A NOTE ON RESONANCE ANGULAR DISTRIBUTIONS WITH APPLICATION TO NUCLEON–ANTINUCLEON SCATTERING AND ANNIHILATION

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Abstract

Angular distributions have been calculated for the reactions $\bar{N}N \to \bar{N}N$ and $\bar{N}N \to 2$ pseudoscalar mesons by assuming the formation of an intermediate isolated state. The scattering distributions are only found to have the characteristic oscillatory shape for unnatural parity resonances whereas those for natural parity states are smooth. The annihilation distributions display an exact degeneracy with respect to the interchange of orbital and total angular momenta. For both reactions, the width of the forward and backward peak is approximately inversely proportional to the resonance spin.

I. INTRODUCTION

The spin of intermediate resonant states in two-body reactions is reflected in the angular distribution of the decay products, to an extent which depends on the channel spin multiplicities. In the simplest example of spinless particles, the resonant amplitude is proportional to the Legendre polynomial $\mathrm{P}_l(\cos\theta)$, where l is the relative orbital angular momentum and the spin of the resonance. In the scattering of two spin-$\frac{1}{2}$ particles, however, in general there is interference between singlet and triplet spin states as well as between the various magnetic substates, so that the dependence on the resonance spin J is obscured. This paper considers the angular distributions for two cases of practical interest: (*a*) elastic scattering of two spin-$\frac{1}{2}$ particles and (*b*) the production of two spinless particles from spin-$\frac{1}{2}$ projectiles. The distributions are calculated assuming that the intermediate state is a pure two-particle state, but the effects of some possible mixing mechanisms are also discussed.

II. ANGULAR DISTRIBUTIONS

The distribution, in the centre of mass system, of polar angles θ about the incident direction and proceeding through an intermediate state of spin J and definite isospin may be expressed as

$$W(\theta) = \sum_{m,m'} |A_{mm'}|^2 . \tag{1}$$

The amplitudes are given by

$$A_{mm'}(\theta) = \sum_{l,l';\, s,s'} \gamma_{ls}\gamma_{l's'}(l\,s\,0\,m|J\,m)(l'\,s'\,m-m'\,m'|J\,m)\,Y_{l'}^{m-m'}(\theta,\phi) \tag{2}$$

where l, l' and s, s' are the orbital angular momenta and channel spins respectively of the incident and outgoing systems, m, m' are the components of s, s', the constants γ

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measure the coupling strengths of the resonances to the appropriate channels, and the quantities in parentheses are Clebsch–Gordan coefficients. The dependence on the unknown coupling constants can be removed by assuming the dominance of single two-particle states in both channels so that only one term of the sum in equation (2) remains.

TABLE 1

ANGULAR DISTRIBUTIONS FOR SPIN-$\frac{1}{2}$ PARTICLES IN RESONANT SCATTERING AND ANNIHILATION

The coefficients a of the Legendre polynomials in the distributions

$$W(\theta) = A_0\{1 + \Sigma\, a_{2r} P_{2r}(\cos\theta)\}$$

are given for resonant scattering of spin-$\frac{1}{2}$ particles (where the summation is from $r = 1$ to J) and for the annihilation of a spin-$\frac{1}{2}$ fermion–antifermion pair into two scalar or pseudoscalar bosons (summation from $r = 1$ to $K = \min(l, J)$)

(1) *Scattering with $J = l$*

$s = 0$ state	a_2	a_4	a_6	a_8	a_{10}	$s = 1$ state	a_2	a_4	a_6	a_8	a_{10}
$^1S_0$	0										
$^1P_1$	2·0					$^3P_1$	0·5				
$^1D_2$	1·43	2·57				$^3D_2$	0·357	1·143			
$^1F_3$	1·33	1·64	3·03			$^3F_3$	0·750	0·045	1·705		
$^1G_4$	1·30	1·46	1·82	3·43		$^3G_4$	0·938	0·364	0·004	2·193	
$^1H_5$	1·29	1·39	1·57	1·98	3·78	$^3H_5$	1·038	0·615	0·141	0·079	2·626

(2) *Scattering with $J = l \pm 1$*

$s = 1$ state	a_2	a_4	a_6	a_8
$^3S_1$	0			
$^3P_0$	0			
$^3D_1$	0·5			
$^3P_2$	0·7			
$^3F_2$	0·914	0·286		
$^3D_3$	0·980	0·449		
$^3G_3$	1·063	0·676	0·189	
$^3F_4$	1·091	0·760	0·316	
$^3H_4$	1·131	0·881	0·517	0·137
$^3G_5$	1·146	0·927	0·599	0·237

(3) *Annihilation*

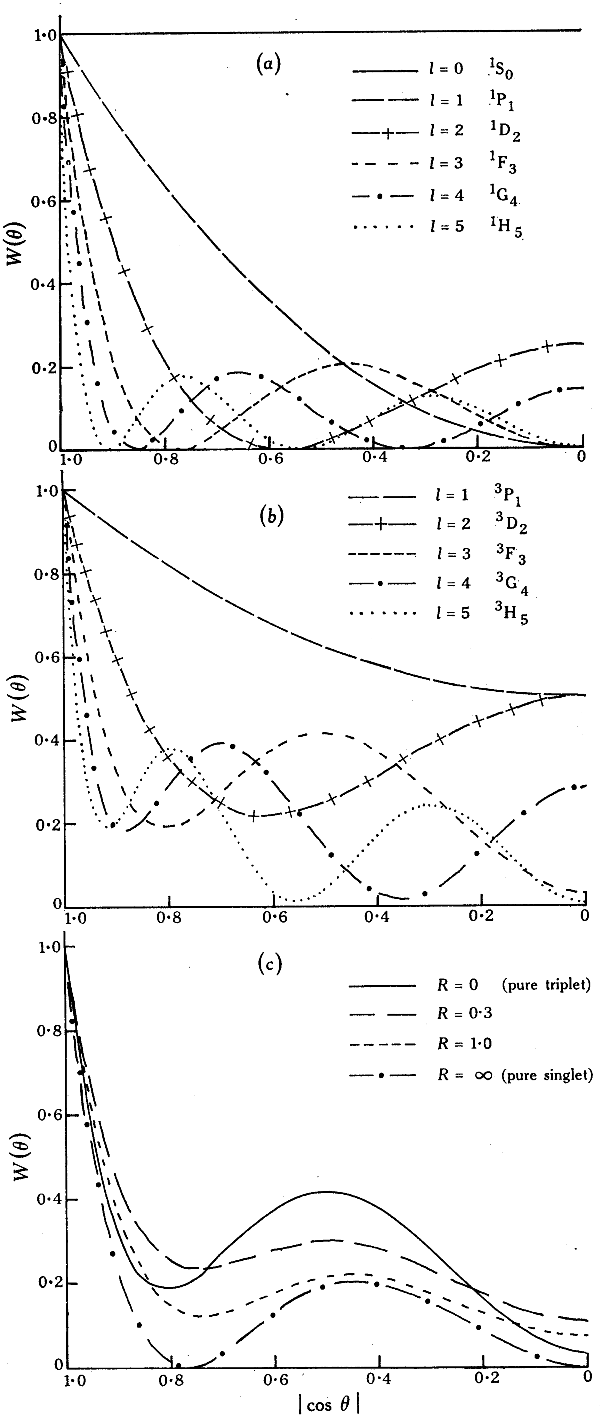
$f, \bar{f}$ states*	a_2	a_4	a_6	a_8	a_{10}
$^3S_1$, $^3P_0$	0				
$^3P_2$, $^3D_1$	1·0				
$^3D_3$, $^3F_2$	1·143	0·857			
$^3F_4$, $^3G_3$	1·190	1·052	0·758		
$^3G_5$, $^3H_4$	1·212	1·133	0·970	0·685	
$^3H_6$, $^3I_5$	1·224	1·175	1·070	0·902	0·630

* States in fermion–antifermion system.

(a) *Scattering of Spin-$\frac{1}{2}$ Particles*

Such reactions allow both singlet and triplet scattering states and link states of equal intrinsic parity, so that the orbital angular momenta l and l' must differ by 0 or 2. Both unnatural and natural parity resonances can be formed via the couplings $J = l$ and $J = l \pm 1$ and the respective angular distributions have significantly different shapes.

Fig. 1 (*opposite*).—Normalized angular distributions for the resonant scattering of spin-$\frac{1}{2}$ particles with $J = l$ in (*a*) the singlet state ($s = 0$), (*b*) the triplet state ($s = 1$), and (*c*) a mixed singlet and triplet state ($J = 3$).


(a)
l = 0 ¹S₀
l = 1 ¹P₁
l = 2 ¹D₂
l = 3 ¹F₃
l = 4 ¹G₄
l = 5 ¹H₅
(b)
l = 1 ³P₁
l = 2 ³D₂
l = 3 ³F₃
l = 4 ³G₄
l = 5 ³H₅
(c)
R = 0 (pure triplet)
R = 0·3
R = 1·0
R = ∞ (pure singlet)
W(θ)
1·0
0·8
0·6
0·4
0·2
0
|cos θ|

(1) $J = l$

The singlet and triplet distributions in this case are given respectively by

$$W_s(\theta) = |\mathrm{P}_J(\cos\theta)|^2 \tag{3a}$$

and

$$W_t(\theta) = \sum_{m,m'} |(J\,1\,0\,m|J\,m)(J\,1\,m-m'\,m'|J\,m)\,Y_J^{m-m'}|^2. \tag{3b}$$

They are listed in section (1) of Table 1 as series of Legendre polynomials and are displayed in Figures 1(*a*) and 1(*b*) for $J \leqslant 5$. The distributions are dominated by the highest order Legendre polynomial P_{2J} and consequently display a sharp peaking at $\cos\theta = 1$ which is followed by an oscillating structure. The triplet case has maxima and minima slightly shifted in phase from the maxima and zeros of $|\mathrm{P}_J|^2$, and hence of the singlet distribution, and also shows secondary maxima approximately twice the height of those in the singlet distributions.

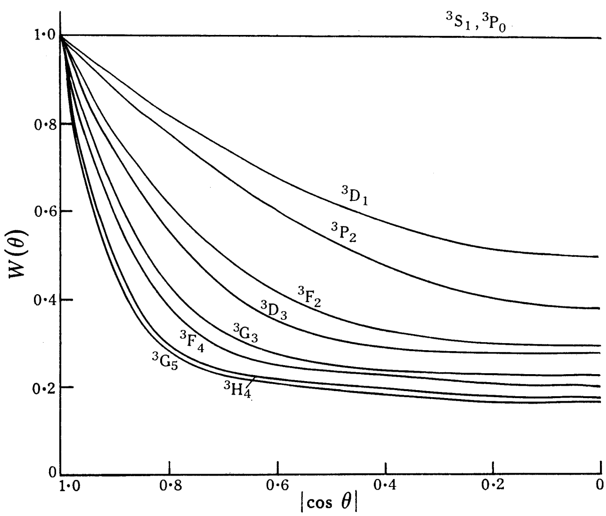


Fig. 2.—Normalized angular distributions for the resonant scattering of spin-$\frac{1}{2}$ particles with $J = l\pm 1$ in the triplet state ($s = 1$).

If spin mixing of the resonance is accounted for, the singlet, triplet, and spin-flip ($s \neq s'$) amplitudes are added coherently and the distribution depends on the parameter $R = \gamma_0^2/\gamma_1^2$ which expresses the relative strengths of the singlet and triplet channels. Figure 1(*c*) shows that the general features of the distributions are not significantly changed except for the transition region near $R = 0\cdot 3$ where the structure is weakened.

(2) $J = l\pm 1$

For this coupling, only triplet states are permitted and, in general, the orbital angular momenta will be equal in the ingoing and outgoing channels. The distributions are given in section (2) of Table 1 and Figure 2, where it can be seen that, in contrast to the previous case, they are dominated by the polynomial P_2 and thus are quite smooth and less sharply peaked. It is also evident that the distributions are weakly grouped into (l,J) interchange pairs, such as $^3F_2$ (3,2) and $^3D_3$ (2,3), in analogy to a corresponding case with exact degeneracy in the two spin-zero final state as discussed in subsection (*b*) below. Amplitudes calculated by allowing mixed l values ($l' = l\pm 2$) give very similar distributions and the main effect of such interference is to weaken the pairing effect observed in Figure 2.

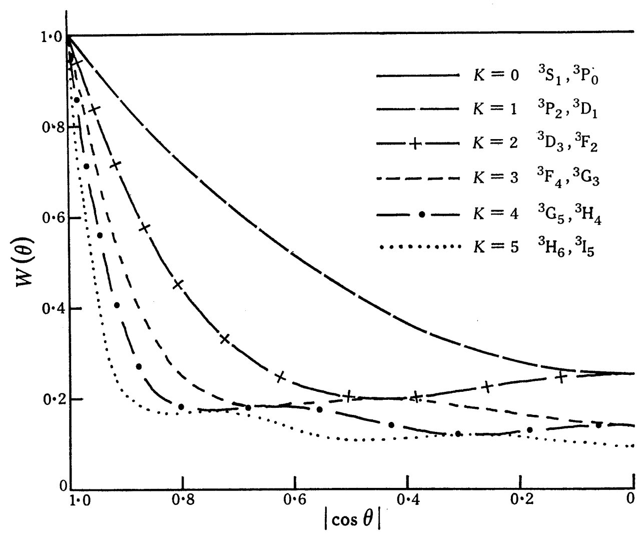


Fig. 3.—Normalized angular distributions for the annihilation of a spin-$\frac{1}{2}$ fermion–antifermion pair into two scalar or pseudoscalar bosons. The spectroscopic notation refers to the fermion–antifermion pair and K is the minimum value of l and J. (Note that the reaction $\bar{N}N \rightarrow 2\pi^0$ can only proceed through even J states.)

(*b*) *Formation of Spinless Particles from Spin-$\frac{1}{2}$ Projectiles*

For these reactions the intrinsic parity of the initial and final states may be the same or opposite. The former case is relevant to a few nuclear reactions (e.g. $^{28}Si(\alpha, p)^{31}P$ and the inverse reaction) and includes channel spins $s = 0, 1$ for the spin-$\frac{1}{2}$ projectiles with $J = l = l'$, giving the following (unnormalized) distributions

$$W(\theta) = |P_J(\cos\theta)|^2, \quad s = 0; \qquad W(\theta) = |P_J^1(\cos\theta)|^2, \quad s = 1. \tag{4}$$

When the intrinsic parities of the ingoing and outgoing channels are opposite, the orbital angular momenta must differ by one unit, so that $J = l' = l\pm 1$ and $s = 1$. The distribution is then given by

$$W_{lJ}(\theta) = \sum_m |(l\,1\,0\,m|J\,m)\, Y_J^m|^2 = \frac{1}{2\pi}\sum_m (l\,1\,0\,m|J\,m)^2 \frac{2J+1}{2}\frac{(J-|m|)!}{(J+|m|)!}|P_J^m|^2 . \tag{5}$$

Using the recurrence formulae (Minami 1954)

$$P^1_{l+1} = \{(l+1)/\sin\theta\}\{(P_l-(\cos\theta)P_{l+1})\}, \tag{6a}$$

$$P^1_l = \{(l+1)/\sin\theta\}\{(\cos\theta)P_l-P_{l+1})\}, \tag{6b}$$

equation (5) leads to the expressions

$$W_{l,l+1} = \frac{1}{4\pi}\frac{(l+1)(2l+3)}{2l+1}\operatorname{cosec}^2\theta\{P_l^2-2(\cos\theta)P_lP_{l+1}+P_{l+1}^2\}, \tag{7a}$$

and

$$W_{l+1,l} = \frac{1}{4\pi}\frac{(l+1)(2l+1)}{2l+3}\operatorname{cosec}^2\theta\{P_l^2-2(\cos\theta)P_lP_{l+1}+P_{l+1}^2\}, \tag{7b}$$

and so the distributions are equal in shape under the interchange of l and J. (Minami (1954) has pointed out a similar ambiguity in πN scattering.) The distributions are

TABLE 2

PEAK WIDTHS FOR SCATTERING AND ANNIHILATION DISTRIBUTIONS

$K = \min(l, J)$

$s = 0$ state	$\cos\theta_{\frac{1}{2}}$	$\theta_{\frac{1}{2}}$ (deg)	$(J+1)\theta_{\frac{1}{2}}$	$s = 1$ state	$\cos\theta_{\frac{1}{2}}$	$\theta_{\frac{1}{2}}$ (deg)	$J\theta_{\frac{1}{2}}$
			(1) *Scattering with $J = l$*				
$^1S_0$	—						
$^1P_1$	0·705	45·1	90·2	$^3P_1$	0	90	90
$^1D_2$	0·900	25·9	77·7	$^3D_2$	0·865	30·1	60·2
$^1F_3$	0·950	18·2	72·8	$^3F_3$	0·938	20·3	60·9
$^1G_4$	0·968	14·5	72·5	$^3G_4$	0·964	15·6	62·4
$^1H_5$	0·975	12·9	77·4	$^3H_5$	0·979	11·8	59·0

(2) *Scattering with $J = l\pm1$*

$s = 1$ state	K	$\cos\theta_{\frac{1}{2}}$	$\theta_{\frac{1}{2}}$ (deg)	$J\theta_{\frac{1}{2}}$	$\frac{1}{2}(J+K)\theta_{\frac{1}{2}}$
$^3S_1$	0	—			
$^3P_0$	0	—			
$^3D_1$	1	0	90	90	90
$^3P_2$	1	0·440	63·9	127·8	95·9
$^3F_2$	2	0·700	45·7	91·4	91·4
$^3D_3$	2	0·760	40·5	121·5	101·3
$^3G_3$	3	0·844	32·4	97·2	97·2
$^3F_4$	3	0·866	30·0	120·0	105·0
$^3H_4$	4	0·901	25·9	103·6	103·6
$^3G_5$	4	0·915	23·8	119·0	107·1

(3) *Annihilation*

$f, \bar{f}$ states*	K	$\cos\theta_{\frac{1}{2}}$	$\theta_{\frac{1}{2}}$ (deg)	$(K+1)\theta_{\frac{1}{2}}$
$^3S_1$, $^3P_0$	0	—		
$^3P_2$, $^3D_1$	1	0·580	54·6	109·2
$^3D_3$, $^3F_2$	2	0·834	33·4	100·2
$^3F_4$, $^3G_3$	3	0·905	25·1	100·4
$^3G_5$, $^3H_4$	4	0·938	20·3	101·5
$^3H_6$, $^3I_5$	5	0·959	16·5	99·0

* States in fermion–antifermion system.

given in section (3) of Table 1 and Figure 3, where they are labelled by the single parameter K which is defined as the minimum value of l and J. The mild oscillations arise from more equally contributing Legendre polynomials than were observed in subsection (*a*) above. Note also the contiguous turning points in the consecutive distributions.

III. Applications

Resonance contributions with angular momenta $l' \geqslant 1$ to an experimental angular distribution will often be discernible above the background yield only by means of the strong peaking at $\theta = 0°$ and 180°. It can be seen in the figures that the width of the peak depends systematically on the angular momenta concerned, so that a measurement of the slope can be used to restrict the resonance parameters to a narrow range of possibilities. Table 2 lists the values of the angle $\theta_{\frac{1}{2}}$ at the half-height of the peak (that is, $\sigma(\theta_{\frac{1}{2}}) = \frac{1}{2}\sigma_{\max}$) for each of the three classes of reactions discussed above. It can be seen that $\theta_{\frac{1}{2}}$ bears an approximate inverse proportionality to the resonance spin with the proportionality constant dependent on the reaction type. Furthermore, in the scattering case, the constant also depends independently on the parity and intrinsic spin (or G-parity for mesonic resonances) of the resonance. In principle, the level of the resonant term away from the $\cos\theta = 1$ peak, and the intensity of the oscillations there, are indicative firstly of the reaction type and secondly of the resonance spin. However, in general such parameters are immeasurable because of the background.

Table 3

Observed peak widths near resonances in $\bar{\mathrm{p}}\mathrm{p}$ annihilation

Data from Nicholson *et al.* (1969)

Reaction	$\bar{\mathrm{p}}$ momentum (GeV/c)	$\cos\theta_{\frac{1}{2}}$	Possible spins K	Possible spins J
$\bar{\mathrm{p}}\mathrm{p} \to \pi^+\pi^-$	0·70	$0 \cdot 88 \pm 0 \cdot 01$	(2), 3	(2), 3, 4
	1·99	$0 \cdot 96 \pm 0 \cdot 01$	5	5, 6
$\bar{\mathrm{p}}\mathrm{p} \to \mathrm{K}^+\mathrm{K}^-$	0·70	$0 \cdot 90 \pm 0 \cdot 01$	3	3, 4
	2·40	$0 \cdot 90 \pm 0 \cdot 01$	3	3, 4

We shall now apply the above results to observations of heavy boson resonances in nucleon–antinucleon annihilation near 1930 MeV (the so-called S region). There are no scattering data available which show peaking at 180°, while $\theta = 0°$ is totally dominated by diffraction and Coulomb scattering effects. (Although the data of Cline *et al.* (1968) do display an energy-dependent peaking in the backward hemisphere, the cross section does not increase monotonically to 180°, but in fact peaks at $\cos\theta = 0 \cdot 8$, and thus the above analysis appears to be violated and a peak width $\theta_{\frac{1}{2}}$ cannot be defined.)

Nicholson *et al.* (1969) have published angular distributions for the reactions $\bar{\mathrm{p}}\mathrm{p} \to \pi^+\pi^-$ and $\bar{\mathrm{p}}\mathrm{p} \to \mathrm{K}^+\bar{\mathrm{K}}^-$ over a range of energies which, they claim, embraces two resonances. The above formulae are applicable only where interference is minimized, i.e. on the high and low energy extremes of the resonant region. Table 3 shows that the $\pi^+\pi^-$ case suggests spins of $J = 3, 4$ (lower resonance) and $J = 5, 6$ (higher resonance) in agreement with the results of Nicholson *et al.*, while the kaon final channel data are more consistent with $J = 3, 4$ for both resonances.

IV. Conclusions

The above results show that single resonance angular distributions for nucleon–antinucleon scattering have an oscillating shape like $\mathrm{P}_{2J}(\cos\theta)$ only for unnatural parity states, while interference between Legendre polynomials removes the structure for natural parity states and $P_2(\cos\theta)$ becomes the dominant term. In the case of nucleon–antinucleon annihilations into two pseudoscalar mesons, where only unnatural parity states can be formed, the distributions are exactly degenerate with respect to interchange of l and J and, furthermore, consecutive non-degenerate distributions have opposite phase with respect to turning points.

All the distributions are peaked at $\theta = 0°$ and 180° and the peak width, as measured by $\theta_{\frac{1}{2}}$, is inversely proportional to the spin. However, the proportionality constant for $\bar{N}N$ scattering further depends on the parity and intrinsic spin of the resonance.

V. References

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