A METHOD FOR DETERMINING THE ELECTRON DENSITY DISTRIBUTION ABOUT THE F_2 PEAK OF THE IONOSPHERE

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Abstract

A method is given for extending the analysis of topside ionograms to yield ionospheric electron density profiles down to and below $h_{\max}$, the peak of the F_2 layer, by analysis of the ground echoes. Calculations using model ionospheres indicate that the accuracy of the method is not seriously affected by the assumptions made or by the limited accuracy with which ionograms can be scaled.

I. Introduction

The problem of determining electron density distributions in the ionosphere from bottomside and topside ionograms has received considerable attention in recent years (see e.g. Wright and Smith 1967; Jackson 1969). Although the basic methods give the electron density profile over a large range in height, the profiles usually do not extend to $h_{\max}$, the peak of the F_2 layer. This is unfortunate because a knowledge of the variations of $h_{\max}$ provides a critical test of F-region theory. Current practice is to estimate $h_{\max}$ by extrapolating the calculated profile according to a model layer: usually a parabolic layer for the bottomside of the F_2 layer and a Chapman layer for the topside.

In the case of topside ionograms the basic methods of analysis can in principle be extended to obtain the complete electron density profile by utilizing the ground echo traces (Dyson 1967). In practice it is more realistic to limit the analysis to a determination of the shape of the F_2 layer, and hence determine $h_{\max}$, rather than to attempt a calculation of the complete electron density profile. This paper presents a method for obtaining the shape of the F_2 layer in the vicinity of the F_2 peak by using the virtual-height–frequency characteristics of topside ionogram ground echoes. Results are also compared with calculations using model ionospheric layers to give an indication of the accuracy of the method.

II. Method

For a radio wave propagating vertically through the ionosphere from height h_1 to height h_2, the group path is given by

$$P' = \int_{h_1}^{h_2} \mu'(f, f_H, \theta, N)\, \mathrm{d}h\,, \tag{1}$$

where the group refractive index μ' is a function of the frequency of propagation f, the gyrofrequency f_H, the dip angle θ, and the electron density N. Both f_H and N are functions of h.

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Figure 1 shows a schematic diagram of a topside ionogram and the corresponding electron density profile. In the usual analysis of topside ionograms the extraordinary ray trace is used to determine the electron density profile from the height of the satellite down to some lower height h_L (above h_{max}), which corresponds to the observed high frequency cutoff for the extraordinary trace. The gyrofrequency is a known function of height so that, once the electron density profile is determined from the satellite height down to h_L, equation (1) can be used to determine, for any frequency, the group path P'_t due to the ionization between the satellite height and h_L. If this is done for frequencies at which ground echoes occur, the group path due to the ionization below h_L is given by the difference between the measured group path P'_m and P'_t as

$$P'_r = P'_m - P'_t = \int_{h_b}^{h_L} \mu' \, dh + h_b = \int_{h_{max}}^{h_L} \mu' \, dh + \int_{h_b}^{h_{max}} \mu' \, dh + h_b , \tag{2}$$

where h_b is the base height of the ionosphere.

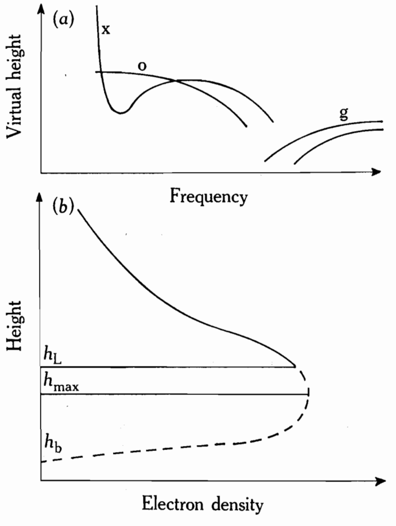

Fig. 1.—Schematic diagrams of:
(a) a topside ionogram with the ordinary (o) and extraordinary (x) wave traces and the ground echoes (g), and
(b) the corresponding electron density profile showing the heights at the observational cutoff h_L, the peak of the F_2 layer h_{max}, and the base of the ionosphere h_b.

Since h_L is known, equation (2) can be simplified by writing it in terms of the group retardation R rather than the group delay. Equation (2) then becomes

$$R = \int_{h_{max}}^{h_L} (\mu' - 1) \, dh + \int_{h_b}^{h_{max}} (\mu' - 1) \, dh . \tag{2a}$$

The variation of electron density with height may be written as $h = F(X)$, where $X = (f_N/f)^2$, f_N being the plasma frequency. Making this substitution in equation (2a) gives

$$R = \int_{X_C}^{X_L} (\mu' - 1) \frac{dh}{dX} \, dX + \int_0^{X_C} (\mu' - 1) \frac{dh}{dX} \, dX , \tag{3}$$

where X_C and X_L are the values of X at h_{max} and h_L respectively. The advantage of making this substitution is that the limits of the integrals are now known since

f_0F_2 can be read from the topside ionogram to give X_C. It is, however, necessary to postulate the form of the function $h = F(X)$ in order to proceed further.

Below h_{max} the variation of X with height may be quite complicated during the daytime when substantial D, E, and F_1 layers can exist. However, if we initially consider the night-time case, underlying layers may be neglected and the F_2 layer can be considered to be parabolic, i.e. it can be represented by the equation

$$X = X_C\{1-(h-h_{max})^2/4H^2\}, \tag{4}$$

where H is the scale height of the layer. Then equation (3) becomes

$$R = -H\int_{X_C}^{X_L} \frac{(\mu'-1)\,dX}{X_C(1-X/X_C)^{\frac{1}{2}}} + H\int_0^{X_C} \frac{(\mu'-1)\,dX}{X_C(1-X/X_C)^{\frac{1}{2}}}. \tag{5a}$$

The integrals in equation (5a) can be evaluated if the gyrofrequency is considered constant up to h_L, an assumption that is made in the analysis of bottomside ionograms. Thus the only unknown in equation (5a) is H and we can write the equation as

$$R = AH. \tag{5b}$$

If the retardation due to the ionization below h_L is determined for a number of frequencies on the ordinary (o) and extraordinary (x) ground echo traces then we may write the series of simultaneous equations

$$\begin{array}{ll}
R_o(f_1) = A_1H, & R_x(f_{j+1}) = A_{j+1}H, \\
R_o(f_2) = A_2H, & R_x(f_{j+2}) = A_{j+2}H, \\
\quad\vdots & \quad\vdots \\
R_o(f_j) = A_jH, & R_x(f_{j+n}) = A_{j+n}H.
\end{array}$$

From these equations a least squares solution for H can be obtained. Once H is determined, h_{max} can be calculated from equation (4).

As demonstrated by equation (5b), H can be determined even if only one measurement of the retardation is available. However, there are two main advantages to be gained from using more than one measurement and finding the least squares solution: (1) minimization of the effect of rounding errors, which arise when an ionogram is scaled, and (2) optimization of the value of f_0F_2, and hence X_C, (already known to within definite limits) for the given set of data points.

During the daytime when considerable underlying ionization occurs, the same method may still be used since most of the group retardation occurs near the F_2 peak. There is, of course, a decrease in accuracy (see Section III).

In outlining the method, the electron density profile of the F_2 layer has been assumed to be parabolic because this is generally believed to be approximately its basic shape. However, the method is not limited to this assumption and other forms of the function $h = F(X)$ can be used. By this means it may be possible to determine if the assumption of a parabolic shape for the F_2 layer is reliable.

III. Test of Method

The method has been tested in the following way. A model for the F_2 layer was chosen which consisted of a Chapman layer for the topside portion and a parabolic layer for the bottomside. The last known point on the topside ionosphere was taken to be half a scale height above the F_2 peak and the layer was considered to be at the equator. The ordinary wave retardation was calculated at 10 chosen frequencies for which the values of X_C lay between 0·7 and 0·925 and the results were rounded to the nearest 10 km to simulate the scaling accuracy of ionograms. Values of H and h_{max} were then calculated by the method outlined in the previous section. Differences between the calculated and model values therefore indicate the errors due to scaling inaccuracies and the assumption that the layer is purely parabolic. The resulting values which are listed below show that the errors are quite small.

	H (km)	$h_L - h_{max}$ (km)	H (km)	$h_L - h_{max}$ (km)
Model	40·0	20·0	80·0	40·0
Calculated	41·5	19·0	81·8	37·4

The effect of underlying ionization was also determined by using the same approach as outlined above but with the addition of parabolic F_1 and E layers to the model. Values of the parameters for two daytime models are given in Table 1 along

Table 1

RESULTS FOR DAYTIME MODELS

Model	Model parameters							Calculated	
	F_1 ht. (km)	$\frac{f_0F_1}{f_0F_2}$	E ht. (km)	$\frac{f_0E}{f_0F_2}$	h_b (km)	h_{max} (km)	H (km)	h_{max} (km)	H (km)
1	200	0·8	120	0·64	100	260	50	256	63
2	180	0·65	120	0·32	100	260	52·5	259	59

with values for the scale and peak heights of the F_2 layer calculated on the assumption that the retardation is due solely to a parabolic F_2 layer. For model 1, the calculated value of H is over 25% too high, and that of h_{max} is 4 km too low. In this model the ratio of the critical frequency of each lower layer to f_0F_2 is quite high, a situation which only occurs at very low sunspot numbers (Davies 1965). For model 2, the calculated value of H is about 12% too high and h_{max} is in error by 1 km; this model is typical of the mid-latitude ionosphere at noon when the sunspot number is 40 (Davies 1965). These results indicate that even during the daytime the analysis of ground echoes will give a good description of the F_2 layer near its peak height, provided that the sunspot number is greater than 40.

IV. Conclusions

A method has been outlined for extending the analysis of topside ionograms to yield electron density profiles to give the shape of the F_2 layer and hence h_{max} by including the analysis of ground echoes. Tests with model ionospheres have indicated that the assumptions made and the rounding errors which occur in scaling ionograms do not introduce large errors. The method should prove most useful in determining latitudinal variations of h_{max} and changes in the shape of the F_2 layer during magnetic storms.

V. Acknowledgment

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VI. References

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