EFFECT OF RAPIDLY RISING PROTON–PROTON TOTAL CROSS SECTIONS ON IDEALIZED EXTENSIVE AIR SHOWERS

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Abstract

A simulation of extensive air showers above 10^{13} eV in which proton–proton scattering takes place partly through a medium-strong interaction is reported. In previous papers the simulation has been shown to be in fair agreement with observational data. The present version includes for the first time the assumption that the total cross section for proton–proton scattering increases with energy, as concluded in a recent paper by Yodh, Pal, and Trefil. The effect of the assumption is to make a noticeably better agreement between the simulation and the data.

I. Introduction

Among the most remarkable properties of high-energy cosmic ray showers, where the primary particle is estimated to have an energy greater than 10^{14} eV, is that the early secondaries are observed to have unusually large components of momenta transverse to the axis of their shower (McCusker, Peak, and Rathgeber 1969). In a different context, Abarbanel, Drell, and Gilman (1969) have brought forward some evidence for the existence of a medium-strong contact interaction between protons which is easiest to observe at very high energies. This interaction has all the right properties to be the interaction which breaks the symmetry SU(3) among hadrons (Ne'eman 1968), and therefore any further evidence for its existence is of considerable interest.

Two previous papers (Campbell 1969, 1972; hereinafter referred to as papers I and II respectively) have examined the possibility of a connection between the medium-strong proton–proton (pp) interaction and cosmic ray data. The evidence for the connection is indirect but persuasive. Numerical simulations of extensive air showers in models which incorporate the contact interaction for pp collisions can produce distributions of any of the experimentally observed quantities. In paper II the computed behaviour of a measure of transverse momenta of the secondaries has been plotted against primary energy, and displayed on the same scales as a collection of points representing observed extensive air showers. The result makes up part of Figure 1 here. A computed curve from paper II behaves almost like a lower envelope

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of the observed distribution. This is not entirely satisfactory, but it stands in strong contrast to the common result of several conventional theories, which is that the measure of momentum should stay constant at about $0\cdot5$ $(\mathrm{GeV}/c)^2$ as the energy increases (Heisenberg 1966).

Certain implicit assumptions are contained in the models used for the numerical simulation. In particular, the normalization of the differential pp cross section in papers I and II implies that the total cross section remains constant with energy once it has reached its asymptotic value of about $4\cdot2\times10^{-26}$ cm^2. Previously there has been no good reason to question this assumption but recent experiments have cast doubt on the existence of Asymptopia for pp scattering. Yodh, Pal, and Trefil (1972) have given a detailed summary of the doubts together with an alternative form of the cross section which increases with energy. The work reported below is a short examination of the effect which their increasing cross section has on the simulation of extensive air showers. In brief, it makes the correspondence between computed behaviour and observed behaviour quite respectable at the energies where observations are most reliable. Section II summarizes the theory from papers I and II and describes the necessary changes which the increasing cross section brings about. The examination concludes with a discussion in Section III.

II. Theory and Consequences

The basic "extensive air shower" is treated at some length in paper II. It evolves through the reactions:

$$\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+n\pi, \tag{1}$$

$$\pi^{\pm} \rightarrow \mu^{\pm}+\mathrm{e}^{\mp}+\nu, \qquad \pi^0 \rightarrow \text{"e.m. cascade"}. \tag{2a, b}$$

Papers I and II contain the details of the simulation. For an examination of the cores of the simulated showers, the reaction (2a) is discarded and only the neutral pions (2b) are considered. In the initial reaction (1), $n = 0$ recovers the elastic-scattering case and $n > 0$ represents the general process which is governed by a differential cross section that factors into an $n = 0$ part and a part for the production of n pions from the vacuum. The differential cross section for elastic scattering follows from the T-matrix element

$$g^2 G^2(t)\, \bar{u}(p_4)\, \gamma_\mu u(p_2)\, \bar{u}(p_3)\, \gamma^\mu u(p_1) + B(s,t), \tag{3}$$

where protons of 4-momenta p_1 and p_2 and mass m scatter to 4-momenta of p_3 and p_4. In the expression (3), the Lorentz-invariant scalar $s = (p_1+p_2)^2$ measures the energy for the reaction and $t = (p_1-p_3)^2$ measures the momentum transfer. The symbols u, $\bar{u}$, and γ are to be read as spinors, adjoint spinors, and Dirac matrices respectively. The medium-strong interaction is carried in the first term of (3), where g is the coupling constant for that interaction ($g^2/4\pi \approx 5$) and $G(t)$ is the magnetic form factor of the proton; the usual strong-interaction effects are abbreviated by $B(s,t)$. The first term of (3), a contact interaction where one power of $G(t)$ arises from each proton, is generally more complicated (Abarbanel, Drell, and Gilman 1969) than the simple vector–vector expression given here, but this term is the asymptotic result which survives when

$$s \gg -t > m^2. \tag{4}$$

Fortunately, the conditions (4) are readily applicable to extensive air showers.

The T-matrix element (3) does not lead to a closed analytic expression for the differential scattering cross section, but the techniques which may be used to find such a cross section for numerical simulation are outlined in paper I. A very approximate summary of the procedure, however, is given by

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \left(G^4(t) + f(t)\exp[2\{\alpha(t)-1\}\log(s/s_0)]\right)\frac{\mathrm{d}\sigma(s,0)}{\mathrm{d}t}, \tag{5}$$

where the structure of $f(t)$ depends on $B(s,t)$, $\alpha(t)$ is the vacuum (Pomeranchuk) Regge trajectory function, and s_0 is a normalizing constant of 1 $(\mathrm{GeV})^2$. In equation (5), the differential scattering cross section $\mathrm{d}\sigma(s,0)/\mathrm{d}t$ for zero momentum transfer factors out. $G^4(t)$ is the energy-independent (s-independent) contribution to $\mathrm{d}\sigma(s,t)/\mathrm{d}t$, while the details of the behaviour of the regular strong interactions in the second term depend on s. In particular, if $\alpha(t)$ is less than unity for physical $t \leqslant 0$, which is supported by the experimental evidence, then the second term decreases as the energy increases. The diffraction peak at $t = 0$ in the differential cross section becomes narrower when s increases in Regge models, as a consequence of the condition $\alpha(t) < 1$, but in (5) the presence of the contact interaction means that the shrinking goes on only until the unshrinkable part in $G^4(t)$ is uncovered. Papers I and II discuss the physical implications of this limit and show how the idea fits in naturally with the observed behaviour of transverse momenta in the extensive air showers.

Paper II is devoted mainly to the replacement of a two-stage pion-production mechanism by the more realistic reaction (1). It is again an approximation that the differential cross section in (5) for $n = 0$ is then multiplied by a factor for the production of the n pions from the vacuum, but the conditions (4) are just what is needed for the approximation to be trustworthy. Under these circumstances the cross section for the reaction (1) written in terms of (5) is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\mathrm{pp} \to \mathrm{pp}+n\pi) = \frac{\mathrm{d}\sigma}{\mathrm{d}t}(\mathrm{pp} \to \mathrm{pp}) \exp(-3x)\, \mathrm{J}_0(2\mathrm{i}x)\frac{x^{n_0}}{n_0!}, \tag{6}$$

where J_0 is a Bessel function, the average number of charged pions x is a function of s, and the process described by (6) includes n_0 neutral pions.

It is possible to evaluate the total cross section $\sigma(\mathrm{p}+\mathrm{p} \to Z)$ from (6), assuming that Z denotes any state composed ultimately of two protons plus pions, via the equation

$$\sigma(s) = \sum_{n=0}^{\infty} \int_{-\infty}^{0} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\mathrm{pp} \to \mathrm{pp}+n\pi)\right)(s,t)\, \mathrm{d}t. \tag{7}$$

Assume also, for the sake of illustration, that s is so large that only the contact-interaction contribution to the elastic scattering is important. Then, given the well-known fit to the data on the magnetic form factor $G(t)$ by the expression

$(1-at)^{-2}$, the substitution of (5) and (6) into (7) generates the equation

$$\sigma(s) = \tfrac{1}{7}a^{-1}\exp(-2x)\,\mathrm{J}_0(2ix)\,\mathrm{d}\sigma(s,0)/\mathrm{d}t\,, \tag{8}$$

where $\mathrm{d}\sigma(s,0)/\mathrm{d}t$ is evidently only a function of s. The constant a is $\sqrt{2}$ $(\mathrm{GeV}/c)^{-2}$. Some doubt exists about the precise behaviour of n_0 as a function of n in equation (6), but luckily there is agreement from cosmic ray data (e.g. Avakian and Pleshko 1968) that n_0 may be represented simply as a number N times n. Since that is so, equation (8) follows from (7) without dependence on N.

The essential message of (8), which is repeated in less obvious form in the numerical simulations based on (3), is that $\mathrm{d}\sigma(s,0)/\mathrm{d}t$ depends strongly on the assumed structure of $\sigma(s)$. Until recently, it was accepted that $\sigma(s)$ was equal to $4\cdot 2\times 10^{-26}$ cm^2 (42 mb) for all s above about 60 (GeV)^2. However, a closer examination of the evidence by Yodh, Pal, and Trefil (1972) has led to the conclusion that the only acceptable total cross sections are those which increase with s at least as fast as

$$\sigma(s) = 38\cdot 8 + 0\cdot 4\log^2(s/s_0) \quad \mathrm{mb}\,. \tag{9}$$

Moreover, this rate of increase with s is the greatest allowed by the Froissart bound (Martin 1966*a*, 1966*b*) from unitarity and some quite general arguments in quantum field theory. Thus the relation (9) itself may be used as a suitable version of $\sigma(s)$ in (8).

In order to find $\mathrm{d}\sigma(s,0)/\mathrm{d}t$ from (8), it is necessary to know how the average multiplicity x of pions depends on s. At the highest energies available from accelerators until a short time ago (e.g. 3×10^{10} eV, corresponding to $s = 60$ (GeV)^2 through $s = 2M(M+E)$), and over the next two orders of magnitude in energy from cosmic ray data, the general belief is that x is logarithmic in s. This is consistent with the higher energy data of McCusker, Peak, and Rathgeber (1969), whose Figure 5 in effect records the ratio of single-cored to multiple-cored showers for primary energies between 2×10^{13} and 2×10^{16} eV. The interpretation of the data for this purpose is indirect: one must assume the relationship $n_0 = Nn$ quoted above, assume that each core is evidence of a cascade (2b) started by a high-energy π^0, and finally accept the view that the number of observed leptons at ground level (the number plotted on the horizontal axis of Figure 5 of the paper by McCusker, Peak, and Rathgeber) times 2×10^9 is the primary energy in electron-volts. I have been unable to identify the original justification of this factor, but several previous papers have treated it as a matter of course, the most recent of which are by Yodh, Wayland, and Pal (1971) and Allan (1972). If all the assumptions are legitimate then so is the dependence of x on s given by a single power of $\log(s/s_0)$. This form of x has been used for the computations repeated in paper II and here.

In the present computations, the consequence of (9) and the logarithmic behaviour of $x(s)$ for (8) is that $\mathrm{d}\sigma(s,0)/\mathrm{d}t$ increases ultimately like $s^{2b}\log^2 s$, where $x(s)-x(s_0) = b\log(s/s_0)$ and b is small and positive. This can be compared with s^{2b} in paper II, although at the energies considered in both computations ($E \leqslant 2\times 10^{16}$ eV) the detailed behaviour of the differential cross section requires all terms and not just the leading term to be taken into account. With the change represented by equations (7), (8), and (9), the theory and subsequent computations are as outlined in papers I and II.

III. Results and Discussion

To say that the transverse momenta of secondaries in a cosmic ray shower are higher than expected is to allow that some unusual process tends to displace each secondary, at the points of production or scattering, through a larger angle than that deduced from conventional reasoning. If the T-matrix element for pp scattering is an expression $B(s,t)$ which contains the conventional information about a shrinking diffraction peak, then the inclusion of the non-shrinking contact-interaction term in (3) provides naturally for the "unusual process" which increases the transverse momenta of secondaries as soon as s is large enough for the contact term to dominate in the differential cross section. This question is considered in paper I.

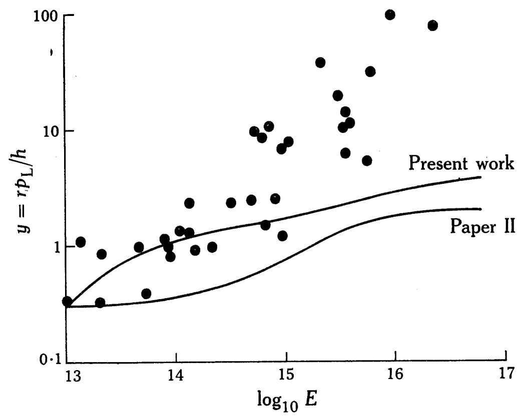

Fig. 1.—Plot of mean values of y (in GeV/c) against primary energy for the simulated showers. The curve C of paper II (curve [57] in that paper) and the curve derived from the computations described in the present work are compared with representative observational points from McCusker, Peak, and Rathgeber (1969).

Now suppose that the basic $\mathrm{d}\sigma(s,t)/\mathrm{d}t$ for reaction (1) is increased by a factor depending only on s, e.g. by the adoption of the new $\sigma(s)$ given by the relation (9). Changes in the average angle for a single scattering follow only from changes in the t-dependence of $\mathrm{d}\sigma(s,t)/\mathrm{d}t$ so that at first sight it seems that the increase has no effect on the geometry of showers. However, the important phrase is "single scattering". Even in the computations of papers I and II, the protons of reaction (1) undergo multiple scatterings, and the effect of (9) here is to increase the number of scatterings in a given shower. Thus cascades (2b) associated with neutral pions from the final events of type (1) in each shower tend to have larger transverse momenta here than in either of the earlier computations.

This qualitative argument is supported by the actual simulation. Figure 1 is almost the same as the figure in paper II, except that only one of the continuous curves representing a simulation from that paper (with $G(t) = (1-at)^{-2}$) is retained for comparison with the curve extracted from the present work. The quantity y in Figure 1, which is a measure of transverse momentum, is equal to rp_{L}/h, where p_{L} is the component of momentum of a π^0 in (2b) along the direction of the original proton primary, r is the separation of the cores of a pair of cascades at ground level, and h is the altitude of production of the pion. All other details needed for the understanding of the computation and Figure 1 are given in papers I and II.

When the relation (9) is used to normalize $\mathrm{d}\sigma(s,t)/\mathrm{d}t$, the resulting summary curve of the computation is notably closer than the curve C of paper II to the experimental points shown in Figure 1. In particular, it rises faster than C over energies between 10^{13} and 10^{14} eV and then lies within the range of the observed points, rather than just below, over the next order of magnitude in energy. Thus this simplified model of extensive air showers, together with the new ingredient of a realistic pp total cross section, brings the simulation directly into line with observations for primary energies up to 10^{15} eV. The apparent horizontal break in Figure 1 near $y = 4$ GeV/c and $E = 10^{15}$ eV indicates a possible depletion of proton primaries in favour of heavier nuclei (McCusker, Peak, and Rathgeber 1969), so that curves may not be required to match the observed points for the higher energies.

Although Figure 1 provides some additional support, probably from an unexpected quarter, for the cross section of Yodh, Pal, and Trefil (1972), its main interest is in the reinforcement it gives to the circumstantial evidence in cosmic ray data that a separate medium-strong interaction exists.

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