OBSERVATIONS OF THE BACKSCATTER OF ULTRASONIC WAVES FROM A ROUGH WATER SURFACE

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[Manuscript received 24 December 1971]

Abstract

An experiment is described in which measurements are made of the Doppler frequency shift imposed on an acoustic signal by resonant backscatter from a wind-generated rough water surface. In particular, for the case where the water waves have plane wave fronts and move in a single direction across the surface, the effects on the Doppler shift of varying the horizontal angle of incidence of the acoustic beam with respect to this direction of movement are studied. Some simple theoretical concepts are invoked in an attempt to explain the apparent dependence of the Doppler frequency shift on the azimuth angle measured from the acoustic beam radial direction. Because of the analogy which exists between the scattering of acoustic and electromagnetic waves from the sea surface, it is proposed that a model employing a procedure similar to that described here would be of use in interpreting data gained in large-scale ocean backscatter experiments.

I. Introduction

Much interest in recent years has centred on the process of the backscattering of both electromagnetic and acoustic waves from the sea surface. In particular, it has been felt that certain information regarding the spatial and temporal characteristics of a water wave surface can be gained through careful analysis of the backscattered signal. As has been pointed out by Beckmann and Spizzichino (1963), the scattering of electromagnetic and acoustic waves can be regarded as exactly analogous processes provided the boundary conditions are equivalent. This will be the case when the electromagnetic waves are vertically polarized and scattered from a perfectly conducting surface (to which the sea surface is a good approximation), and when the source of sound waves is in air, i.e. the scattering surface is approximately rigid. The results of work with both these media will be equivalent for these experimental conditions, and accordingly may be compared directly.

Following the pioneering work of Crombie (1955) and others, it has now become accepted that electromagnetic waves will be backscattered preferentially from that component of the total sea wave spectrum for which the Bragg resonant condition applies, i.e. where the sea wavelength is equal to an integral multiple of half the radio wavelength. The backscattered signal has then a Doppler frequency shift imposed on it by the moving sea wave scatterer. The process will be most effective

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for radio waves in the h.f. radio band, where the radio wavelength more nearly matches those sea wavelengths which form the dominant part of the total sea wave spectrum.

The analogy which exists between the scattering of acoustic and electromagnetic waves, as pointed out above, indicates that much could be learned regarding the scattering process through the use of acoustic models in the laboratory. Such a model would appear to be relevant, and most effective, when backscatter occurred from capillary waves generated in a ripple tank. For resonance, the acoustic wavelength would then have to be less than about 3 cm, necessitating a transmitter frequency of greater than 13 kHz. Liebermann (1963) has performed an experiment along these lines, using an ultrasonic wave of frequency 78 kHz (wavelength $\lambda = 0 \cdot 44$ cm) backscattered from a water surface on which capillary waves with wavelengths in the range $0 \cdot 19$–$0 \cdot 36$ cm were generated in a random fashion using an electrodynamic transducer. Liebermann's experiment showed conclusively that the acoustic waves were backscattered preferentially from that water wave component having a wavelength given by

$$d = \lambda/(\cos\alpha + \cos\psi),$$

where d is the water wavelength, λ is the wavelength of the incident acoustic wave, and α and ψ are respectively the angles of elevation of the transmitter and receiver.

Liebermann's (1963) experiment, however, employed only capillary waves which were propagating radially along the transmitter beam from a point source, i.e. they were essentially isotropic in direction of propagation, with curved wave fronts. In fact, as far as can be determined from the literature, all experiments which have been performed so far using either tank models or the actual sea surface have assumed that the water waves propagated radially with respect to the transmitter–receiver direction.

Since sea surface waves are by no means isotropic, but propagate in a mean direction which is aligned, to a greater or lesser extent, with the mean wind, the question now arises as to what happens to the backscattered radiation when this mean direction of propagation is at some azimuthal angle θ to the irradiating beam, either acoustic or electromagnetic. There is of course a certain angular spread in actual sea waves about the mean direction which is dependent on, among other things, the wavelength of the particular wave component and the wind speed. As this angular spread will not affect the general principles involved, it will be assumed for the present that all waves propagate approximately in the same direction.

This problem can best be investigated through the use of a laboratory model similar to that employed by Liebermann (1963), but differing in one important aspect, namely that the capillary water waves should propagate with approximately plane wave fronts in a single direction down the tank. The angles of incidence, both horizontal and vertical, of the irradiating beam can then be varied by varying the positions of the transmitter or receiver or both. Such an experiment is described below, in which the angular dependence of the Doppler frequency shift associated with the resonant backscatter of ultrasonic waves from a one-dimensionally rough capillary wave surface is investigated. In particular, a plot is obtained of Doppler frequency shift against azimuth angle θ and an attempt is made to explain the resulting curve using simple theoretical concepts.

II. Equipment

A schematic diagram of the experimental layout is shown in Figure 1. The arrangement was equivalent to a monostatic radar, since the difference in elevation of the transmitter and receiver above the water surface was less than the uncertainty involved in measuring this angle. A capillary wave surface, rough in one dimension, was produced on the water by a steady airflow from a large fan. Unfortunately, this airflow was not constant across the entire width of the tank and as a result the wave fronts produced had slight curvature towards the edges of the tank. However, the high directivity of the transmitting–receiving transducer beam patterns reduced this effect somewhat. The ends of the water tank were sloped at an angle of $\sim 45°$ in order to eliminate undesirable wave reflections and standing wave effects.

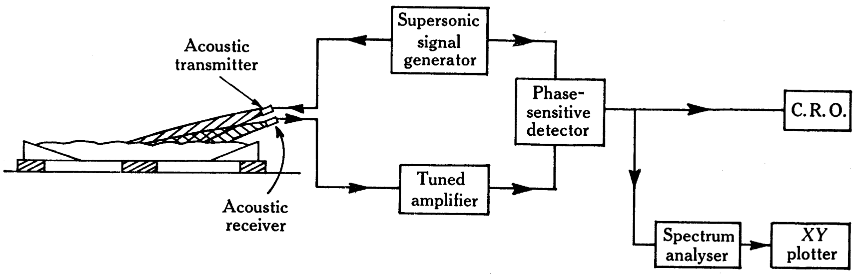

Fig. 1.—Schematic diagram of the experimental arrangement for observing acoustic wave backscatter.

The receiver and transmitter were similar crystal transducers with a frequency response which peaked sharply at 36 kHz. As mentioned above, they were both highly directional. The operating frequency f generally employed was 36 kHz to correspond with the maximum power output, although other frequencies in the range 35–40 kHz were employed at times. The signal from the receiving transducer was passed via a specially constructed tuned amplifier and filter to a phase discriminator, where its phase was matched with that of the output signal. The output from the discriminator was a phase–time relationship which could be examined visually on a C.R.O. and was also passed directly to the spectrum analyser (Fenlow SA2). The advantages of this real-time analysis as against recording the phase–time signal on magnetic tape and subsequently analysing via a tape loop are obvious. However, since the production of a single spectrum over a suitable frequency range required anything from 20 to 50 min, it was difficult to maintain exactly constant experimental conditions for the duration of an observation period.

The elevation angle α of the transmitter and receiver with respect to the horizontal was kept constant throughout the main part of the experiment at a value of $20° \pm 5°$ whilst the azimuth angle θ was varied. A few spectra were obtained also for different values of α, mainly as an additional check on the results and also to check on the dependence of the Doppler frequency shift δf on the angle of elevation.

III. Results

Three spectra which are typical of the output of the spectrum analyser during this experiment are shown in Figure 2. For experimental parameters of $\alpha = 20° \pm 5°$ and $f = 36$ kHz, Figure 2(a) contains a frequency peak which was used to locate one

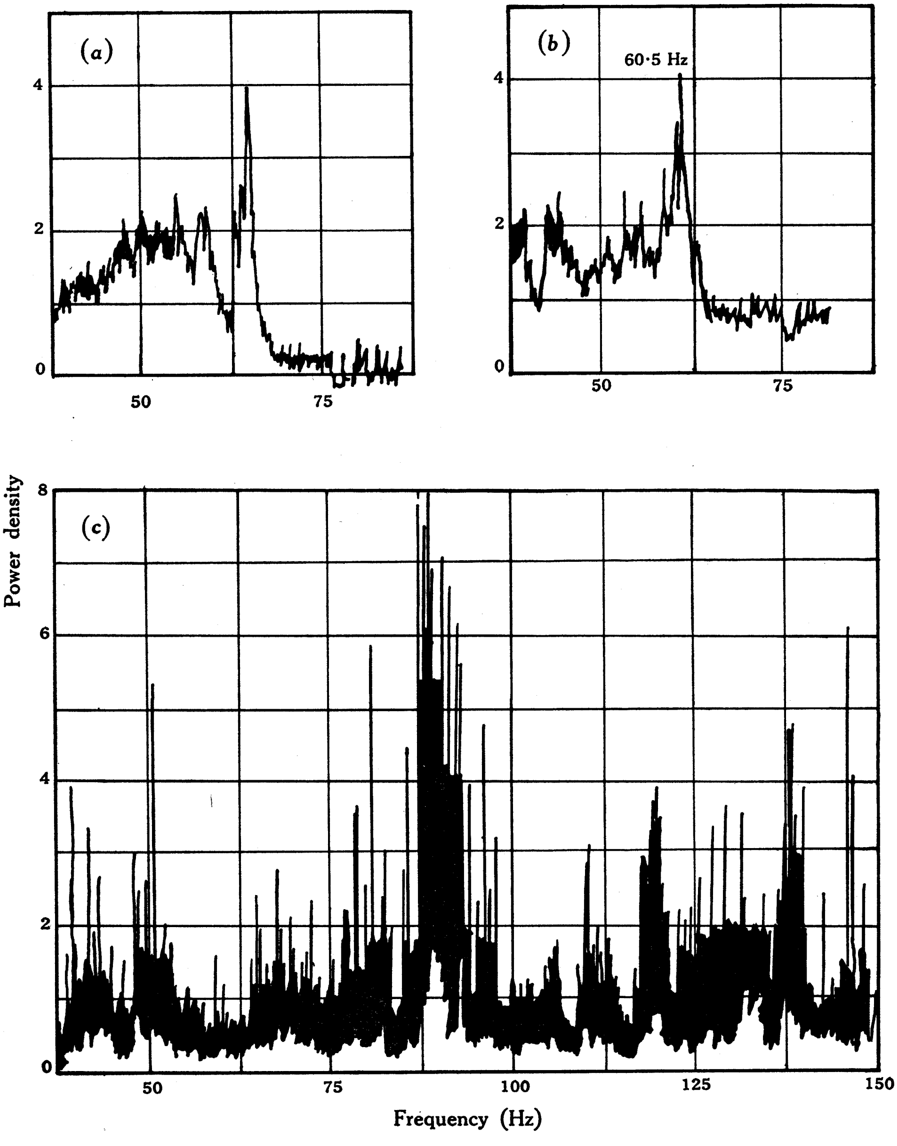

Fig. 2.—Selected frequency spectra illustrating the occurrence of Doppler frequency peaks for different experimental conditions:

Spectrum	Frequency range (Hz)	Bandwidth (Hz)	Smoothing time (s)	α (deg)	θ (deg)	f (kHz)
(*a*)	37·5–208	0·3	15	20 ± 5	20 ± 5	36
(*b*)	37·5–208	0·3	15	20 ± 5	0 ± 5	37
(*c*)	37·5–208	0·3	2	30 ± 5	30 ± 5	36

of the points in Figure 3 (see below), while Figures 2(b) and 2(c) have been included to illustrate the spectra obtained when the other experimental parameters were varied. It should be noted that Figures 2(a) and 2(b) were obtained for an analyser integration time (or smoothing time, i.e. the time spent in integrating the spectral power density at each frequency step in the total frequency scan) of 15 s, while Figure 2(c) was for an integration time of 2 s and thus contains much more noise detail.

Figure 3, which shows a plot of the Doppler frequency shift δf against the azimuth angle θ, was derived from Figure 2(a) and a series of similar spectra. The smooth curve in this figure is represented by a theoretical relation which is considered in the following section.

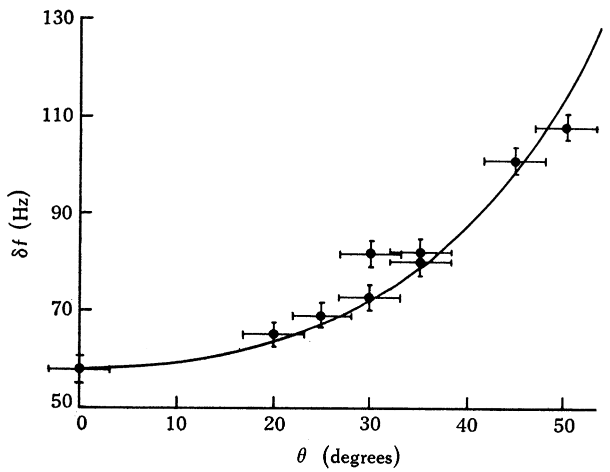

Fig. 3.—Graph of the Doppler frequency shift δf plotted against the azimuth angle θ for experimental parameters $\alpha = 20° \pm 5°$ and $f = 36$ kHz. The smooth curve is a plot of equation (8) with $\alpha = 20°$, $(\delta f)_0 = 64$ Hz, and $m = 1$.

IV. Possible Interpretation

Although any wind-generated wave system of sufficient excitation time will, in principle, contain complete spectra of wavelengths and wave heights (which have an approximately Gaussian distribution), it is generally accepted that backscatter will occur preferentially from that component of the total wave system with a wavelength which satisfies the Bragg resonant condition of an equivalent diffraction grating. This will apply equally to capillary waves and to larger gravity waves, where capillary waves are defined as waves for which the effects of gravity may be neglected. (This occurs in practice for wavelengths less than 0·85 cm; see e.g. Kinsman (1965).) The propagation characteristics of this resonant component may be obtained through solution of the relevant hydrodynamic equations. If a sinusoidal profile is assumed and only small amplitude waves are considered, a first-order linear solution for the phase velocity c of this component gives (Kinsman 1965)

$$c = (S'K)^{\frac{1}{2}}, \tag{1}$$

where $S' = S/\rho$, with S the surface tension and ρ the density of the water, and $K = 2\pi/L$ is the wave number, with L the wavelength of the water wave. When the small amplitude assumption is abandoned, Crapper (1957) provides an exact solution.

However, the phase velocity of Crapper's wave differs from that given by equation (1) by less than 5% when the ratio of wave height to wavelength is less than 0·1, and therefore, for the capillary waves generated in this experiment, equation (1) provides a reasonable approximation. It should be noted that small gravity waves as well as capillary waves were generated on the ripple tank by the air flow but, as the gravity wavelengths were far too large to match the resonance condition required, these components were not considered further.

The experiment demonstrated in fact that, because of the apparent selectivity of the backscatter process, the resonant capillary wave could be treated completely separately from the rest of the surface. In order to determine which component of the total surface actually is the resonant component, and then to calculate the Doppler shift associated with it, consider Figure 4. Here the incident acoustic wave is represented by its propagation constant $\boldsymbol{k}_1$ (wavelength λ) and that part of $\boldsymbol{k}_1$ backscattered from two adjacent wave crests, by $\boldsymbol{k}_2$. For a capillary water wave of

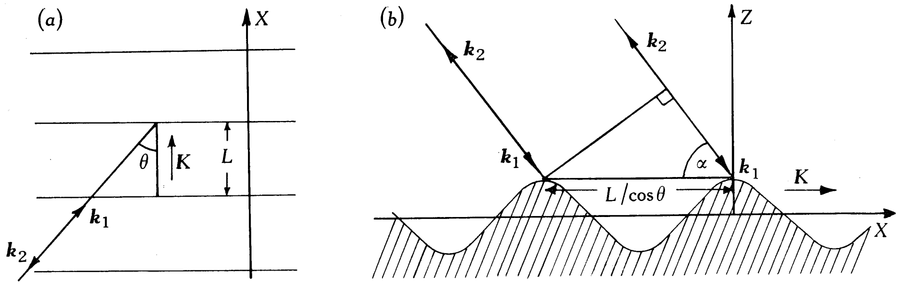

Fig. 4.—Illustrating (a) the horizontal wavelength component and (b) the vertical wavelength component for backscatter.

wavelength L, its wavelength component in the direction of backscatter will be (resolving both horizontally and vertically)

$$L' = L\cos\alpha/\cos\theta\,. \tag{2}$$

For resonance to occur then acoustic waves backscattered from adjacent water wave crests must be in phase, i.e.

$$2L\cos\alpha/\cos\theta = m\lambda\,, \qquad m = 1,2,3,\dots. \tag{3}$$

The Doppler equation for backscatter is

$$\delta f = 2v_{\mathrm{p}}/\lambda\,, \tag{4}$$

where δf is the Doppler frequency shift and v_{p} the phase velocity component in the backscatter direction. Now for a capillary wave of wavelength L the phase velocity is given by equation (1), and from equation (3)

$$c = (2\pi S'\,.\,2\cos\alpha/m\lambda\cos\theta)^{\frac{1}{2}}\,. \tag{5}$$

The component of this *phase* velocity in the backscatter direction then becomes

$$v_{\mathrm{p}} = c\cos\alpha/\cos\theta. \tag{6}$$

Substituting equation (5) into (6) and then into (4), the Doppler frequency shift finally may be written as

$$\delta f = (2\pi S')^{\frac{1}{2}}(2/\lambda)^{3/2}m^{-\frac{1}{2}}(\cos\alpha/\cos\theta)^{3/2}.$$

For first-order scattering ($m = 1$), grazing incidence ($\alpha = 0$), and radially propagating waves ($\theta = 0$), this becomes

$$(\delta f)_0 = (2\pi S')^{\frac{1}{2}}(2/\lambda)^{3/2}, \tag{7}$$

and we have

$$\delta f = (\delta f)_0\, m^{-\frac{1}{2}}(\cos\alpha/\cos\theta)^{3/2}. \tag{8}$$

Thus equation (8) represents the Doppler frequency shift expected in the signal backscattered from what is essentially a random rough surface. As can be seen, it varies with both elevation and azimuth angles. For an ultrasonic frequency of 36 kHz ($\lambda = 0\cdot97$ cm) and a fresh water surface tension of 73 dyn cm^{-1}, $(\delta f)_0 = 64$ Hz, and for this value of $(\delta f)_0$, with $m = 1$ and $\alpha = 20°$, the smooth curve shown in Figure 3 is a plot of equation (8).

The closeness of fit between the experimental points and the theoretical curve in Figure 3 is somewhat surprising in view of the approximate and very simplified nature of the theoretical calculations involved. Nevertheless, it does seem to indicate the general validity of equation (8), and the steps used in deriving it (in particular equation (6); see Section V).

V. Discussion and Conclusions

A set of experimental points has been obtained from the present acoustical data which indicates a relationship between the Doppler frequency shift imposed on a backscattered signal and the horizontal angle between the transmitted beam radial and the propagation vector of the surface waves. A simplified theoretical treatment has shown that these points can be fitted by a function of the form

$$\delta f \propto (\cos\theta)^{-3/2},$$

where preferential backscatter occurs for the Bragg-type resonant condition.

Because of the analogy between acoustic backscatter as described above and the backscatter of electromagnetic waves from the sea surface, it is evident that the results of this experiment can be compared with others involving h.f. radio waves, provided full allowance is made for the difference in phase velocity between capillary and gravity water waves. If the gravity wave phase velocity

$$c = (g/K)^{\frac{1}{2}}$$

(where g is the acceleration due to gravity) is used in place of equation (1) and a line of reasoning identical with that of Section IV is followed, then the Doppler frequency shift expected in the backscatter of acoustic *or* radio waves from a gravity wave water surface is given by

$$\delta f = (\delta f)_0 \, m^{\frac{1}{2}} (\cos\alpha / \cos\theta)^{\frac{1}{2}}, \tag{9}$$

where

$$(\delta f)_0 = (g/2\pi)^{\frac{1}{2}} (2/\lambda)^{\frac{1}{2}}.$$

Thus comparisons based on the analogy between the backscatter of acoustic and electromagnetic waves at a water surface must be made through equation (8) when Bragg resonance occurs with capillary waves, and through equation (9) for resonance with gravity waves.

In this context, the results of Ward (1969) may be compared with equation (9), and hence indirectly with the results of this model. Using a 21·84 MHz radar, his results evinced some support for a relation of the form

$$\delta f \propto (\cos\theta)^{3/2} \tag{10}$$

for resonant backscatter from gravity sea waves. In his derivation of equation (10), Ward used reasoning similar to that of Section IV above, with the exception that in place of equation (6), he used the relation

$$v_{\mathrm{p}} = c\cos\alpha\cos\theta \tag{11}$$

for the phase velocity component of the surface wave in the backscatter direction. It is at this point that the differences between equations (9) and (10) arise. The results reported here appear to provide somewhat stronger evidence in support of equation (6) than do those of Ward with respect to equation (11).

Work has also been conducted on the backscatter of electromagnetic waves at frequencies ranging from 428 to 8910 MHz from capillary and small gravity waves (Wright 1968; Valenzuela and Laing 1970), particularly with regard to the problem of radar sea clutter. Both the papers cited demonstrated the expected dependence of Doppler frequency shift on elevation angle, but neither looked for, or found, any dependence on azimuth angle. It is suggested here, however, that when such experiments are conducted on the open sea (e.g. Valenzuela and Laing 1970) the capillary wave resonant scatterers will be moving nearly isotropically (at least in the forward half-plane, for most wind speeds) and thus little angular dependence would be expected.

A further point concerns the bandwidth of the Doppler spectra. Valenzuela and Laing (1970) found in their radar work that this is dependent on both wave height (wind speed) and wind direction, and conclude that explanations for this may lie in the fluid velocities of the water surface and in the presence of spray (particularly at higher wind speeds) above the surface. The results of the acoustic experiment described above suggest a further possible source for this spectral broadening, namely the angular spread of capillary waves in the wave tank. It has been observed in Section II that the waves were not quite unidirectional but contained a small angular spread which increased towards the edges of the tank. This could be expected to result in a small frequency spread in the backscattered signal about the resonant Doppler frequency. Examination of Figures 2(*a*)–2(*c*) reveals this anticipated bandwidth, which is always less than about 5 Hz, increasing slightly with azimuth angle (as a result of the greater contamination of returns from near the tank walls at larger azimuth). The bandwidth would be much greater for backscatter at the sea surface, and may even be used as a rough measure of wave height by way of empirical relationships such as that found by Valenzuela and Laing (1970).

Finally, some recent Russian work is also of interest. Using a radar with an electromagnetic wavelength of 3·2 cm ($\sim$ 9400 MHz), Mel'nichuk and Chernikov (1971) measured certain characteristics of the signal backscattered from the sea surface. It is evident that, for this wavelength, resonant interactions would occur with sea waves which lie in the transition region between capillary and gravity waves. Nevertheless, using arguments along the lines of Section IV above, one would still expect an inverse dependence of the Doppler frequency shift on the cosine of the azimuth angle for this particular case. To a first approximation, however, a relation of the form

$$\delta f \propto \cos\theta$$

was found experimentally. It is suggested here that this result may not be at variance with the relevant theory, as outlined above, because of the special conditions applying to the experiment. As reported by Mel'nichuk and Chernikov, the short water waves contributing to the resonant interaction were in fact riding on the surface of much larger gravity waves, for whose water particles the *orbital* and *surface drift* velocities combined were of the order of 1 m s^{-1}. This velocity is nearly an order of magnitude greater than the phase velocity of the resonant waves ($<$ 20 cm s^{-1}), and thus would produce the major contribution to the Doppler shift in the radar wave backscattered from the moving water surface. In particular the Doppler shift imposed by the moving water particles (as opposed to that produced by the water waves) would be expected to vary linearly with $\cos\theta$ (as was observed) and effectively mask the variation contributed by the resonant interaction. This special condition will not apply either to the present work or to Ward's (1969) h.f. backscatter, since the orbital velocity of the water particles for any given wave frequency component will, in general, be an order of magnitude less than the phase velocity for that particular component.

Thus it is obvious that the resonant backscatter of electromagnetic waves from the sea surface is capable of providing certain valuable meteorological and oceanographic data (see also Ward 1969; Hasselmann 1971). The interpretation of these data can be aided by models using acoustic wave backscatter similar to that outlined here.

VI. Acknowledgments

I should like to thank Professor J. F. Ward for much fruitful discussion on the work, and also Mr. D. Christy and Mr. D. Galloway for their help in constructing and operating the equipment. The work was supported in part by a contract with the Commonwealth Bureau of Meteorology and partly by the Radio Research Board of Australia.

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