TRANSFORMATION OF BREMSSTRAHLUNG PHOTONEUTRON DATA TO DATA CORRESPONDING TO MONOCHROMATIC GAMMA EXCITATION

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Abstract

Using the count-ratio technique described by Thies and Böttcher one can coarsely measure with a multi-BF_3-counter neutron detection system the 4π-integrated neutron spectrum, neutron mean energies, and related functions in photoneutron experiments using bremsstrahlung. It is shown here how the bremsstrahlung photoneutron data can be used to calculate the equivalent data that correspond to an idealized experiment using monochromatic γ-rays. Simple formulae are derived which permit an estimate to be made of the accuracy of the tıansformed neutron data.

I. INTRODUCTION

Thies and Böttcher (1969) describe a method of using a multi-BF_3-counter neutron detection system as a "high efficiency coarse 4π fast neutron spectrometer". Such a system consists in principle of an array of BF_3 counters which is inserted into a large paraffin moderator in a precalculated geometry as shown diagrammatically in Figure 1. Use is made of the fact that the probability for an individual counter to detect neutrons is a function of counter position as well as of the energy with which these neutrons were emitted initially into the moderator. If the counts of each counter are recorded individually, one can reconstruct in crude approximation from this information the energy spectrum with which the neutrons initially entered the moderator, the mean energy of these neutrons, and related functions. In particular the measurement of neutron mean energies using the ratio of counts from counters in two selected positions only, an "inner" and an "outer" position, has been discussed in detail by Barrett and Thies (1971). The above methods can be used conveniently to obtain information on neutron energies from photonuclear reactions. If bremsstrahlung is used, the observed neutron data are functions of the peak bremsstrahlung energy E_0.

In this paper we show how bremsstrahlung neutron data from "energy sweeping" (Thies *et al.* 1972) can be converted to neutron data as they would be obtained in an equivalent experiment with monochromatic photons, i.e. if $\hat{E}_n(E_0)$ is the mean energy of the neutrons emitted in the bremsstrahlung experiment, the transformation calculation evaluates $\bar{E}_n(h\nu)$, the mean energy of the neutrons that would have been emitted after excitation by monochromatic photons of energy $h\nu$. The propagation of errors in these transformation calculations is not a trivial problem, and some simple expressions which allow an adequate estimate of these errors are given here.

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II. Analysis of Transformation

(*a*) *Monochromatic Gamma Emission*

We first consider the connection between photoneutron emission and count ratios for the case of an idealized experiment using monochromatic γ-rays. This will help to define our peculiar notation which, with appropriate modifications, will be applied to describe the bremsstrahlung experiment.

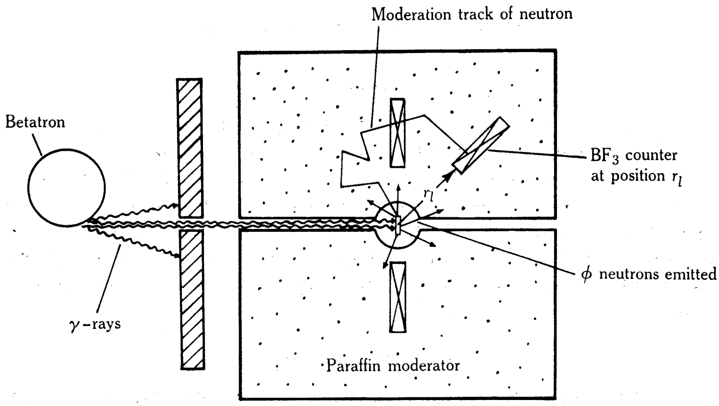

Fig. 1.—Schematic diagram of the detection of photoneutrons by a multi-BF_3-counter system.

Consider Figure 1. We assume that monochromatic photons of energy $h\nu$ hit the target which is located at the centre of the moderator cavity, and that these photons eject ${}^{M}\phi$ neutrons from the target. Figure 2 diagrammatically shows the excitation and decay mechanism involved. The emitted neutrons are polychromatic in general. The normalized spectral distribution function of the photoneutrons is a function of $h\nu$, which we shall denote by $m(h\nu, E_{\mathrm{n}})$, where the neutron energy E_{n} can have any value between 0 and $h\nu - E_{\mathrm{th}}$, E_{th} being the threshold energy. If $\mathscr{G}(\boldsymbol{r}_l, E_{\mathrm{n}})$ is the probability that a neutron of energy E_{n} emitted into the moderator is detected by the lth counter located at $\boldsymbol{r}_l$ then ${}^{M}I(\boldsymbol{r}_l, h\nu)$, the number of neutrons recorded by the counter at $\boldsymbol{r}_l$, is given by

$$ {}^{M}I(\boldsymbol{r}_l, h\nu) = {}^{M}\phi \int_0^{h\nu - E_{\mathrm{th}}} \mathscr{G}(\boldsymbol{r}_l, E_{\mathrm{n}})\, m(h\nu, E_{\mathrm{n}})\, \mathrm{d}E_{\mathrm{n}}. \tag{1} $$

We define the detection efficiency of the lth counter for photoneutrons ejected by photons of energy $h\nu$ by

$$ {}^{M}\eta(\boldsymbol{r}_l, h\nu) = {}^{M}I(\boldsymbol{r}_l, h\nu)/{}^{M}\phi. \tag{2} $$

Hence the count ratio of counters located at $\boldsymbol{r}_l$ and $\boldsymbol{r}_m$ is given by

$$ {}^{M}I(\boldsymbol{r}_l, h\nu)/{}^{M}I(\boldsymbol{r}_m, h\nu) = {}^{M}\eta(\boldsymbol{r}_l, h\nu)/{}^{M}\eta(\boldsymbol{r}_m, h\nu). \tag{3} $$

For the case where $h\nu$ assumes the specific value $h\nu_i$, we adopt the shorthand notation for equation (3)

$$ {}^{M}_{l}I_i/{}^{M}_{m}I_i = {}^{M}_{l}\eta_i/{}^{M}_{m}\eta_i. \tag{3a} $$

(b) Bremsstrahlung Gamma Emission

Now consider the realistic situation where the incident photons are polychromatic and have a non-normalized bremsstrahlung spectral distribution function $P(E_0, h\nu)$. Then the photoneutron yield $Y(E_0)$ and the photoneutron cross section $\sigma(h\nu)$ are related by

$$Y(E_0) = \int_{E_{\text{th}}}^{E_0} P(E_0, h\nu)\,\sigma(h\nu)\,\mathrm{d}(h\nu)\,, \tag{4}$$

where E_0 is the peak bremsstrahlung energy and E_{th} is again the neutron threshold

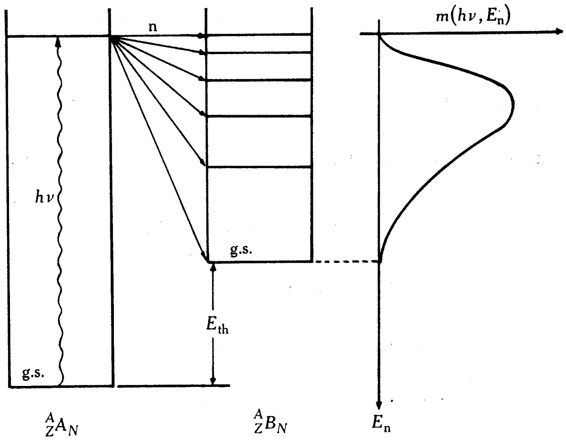

Fig. 2.—Schematic diagram of monochromatic photonuclear excitation and subsequent emission of photoneutrons.

energy. For convenience we define the normalized effective bremsstrahlung distribution function by

$$p(E_0, h\nu) = P(E_0, h\nu)\left(\int_{E_{\text{th}}}^{E_0} P(E_0, h\nu)\,\mathrm{d}(h\nu)\right)^{-1} = P(E_0, h\nu)/g(E_0) \tag{5}$$

and analogously the normalized yield by

$$y(E_0, h\nu) = \int_{E_{\text{th}}}^{E_0} p(E_0, h\nu)\,\sigma(h\nu)\,\mathrm{d}(h\nu)\,. \tag{6}$$

If now $\partial\{{}^{\mathrm{B}}\phi(E_0)\}/\partial t$ is the rate at which neutrons are emitted into the moderator in a particular bremsstrahlung experiment, we can write

$$\partial\{{}^{\mathrm{B}}\phi(E_0)\}/\partial t = K(t)\,\beta(E_0)\,Y(E_0) = K(t)\,\beta(E_0)\,g(E_0)\,y(E_0)\,, \tag{7}$$

where $K(t)$ is a parameter that is independent of E_0 but depends on the target weight, beam geometry, and similar factors, and possibly on the time t (if, for example, the electron-injection intensity of the accelerator varies with time). The function $\beta(E_0)$ in (7) contains the dependence of γ-intensity of the accelerator on E_0 (e.g. in units of γ-rays per electron injected) and this function can be measured readily by comparing the counts per dose from a conventional counting experiment with the counts from an "energy sweeping" experiment (for a detailed discussion see Thies *et al.* 1972). Using the above notation, the number of counts ${}^{\mathrm{B}}_{l}I(E_0,t)$ recorded in the bremsstrahlung experiment by the counter at position r_l after a time interval t is given by

$$ {}^{\mathrm{B}}_{l}I(E_0,t) = \beta(E_0)\,g(E_0)\int_{E_{\mathrm{th}}}^{E_0} p(E_0,h\nu)\,{}^{\mathrm{M}}_{l}\eta(h\nu)\,\sigma(h\nu)\,\mathrm{d}(h\nu)\int_0^t K(t)\,\mathrm{d}t\,. \tag{8} $$

Approximating the first integral in equation (8) by a finite summation and using an analogous shorthand notation to that of Section II(a), we can rewrite equation (8) as

$$ {}^{\mathrm{B}}_{l}I_j(t) = \beta_j\,g_j\sum_{i=1}^{j} p_{ji}\,{}^{\mathrm{M}}_{l}\eta_i\,\sigma_i\,\varDelta_i\int_0^t K(t)\,\mathrm{d}t\,, \tag{9} $$

where

$$ \varDelta_i = E_{0,i}-E_{0,i-1} \qquad \text{and} \qquad h\nu_i = \tfrac{1}{2}(E_{0,i}+E_{0,i-1})\,. \tag{10} $$

Assume now that in the actual experiment counts were taken at the energies

$$ E_0 = E_{01},\, E_{02},\, \dots,\, E_{0j},\, \dots,\, E_{0f}\,. $$

The corresponding counts ${}^{\mathrm{B}}_{l}I_j(t)$ then are (for q counters)

$$ \left.\begin{matrix} {}^{\mathrm{B}}_{1}I_1, & {}^{\mathrm{B}}_{1}I_2, & \dots, & {}^{\mathrm{B}}_{1}I_j, & \dots, & {}^{\mathrm{B}}_{1}I_f, \\ {}^{\mathrm{B}}_{2}I_1, & {}^{\mathrm{B}}_{2}I_2, & \dots, & {}^{\mathrm{B}}_{2}I_j, & \dots, & {}^{\mathrm{B}}_{2}I_f, \\ \vdots & \vdots & & \vdots & & \vdots \\ {}^{\mathrm{B}}_{q}I_1, & {}^{\mathrm{B}}_{q}I_2, & \dots, & {}^{\mathrm{B}}_{q}I_j, & \dots, & {}^{\mathrm{B}}_{q}I_f. \end{matrix}\right\} \tag{11} $$

The dependence on t has been omitted in this tabulation since it is assumed that "energy sweeping" was used, in which case the interval t and hence the integral in (9) were constant for all counts ${}^{\mathrm{B}}_{l}I_j(t)$.

For any fixed value of l and $j = 1,2,3,\dots,f$, equation (9) represents a system of f linear equations in the f variables ${}^{\mathrm{M}}_{l}\eta_i\,\sigma_i\,\varDelta_i$, $i = 1,2,3,\dots,f$. If we define an element of the triangular matrix $\mathbf{p}$ by $(\mathbf{p})_{ji} = p_{ji}$ and correspondingly an element of the inverse matrix $(\mathbf{p}^{-1})$ by $(\mathbf{p}^{-1})_{ij}$, the f solutions of equation (9) can be expressed in the form*

$$ {}^{\mathrm{M}}_{l}\eta_i\,\sigma_i\,\varDelta_i = \left(\int_0^t K(t)\,\mathrm{d}t\right)^{-1}\sum_{j=1}^{i}(\mathbf{p}^{-1})_{ij}\,{}^{\mathrm{B}}_{l}I_j/\beta_j\,g_j\,, \qquad l = 1,2,\dots,q\,. \tag{12} $$

Hence with the $q\times f$ experimental values of the tabulation (11) we can calculate any of the ratios ${}^{\mathrm{M}}_{l}\eta_i/{}^{\mathrm{M}}_{m}\eta_i$, which we shall denote by ${}^{\mathrm{M}}Q(l/m)_i$, and because of equation (3a) we can thus obtain any of the count ratios ${}^{\mathrm{M}}_{l}I_i/{}^{\mathrm{M}}_{m}I_i$, which correspond to the idealized

* The transformation from equation (9) to equation (12) is identical in principle with the one used by Penfold and Leiss (1958). The resolution of the transformed data is limited due to approximations made and to the statistical nature of the data. This point has been discussed in detail by Thies (1961).

experiment with monochromatic photons, from

$$^{\mathrm{M}}Q(l/m)_i = \frac{^{\mathrm{M}}_{l}I_i}{^{\mathrm{M}}_{m}I_i} = \frac{^{\mathrm{M}}_{l}\eta_i}{^{\mathrm{M}}_{m}\eta_i} = \left(\sum_{j=1}^{i} (\mathbf{p}^{-1})_{ij}\, {}^{\mathrm{B}}_{l}I_j/\beta_j g_j\right) \Big/ \left(\sum_{j=1}^{i} (\mathbf{p}^{-1})_{ij}\, {}^{\mathrm{B}}_{m}I_j/\beta_j g_j\right). \tag{12a}$$

These count ratios can then be used to calculate in crude approximation the photoneutron energy spectra $m(h\nu_i, E_{\mathrm{n}})$, the associated neutron mean energies $\bar{E}_{\mathrm{n}}(h\nu_i)$, and related functions, as discussed in detail by Thies and Böttcher (1969). If it is ascertained that the resulting neutron spectra are Maxwellian to a reasonable approximation then the relatively simple and accurate method of Barrett and Thies (1971) can be used to obtain values of $\bar{E}_{\mathrm{n}}(h\nu_i)$. In this method the ratio of the counts from an "outer" (o) to an "inner" (i) counter is obtained as a function of mean neutron energy $\bar{E}_{\mathrm{n}}(h\nu)$ from a calibration-type experiment. Thus for any experimental value $Q(\mathrm{o/i})_i$ the corresponding mean energy $\bar{E}_{\mathrm{n}}(h\nu_i)$ can be read off a calibration chart.

(c) *Errors in Count Ratios*

It remains to calculate or at least estimate the errors in the count ratios, as given by equation (12a), due to statistical errors in the values of ${}^{\mathrm{B}}_{l}I_j$ and ${}^{\mathrm{B}}_{m}I_j$. We denote by $[\![F]\!]$ the standard deviation of any function F. If the counter at $\boldsymbol{r}_l$ is further from the moderator centre than the counter at $\boldsymbol{r}_m$, then the standard errors $[\![{}^{\mathrm{B}}_{l}I_j]\!]$ in general are substantially greater than the $[\![{}^{\mathrm{B}}_{m}I_j]\!]$, as the relative number of counts falls off rapidly with increasing $|\boldsymbol{r}|$. Hence the relative error in the count ratio of equation (12a) can be written in reasonable approximation as

$$\frac{[\![^{\mathrm{M}}Q(l/m)_i]\!]}{^{\mathrm{M}}Q(l/m)_i} = \left(\sum_{j=1}^{i} (\mathbf{p}^{-1})^2_{ij}\, [\![{}^{\mathrm{B}}_{l}I_j/\beta_j g_j]\!]^2\right)^{\frac{1}{2}} \Big/ \left(\sum_{j=1}^{i} (\mathbf{p}^{-1})_{ij}\, {}^{\mathrm{B}}_{l}I_j/\beta_j g_j\right). \tag{13}$$

Since numerically the coefficients $(\mathbf{p}^{-1})_{ij}$ decrease rapidly with decreasing j, whereas $[\![{}^{\mathrm{B}}_{l}I_j/\beta_j g_j]\!]$ varies only slowly with j, equation (13) can be further approximated by

$$\frac{[\![^{\mathrm{M}}Q(l/m)_i]\!]}{^{\mathrm{M}}Q(l/m)_i} = \frac{[\![{}^{\mathrm{B}}_{l}I_i]\!]}{\beta_i g_i}\left(\sum_{j=1}^{i} (\mathbf{p}^{-1})^2_{ij}\right)^{\frac{1}{2}} \Big/ \left(\sum_{j=1}^{i} (\mathbf{p}^{-1})_{ij}\, {}^{\mathrm{B}}_{l}I_j/\beta_j g_j\right). \tag{14}$$

This expression can be simplified further. If we define the normalized "counter yield" corresponding to a counter at $\boldsymbol{r}_l$ by

$${}_{l}z(E_0) = \int_{E_{\mathrm{th}}}^{E_0} p(E_0, h\nu)\, {}^{\mathrm{M}}_{l}\eta(h\nu)\, \sigma(h\nu)\, \mathrm{d}(h\nu)\,, \tag{15}$$

then, with a similar notation to before and replacing the integration in (15) by a finite summation, we obtain the approximation

$${}_{l}z_j = \sum_{i=1}^{j} p_{ji}\, {}^{\mathrm{M}}_{l}\eta_i\, \sigma_i\, \Delta_i\,, \qquad j = 1, 2, \ldots, f\,, \tag{16}$$

and hence

$${}^{\mathrm{M}}_{l}\eta_i\, \sigma_i\, \Delta_i = \sum_{j=1}^{i} (\mathbf{p}^{-1})_{ij}\, {}_{l}z_j\,. \tag{17}$$

Expressing ${}^{\mathrm{B}}_{l}I_j$ in terms of ${}_{l}z_j$ in equation (14) and substituting equation (17) into (14), we obtain

$$\frac{[\![{}^{\mathrm{M}}Q(l/m)_i]\!]}{{}^{\mathrm{M}}Q(l/m)_i} = \frac{[\![{}_{l}z_i]\!]}{{}^{\mathrm{M}}_{l}\eta_i\,\sigma_i\,\Delta_i}\left(\sum_{j=1}^{i}(\mathbf{p}^{-1})^2_{ij}\right)^{\frac{1}{2}}, \tag{18}$$

which for a Schiff-bremsstrahlung spectrum is numerically approximated by

$$\frac{[\![{}^{\mathrm{M}}Q(l/m)_i]\!]}{{}^{\mathrm{M}}Q(l/m)_i} = \frac{20\,E_i\,g_i}{\Delta_i^{3/2}\,\sigma_i}\frac{[\![{}_{l}z_i]\!]}{{}^{\mathrm{M}}_{l}\eta_i}, \tag{19}$$

where E_i is expressed in MeV and σ_i and ${}_{l}z_i$ are in mbarn. In analogy with the detection efficiency ${}^{\mathrm{M}}_{l}\eta_i$ for monochromatic γ-excitation, we define the detection efficiency ${}^{\mathrm{B}}_{l}\eta_i$ for bremsstrahlung excitation by

$${}^{\mathrm{B}}_{l}\eta_i = {}_{l}z_i/y_i\,. \tag{20}$$

Usually ${}^{\mathrm{B}}_{l}\eta_i$ is numerically not very different from ${}^{\mathrm{M}}_{l}\eta_i$, and hence, from (20), equation (19) can be further approximated by

$$\frac{[\![{}^{\mathrm{M}}Q(l/m)_i]\!]}{{}^{\mathrm{M}}Q(l/m)_i} = \frac{20\,E_i\,g_i\,y_i}{\Delta_i^{3/2}\,\sigma_i}\frac{[\![{}_{l}z_i]\!]}{{}_{l}z_i} = \frac{20\,E_i\,g_i\,y_i}{\Delta_i^{3/2}\,\sigma_i}\frac{1}{({}^{\mathrm{B}}_{l}I_i)^{\frac{1}{2}}}, \tag{21}$$

or in terms of the non-normalized yield Y_i

$$\frac{[\![{}^{\mathrm{M}}Q(l/m)_i]\!]}{{}^{\mathrm{M}}Q(l/m)_i} = \frac{20\,E_i\,Y_i}{\Delta_i^{3/2}\,\sigma_i}\frac{1}{({}^{\mathrm{B}}_{l}I_i)^{\frac{1}{2}}}. \tag{22}$$

Equations (21) and (22) can be readily used to estimate, prior to an actual experiment, the statistical errors in the count ratios of existing approximate information on yields and cross sections. For example, in the mean neutron energy experiment of Barrett and Thies (1971) a reasonable estimate for the relative error in derived mean neutron energies $\bar{E}_{\mathrm{n}}(h\nu_i) = \bar{E}_{\mathrm{n}i}$ is given by

$$\frac{[\![\bar{E}_{\mathrm{n}i}]\!]}{\bar{E}_{\mathrm{n}i}} = \frac{20\,E_i\,Y_i}{\Delta_i^{3/2}\,\sigma_i}\frac{1}{({}^{\mathrm{B}}_{l}I_i)^{\frac{1}{2}}}, \tag{23}$$

as here, to a first approximation, $Q(\mathrm{o}/\mathrm{i})_i$ is proportional to $\bar{E}_{\mathrm{n}i}$. Obviously, if an accurate error value is required then equation (12a) must be used.

III. References

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