of electromagnetic waves in a warm magnetoplasma may be derived from the above results. For this purpose we rewrite equation (28) in the form

$$-\left\{(1-\mu^2)-\frac{1}{2}\left(1+\frac{X}{1+\bar{Z}^2}\right)\right\} = \pm\frac{1}{2}\left\{\left(1-\frac{X}{1+\bar{Z}^2}\right)^2+\left(\frac{X\bar{Z}}{1+\bar{Z}^2}\right)^2\right\}^{\frac{1}{2}}. \tag{49}$$

Following Unz (1966), we square both sides of equation (49) and, with the help of equations (22) and (26), obtain

$$(q^2-c'^2)^2(1+\bar{Z}^2)+(q^2-c'^2)(1+\bar{Z}^2+X)+X-X^2\bar{Z}^2/4(1+\bar{Z}^2) = 0\,. \tag{50}$$

The Booker quartic equation can then be obtained by rearranging (50) as

$$(q^2-c'^2)\{(q^2-c'^2)(1+\bar{Z}^2)+(1+\bar{Z}^2)+X\}+X-X^2\bar{Z}^2/4(1+\bar{Z}^2) = 0\,. \tag{51}$$

This expression is similar to that obtained by Unz (1966).

## VI. Propagation in Ionosphere and Magnetosphere

### (a) *Ambient Plasma Models*

In order to study the propagation characteristics of electromagnetic and electroacoustic waves in the ionosphere during day and night times we have taken electron density profiles at quiet solar conditions from Aikin and Blumle (1968) and the effective collision frequency profile from Shukla, Misra, and Singh (1970). For a dipole field configuration in the ionosphere, the variation of the magnetic field has been calculated from the equation

$$B = B_0(1-3h/R_0)\,, \tag{52}$$

where $B_0$ is the magnetic field at ground level at the equator, $R_0$ is the radius of the Earth, and $h$ is the altitude.

To study the propagation of electromagnetic waves in the magnetosphere, we have used the electron density model of Thorne and Kennel (1967), and the variations of electron density along the geomagnetic lines of force have been calculated from their expression

$$N(R,\phi) = 3\cdot5\times10^4\,B(R,\phi)\cos^6\phi(1+3\sin^2\phi)^{-\frac{1}{2}}\,, \tag{53}$$

where $\phi$ is the geomagnetic latitude in degrees and $B(R,\phi)$ is the value of the magnetic field at $\phi$. The variation of the ambient dipole magnetic field with the McIlwain parameter $L$ $(= R/R_0)$ has been computed from the relation

$$B(R,\phi) = (B_0/L^3)(1+3\sin^2\phi)\,, \tag{54}$$

$R$ being the radial distance from the centre of the Earth.

### (b) *Results*

The present study of the propagation of waves in ionospheric and magnetospheric plasmas has been restricted to the case $\omega > \omega_p$, and in this frequency range the refractive index is found to be less than unity, in agreement with previous results. Using the values of electron density, collision frequency, and magnetic field obtained from the selected model of the ionosphere discussed above, the refractive index has been computed and the resulting variation with altitude is plotted in Figure 1(*a*). It is found that during the sunspot minimum period in the daytime the refractive index decreases above 100 km, while at night it increases slowly up to a height of 150 km and then decreases. As electron density decreases during the night-time, this will in turn increase the refractive index. The results are similar to those obtained by Ratcliffe (1959).

The variation of the refractive index in the magnetosphere with the parameter $L$ has been obtained from the selected model and is plotted in Figure 1(*b*). For frequencies $\omega > \omega_p$ it is found that the refractive index is greater at higher altitudes in the equatorial region. Therefore the increase in the refractive index may be attributed to the decrease in electron density. The effective electron collision frequency is of the order of 1 Hz in the magnetosphere (Shukla, Misra, and Singh 1970) and hence the effect of collisions on the propagation of waves in this region is negligible.

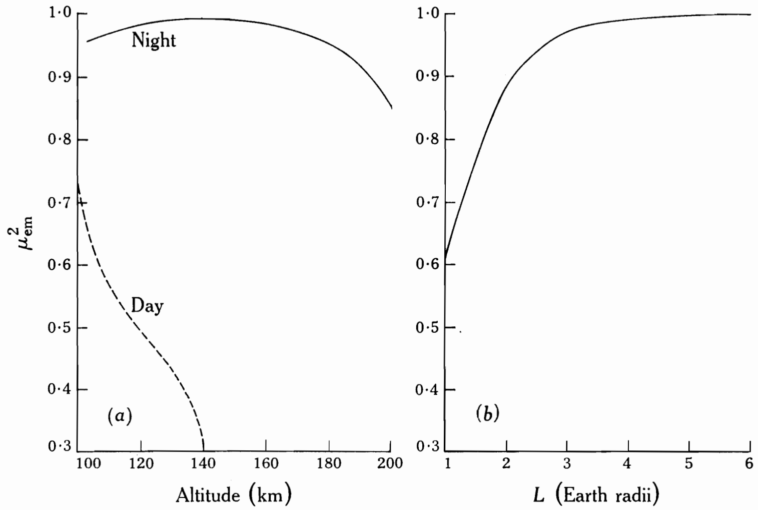


Fig. 1.—Variation of the square of the refractive index $\mu^2_{em}$ for electromagnetic waves with (*a*) altitude during day and night times for a frequency of 5 MHz in the ionosphere, and (*b*) the McIlwain parameter $L$ for a frequency of 1 MHz in the equatorial region of the magnetosphere.

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