

Dipole States in ^{86}Sr , ^{88}Sr and ^{90}Zr

R. P. Robertson

Department of Physics, University of Melbourne, Parkville, Vic. 3052.

Abstract

A shell model method for calculating dipole states in $N > Z$ even nuclei is discussed. By assuming isospin to be a good quantum number, the spectrum resolves into two classes of states differing in isospin. Applications of this model to ^{88}Sr and ^{90}Zr essentially differ from previous calculations by the use of a new realistic force in the residual interaction. Dipole states in ^{86}Sr are also investigated along with the effect of correlating the ground state wavefunction.

1. Introduction

In the present paper, an outline is given of a well-known method due to Goulard *et al.* (1968) for calculating dipole states in $N > Z$ even nuclei. In addition to the conventional particle-hole ($p-h$) modes involving single particle transitions between orbitals of different parity, a particular type of $2p-2h$ excitation is included, which allows the formation of a basis set of states with definite isospin. Mixing between states of different isospin arises mainly from differences between neutron and proton radial wavefunctions and the residual Coulomb interaction. Both of these effects are small (Mahaux and Weidenmuller 1969) and can be ignored. For a charge independent nuclear interaction, the spectrum resolves into states of isospin $T+1$ and T , where the ground state isospin T is assumed to be pure with third component given by $T_3 = \frac{1}{2}(N-Z)$. The model is described in Section 2 and then used in Section 3 to calculate dipole states in three $J^\pi = 0^+$ ground state nuclei. A discussion of the states in ^{86}Sr , ^{88}Sr and ^{90}Zr together with comparisons with experimental spectra for the latter two nuclei are given in Section 4.

2. Theory

The problem is to solve the Schrödinger equation

$$\mathcal{H}|E\rangle = E|E\rangle, \quad (1)$$

where the nuclear eigenstates $|E\rangle$ are to be orthonormal linear combinations of shell model states coupled to a definite isospin and to an angular momentum and parity $J^\pi = 1^-$. Equation (1) is solved using the Hartree-Fock approximation (Eisenberg 1963) by separating the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$. Eigenfunctions of the unperturbed part $\mathcal{H}_0$ comprise the configuration basis while $\mathcal{H}'$ expresses their interactions. In the ground state configuration for the $N > Z$ even nucleus, j represents levels unoccupied by both neutrons and protons and $\bar{j}$ represents the doubly occupied levels. A neutron excess subshell is denoted by j_i , where i labels the subshells. Fig. 1a represents the type of excitation considered in the parent nucleus ($N+1, Z-1$).

The neutron n and proton-hole $\bar{p}$ are coupled to $J^\pi = 1^-$ with quantum numbers j and $\bar{j}$ implicitly understood. The parent-state ket $|n\bar{p}\rangle$ with isospin $T+1$ is rotated in isospin space to form the analogue ket with the same isospin

$$|A\rangle = (T+1)^{-\frac{1}{2}}(|v\rangle + T^{\frac{1}{2}}|n\bar{p}; a\rangle),$$

where

$$|n\bar{p}; a\rangle = \sum_i g_i |n\bar{p}; (p_i \bar{n}_i)_0\rangle \quad \text{with} \quad g_i = \{(2j_i + 1)/2T\}^{\frac{1}{2}}.$$

For simplicity the $2p-2h$ excitation $|n\bar{p}; (p_i \bar{n}_i)_0\rangle$, shown in Fig. 1b, includes only the coupling $J^\pi = 0^+$ between the proton p_i and neutron-hole $\bar{n}_i$. The isovector excitation

$$|v\rangle = (1/\sqrt{2})(|n\bar{n}\rangle - |p\bar{p}\rangle)$$

has the 'active' particles and holes outside the neutron excess region (see Figs 1c and 1d). A configuration orthogonal to $|A\rangle$ is the anti-analogue

$$|\bar{A}\rangle = (T+1)^{-\frac{1}{2}}(-T^{\frac{1}{2}}|v\rangle + |n\bar{p}; a\rangle)$$

with isospin T . In contrast to $|v\rangle$ and $|n\bar{p}; a\rangle$, the isoscalar excitation

$$|s\rangle = (1/\sqrt{2})(|n\bar{n}\rangle + |p\bar{p}\rangle)$$

and the excitation

$$|n\bar{p}; \bar{a}\rangle = \sum_i \bar{g}_i |n\bar{p}; (p_i \bar{n}_i)_0\rangle, \quad \text{where} \quad \sum_i g_i \bar{g}_i = 0,$$

contribute only to T excitations of the nucleus. The basis set is completed by including the T isospin configurations $|p_i \bar{p}\rangle$ and $|n\bar{n}_i\rangle$ shown in Figs 1e and 1f.

Proton and neutron energies for particles in level j are assumed to be related by

$$e_p(j) = e_n(j) + \Delta_C - U(j), \quad (2)$$

and similar relations are assumed to hold for particles in levels j and j_i . The Coulomb displacement energy Δ_C includes the neutron-proton mass difference and is assumed to be level-independent. The symmetry energy $U(j)$ expresses the greater attraction on a proton, relative to a neutron, by the excess neutrons. A hole energy is the negative energy of the particle whose absence corresponds to the hole. The basis configuration energies are evaluated using the equations

$$\langle A | \mathcal{H}_0 | A \rangle = e_n(j) - e_n(\bar{j}) + \frac{1}{2}(T+1) \left((2T+1)U(j) - U(\bar{j}) - \sum_i (2j_i+1)U(j_i) \right), \quad (3a)$$

$$\langle \bar{A} | \mathcal{H}_0 | \bar{A} \rangle = e_n(j) - e_n(\bar{j}) + \frac{1}{2}(T+1) \left((T+2)U(\bar{j}) - TU(j) - T^{-1} \sum_i (2j_i+1)U(j_i) \right), \quad (3b)$$

$$\langle n\bar{p}; \bar{a} | \mathcal{H}_0 | n\bar{p}; \bar{a} \rangle = e_n(j) - e_n(\bar{j}) + \left(U(\bar{j}) - \frac{1}{2}T \sum_{i \neq j} (2j_i+1)U(j_i) \right), \quad (3c)$$

$$\langle s | \mathcal{H}_0 | s \rangle = e_n(j) - e_n(\bar{j}) + \frac{1}{2}\{U(j) - U(\bar{j})\}, \quad (3d)$$

$$\langle p_i \bar{p} | \mathcal{H}_0 | p_i \bar{p} \rangle = e_n(j_i) - e_n(\bar{j}) + U(\bar{j}) - U(j_i), \quad (3e)$$

$$\langle n\bar{n}_i | \mathcal{H}_0 | n\bar{n}_i \rangle = e_n(j) - e_n(j_i). \quad (3f)$$

The perturbing part of the Hamiltonian $\mathcal{H}'$ is assumed to be well represented by a two-body interaction V_{12} . Writing $|\bar{j}_3 j_2; J'0, T'0\rangle$ for hole $\bar{j}_3$ and particle j_2 coupled

to angular momentum J' (projection 0) and isospin T' (projection 0), gives the general p - h matrix element as

$$\langle \bar{j}_3 j_2; J'0, T'0 | V_{12} | \bar{j}_1 j_4; J'0, T'0 \rangle = \sum_{J, T} (2J+1)(2T+1) \left\{ \begin{matrix} j_4 & j_3 & J \\ j_2 & j_1 & J' \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & T' \end{matrix} \right\} \langle j_1 j_2; J0, T0 | V_{12} | j_3 j_4; J0, T0 \rangle. \quad (4)$$

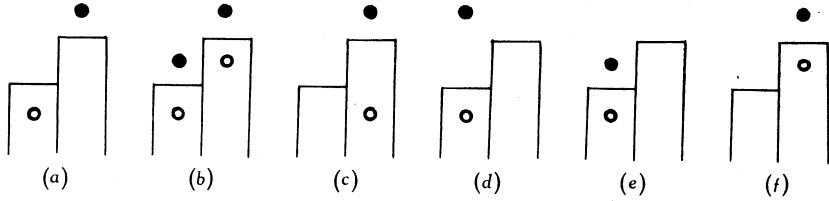


Fig. 1. Excitations forming a basis set. Isospin analogue to (a) the parent state involves (b) $2p$ - $2h$ excitations and (c) and (d) $1p$ - $1h$ excitations. Excitations (e) and (f) contribute only to T -states of the nucleus.

Table 1. Symmetric interactions $\langle | \mathcal{H} - \mathcal{H}_0 | \rangle$ between basis configurations

For convenience quantum numbers in V_s and V_v are suppressed

Configuration	$ A\rangle$	$ \bar{A}\rangle$	$ n\bar{p}; \bar{a}\rangle$	$ s\rangle$	$ p_i \bar{p}\rangle$	$ n\bar{n}_i\rangle$
$ A'\rangle$	D_1	0	0	0	0	0
$ \bar{A}'\rangle$		D_2	0	0	$(T+\frac{1}{2}T)^{\dagger} V_v$	$-(T+\frac{1}{2}T)^{\dagger} V_v$
$ n\bar{p}'; \bar{a}'\rangle$			D_3	0	$\bar{g}_i \hat{j}_i^{-1} V_v$	$-\bar{g}_i \hat{j}_i^{-1} V_v$
$ s'\rangle$				V_s	$(1/\sqrt{2})V_s$	$(1/\sqrt{2})V_s$
$ p_i \bar{p}'\rangle$					$\frac{1}{2}(V_s + V_v)$	$\frac{1}{2}(V_s - V_v)$
$ n\bar{n}_i'\rangle$						$\frac{1}{2}(V_s + V_v)$

For dipole states, $J' = 1$ and the possible isospin couplings $T' = 0$ or 1 correspond to the isoscalar interaction $V_s(j_2 \bar{j}_3; j_4 \bar{j}_1)$ or the isovector interaction $V_v(j_2 \bar{j}_3; j_4 \bar{j}_1)$ respectively. The symmetry energy can be written as

$$U(j) = \sum_i (2j_i + 1) M_{jji}, \quad (5)$$

with similar expressions for $\bar{j}$ and j_i . The monopole charge-exchange interaction

$$M_{j_1 j_2} = (2j_1 + 1)^{-1} (2j_2 + 1) \sum_{J, T} \frac{1}{2} (2J + 1) (-1)^{1+T} \langle j_1 j_2; J0, T0 | V_{12} | j_1 j_2; J0, T0 \rangle. \quad (6)$$

The particularly simple type of $2p$ - $2h$ excitation used has the consequence that only the quantities given by equations (4), (5) and (6) are required to evaluate the configuration interactions. These interactions are given in Table 1, in which

$$\begin{aligned} D_1 &= V_v(j'j'; j\bar{j}) + \frac{1}{2}(T+1)\delta_{jj'}\delta_{j\bar{j}} \left(U(j) + U(\bar{j}) + \sum_i (2j_i + 1) U(j_i) \right), \\ D_2 &= V_v(j'j'; j\bar{j}) + \frac{1}{2}T(T+1)\delta_{jj'}\delta_{j\bar{j}} \left(\sum_i (2j_i + 1) U(j_i) - (2T+1)\{U(j) + U(\bar{j})\} \right), \\ D_3 &= V_v(j'j'; j\bar{j}) + \delta_{jj'}\delta_{j\bar{j}} \left(\sum_{i,i'} \hat{j}_i \hat{j}_{i'} \bar{g}_i \bar{g}_{i'} M_{jji i'} - \sum_i \bar{g}_i^2 (M_{jji} + M_{\bar{j}\bar{j}i}) \right), \end{aligned}$$

where $\hat{j}_i = (2j_i + 1)^{\frac{1}{2}}$.

3. Calculations

The nuclei ^{88}Sr and ^{90}Zr contain a magic number (50) of neutrons filling single particle levels up to and including the $1g_{9/2}$ subshell, and the 38 and 40 protons are assumed to close the $2p_{3/2}$ and $2p_{1/2}$ subshells respectively. Two calculations were performed on ^{86}Sr : in calculation I the ground-state wavefunction included two holes in the $1g_{9/2}$ neutron subshell, whereas in calculation II the ground-state wavefunction had the form $a(1g_{9/2})^{-2} + b(2p_{1/2})^{-2}$. The first excited 0^+ state $-b(1g_{9/2})^{-2} + a(2p_{1/2})^{-2}$ recently identified (Kitching *et al.* 1971) at 2.1 MeV provides, with the condition $a^2 + b^2 = 1$, that $a = 0.94$ and $b = 0.35$. These values are in good agreement with $a = 0.92$ and $b = 0.39$ obtained by Morton (1970).

Table 2. Comparison of unperturbed energies for $1p-1h$ configurations

Configuration Particle Hole	Particle-hole energy* (MeV)					Dipole operator matrix element (fm)	
	P	G	V	H	S	Proton	Neutron
$2(d_{5/2})2(p_{3/2})^{-1}$	7.8	8.1	7.7	8.1	7.7	2.32	-1.81
$2(d_{5/2})1(f_{5/2})^{-1}$	8.2	8.5	8.5	8.1	7.7	-0.32	0.25
$3(s_{1/2})2(p_{3/2})^{-1}$	9.7	10.0	8.7	9.1	10.3	0.96	-0.75
$1(g_{7/2})1(f_{5/2})^{-1}$	11.2	11.1	11.5	11.1		2.51	-1.96
$2(d_{3/2})2(p_{3/2})^{-1}$	10.6	11.3	10.2	10.5		0.78	-0.61
$2(d_{3/2})1(f_{5/2})^{-1}$	11.0	11.7	11.0	10.5		1.24	-0.97
$2(d_{5/2})1(f_{7/2})^{-1}$	12.5	13.1	12.2	13.1	12.3	1.49	-1.16
$3(s_{1/2})2(p_{1/2})^{-1}$	8.3	8.1	7.2		9.5	-0.66	0.52
$1(h_{11/2})1(g_{9/2})^{-1}$	10.1	8.7	10.4			4.45	-3.43
$2(d_{3/2})2(p_{1/2})^{-1}$	9.2	9.4	8.7			1.75	-1.35
$2(f_{7/2})1(g_{9/2})^{-1}$	12.4	12.5				1.73	-1.33
$1(g_{9/2})1(f_{7/2})^{-1}$	7.5	8.3	6.8		9.3	3.60	-2.80
$2(p_{1/2})2(s_{1/2})^{-1}$	9.1	10.5				1.03	-0.81
$2(p_{1/2})1(d_{3/2})^{-1}$	10.4	11.8				0.93	-0.73

* References to energy data: P, present work; G, Goulard *et al.* (1968); V, Vergados and Kuo (1971); H, Hughes (1968); S, Stautberg *et al.* (1967).

The thirteen single-particle levels considered were assumed to have energies that were common to each nucleus. Energies for the fourteen possible dipole $1p-1h$ excitations were chosen after comparison with values obtained in the literature (for references, see Table 2) and examination of their effect on the calculated spectra. With these configurations, seven $T+1 = 7$ analogue states and twenty-eight $T = 6$ normal states form the ^{88}Sr basis while nine $T+1 = 6$ and twenty-one $T = 5$ states contribute to the ^{90}Zr spectrum. Calculation I has eight $T+1 = 6$ and thirty $T = 5$ states with appropriate weighting factors included for transitions involving the partially filled $1g_{9/2}$ neutron subshell. In calculation II, configuration states were generated from both the ground and first excited 0^+ states to give a total of 17 analogue and 51 normal states. Configuration states with different distributions for the two holes in the neutron excess region were considered to be uncoupled except where two states differed only in this respect. The interaction between such pairs of states was found to have a strength of about 0.5 MeV.

In equations (4) and (6), the matrix elements consisting of antisymmetric two-particle wavefunctions are evaluated via the Talmi transformation to relative and

centre-of-mass coordinates. The two-particle matrix element can then be expressed (Shakin *et al.* 1967) as a sum of reduced matrix elements $\langle n^{2S+1}L_J : n'^{2S+1}L'_J \rangle$ which depend only on relative coordinates. Nonzero contributions were taken for n and $n' = 0, 1, 2$ and $L = L' = 0, 1, 2, 3$ (i.e. S, P, D, F) for both $S = 0$ and $S = 1$ and in addition the tensor components $^3F_2 : ^3P_2$ and $^3S_1 : ^3D_1$. The reduced matrix elements used are those obtained by Petris (1970) using a nonlocal potential, which includes the usual noncentral forces with Gaussian and Yukawa radial forms and quadratic momentum dependence. The most suitable force strength investigated corresponded to the harmonic oscillator range parameter $b = 2.1$ fm. This agrees with the value obtained from the rule of thumb (Eisenberg 1963) $\hbar\omega = 41 A^{-\frac{1}{3}}$ MeV for $A = 90$. The symmetry energies calculated for this force strength were ~ 3 MeV, which is in poor agreement with the approximate experimental value of 6 MeV obtained from nucleon separation energies. Such disparity is a common difficulty with realistic forces (J. D. Vergados, personal communication). As in previous calculations, the charge-exchange matrix element is assumed to be independent of quantum numbers, and equation (5) reduces to $U = 2TM$. Furthermore, terms involving U in equation (4) vanish and the configuration interactions are simplified. A value of $M = 0.55$ MeV produced closest agreement between calculated and experimental (Paul *et al.* 1971) separations of $T+1$ and T dipole strength.

The radiative width for the ground state transition is given by

$$\Gamma_{\gamma_0} = \frac{16}{9} \pi e^2 k^2 \langle 0 | Q_1 | E \rangle^2. \quad (7)$$

The dipole operator Q_1 with components

$$Q_1^\mu = \sum_{i=1}^A e_i r_i Y_1^\mu(i)$$

describes a single-particle process, and so only $1p-1h$ components of $|E\rangle$ contribute to the radiative width. Dipole operator matrix elements for each $1p-1h$ configuration (Table 2) were obtained using harmonic oscillator radial wavefunctions ($b = 2.1$ fm.) and effective charges $e_i = N/A$ or $-Z/A$ for neutrons or protons respectively.

4. Results

Radiative widths for dipole states in ^{88}Sr with excitation energy $E_x \geq 12$ MeV are shown in Fig. 2a. In each isospin group there is a high energy concentration of transition strength. The analogues contain 7% of the total strength, which agrees with the value 0.076 provided by the estimate (Goulard and Fallieros 1967)

$$|C_{T+1}|^2 = (T+1)^{-1} (1 - 3T/2A^{2/3}). \quad (8)$$

The giant dipole resonance (hereinafter designated GDR) concentrating 50% of the total strength is placed at 16.64 MeV, in good agreement with a single Lorenz line fit to the natural Sr photoneutron cross section (Lepretre *et al.* 1971) centred at 16.3 MeV. Dominant components in the GDR wavefunction are the $1(g_{7/2})1(f_{5/2})^{-1}$ and $2(d_{5/2})1(f_{7/2})^{-1}$ configurations. The continuous curve in Fig. 2a represents a sum of widths distributed around their positions by Breit-Wigner type functions with spreading width 0.25 MeV. The curve aids comparison with the yield curve from the reaction $^{87}\text{Rb}(p, \gamma_0)^{88}\text{Sr}$ for $\theta_\gamma = 90^\circ$ obtained by Hasinoff *et al.* (1969) (Fig. 2b) and with the reaction cross section for $^{88}\text{Sr}(\gamma, p_0)^{87}\text{Rb}$ (Fig. 2c) obtained

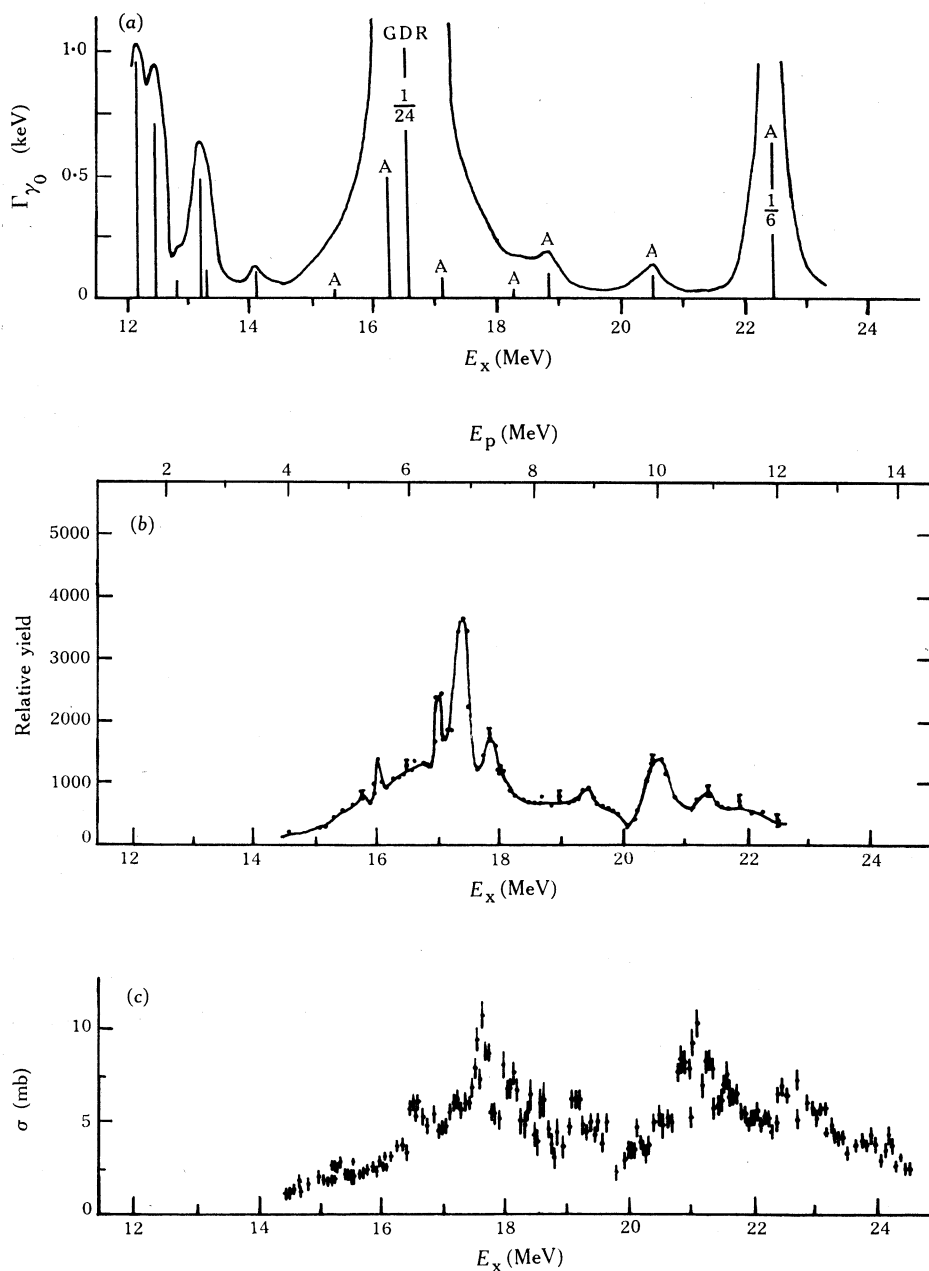


Fig. 2. Comparison of experimental and theoretical results for the dipole transitions $^{88}\text{Sr}(1^- \rightarrow 0^+)$. Part (a) shows the radiative widths for states with excitation energy $E_x \geq 12$ MeV in ^{88}Sr . The analogues are denoted by the symbol A and the continuous curve represents a spreading of widths (see text). Part (b) shows the experimental results of Hasinoff *et al.* (1969) for the reaction $^{87}\text{Rb}(p, \gamma_0)^{88}\text{Sr}$ for $\theta_\gamma = 90^\circ$. Part (c) shows the experimental results of Shoda *et al.* (1969) for the reaction $^{88}\text{Sr}(\gamma, p_0)^{87}\text{Rb}$.

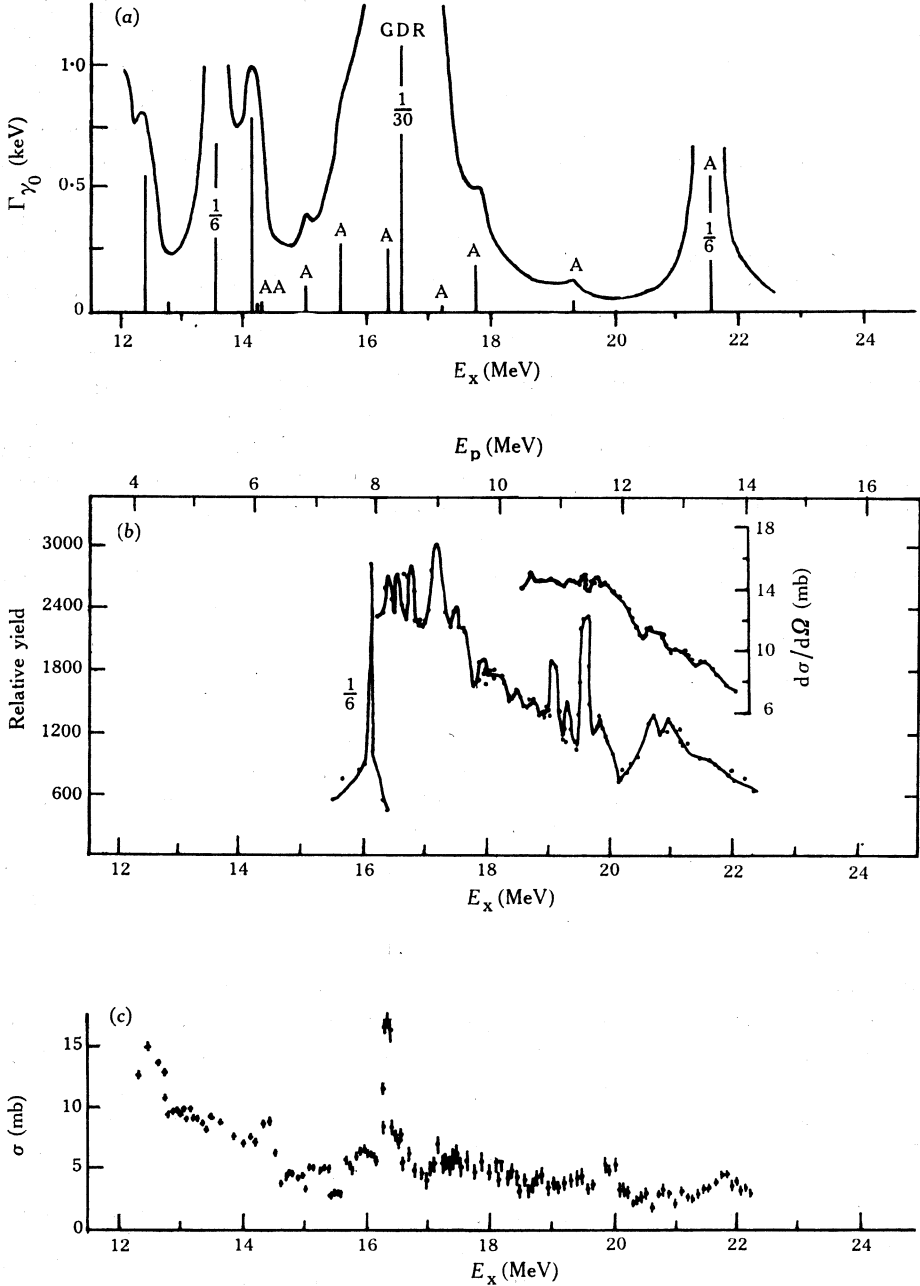


Fig. 3. Comparison of experimental and theoretical results for the dipole transitions $^{90}\text{Zr}(1^- \rightarrow 0^+)$. Part (a) shows the radiative widths for states with excitation energy $E_x \geq 12$ MeV in ^{90}Zr . The analogues are denoted by the symbol A and the continuous curve represents a spreading of widths. Part (b) shows the experimental results of Hasinoff *et al.* (1969) for the reaction $^{89}\text{Y}(p, \gamma_0)^{90}\text{Zr}$ for $\theta_\gamma = 90^\circ$, and the insert shows the experimental differential cross section for $^{89}\text{Y}(p, p_0)^{90}\text{Zr}$ for $\theta_{\text{CM}} = 140.8^\circ$. Part (c) shows the experimental results of Shoda *et al.* (1969) for the reaction $^{90}\text{Zr}(\gamma, p_0)^{89}\text{Y}$.

by Shoda *et al.* (1969). Pronounced experimental structure is present at 21 MeV and at 21.0 and 22.7 MeV respectively. Analogue states with widths of 0.092 and 2.455 keV are predicted at 20.47 and 22.47 MeV to possess amplitudes of -0.71 , 0.55 and 0.62 , 0.66 for the $1(g_{7/2})1(f_{5/2})^{-1}$, $2(d_{5/2})1(f_{7/2})^{-1}$ configurations. Resonances at excitation energies of 15.7 and 16.0 MeV in the (p, γ_0) curve have also been observed in the (p, n) reaction (Zioni *et al.* 1968). These levels compare with the predominantly $2(d_{5/2})2(p_{3/2})^{-1}$ and $2(d_{5/2})1(f_{5/2})^{-1}$ analogue states at 15.44 and 16.24 MeV.

The dipole states in ^{90}Zr are shown in Fig. 3a. The 9% of total strength associated with the analogues agrees with the value 0.091 obtained from equation (8). The GDR containing 66% of the total strength matches line fits to the total photoneutron cross section centred at 16.65 MeV (Lepretre *et al.* 1971) and 16.83 MeV (Berman *et al.* 1967). The experimental results of Hasinoff *et al.* (1969) for the reaction $^{89}\text{Y}(p, \gamma_0)^{90}\text{Zr}$ are shown in Fig. 3b and those of Shoda *et al.* (1969) for the reaction $^{90}\text{Zr}(\gamma, p_0)^{89}\text{Y}$ are shown in Fig. 3c. These results display structure at 21 and 19.2 MeV and at 21.5 and 20 MeV respectively. Analogues calculated at 21.60 and 19.39 MeV compare favourably with these observations. Although the uppermost analogue in both ^{88}Sr and ^{90}Zr is shown to have a large intrinsic γ_0 strength, the observed resonance is weak compared with lower-lying analogues. This results from the low overlap between the major components in these states and the ^{87}Rb and ^{89}Y ground states. The resonances are inhibited even more in a (p, p_0) reaction, as can be seen by comparing yield curves in Fig. 3b. Neutron decay of these states to low-lying analogues in the $(A-1)$ nucleus is isospin allowed, enhancing photoneutron cross sections at these energies. The well-known $(s_{1/2})(p_{1/2})^{-1}$ resonance (Black and Tanner 1964) is predicted by an analogue at 14.26 MeV with width 10 eV. Although the $(d_{3/2})(p_{1/2})^{-1}$ resonance is attributed (Hanna 1969) to the 16.3 in preference to the 15.7 MeV level, analogues at 15.63 and 16.30 MeV have roughly the same amplitudes for this component in the wavefunctions.

Two similar calculations have been done previously for both ^{88}Sr and ^{90}Zr . To investigate ^{88}Sr , Goulard *et al.* (1968) employed the same configuration basis used here and the somewhat crude schematic model approach. The GDR was placed at 15.6 MeV and a strong analogue state predicted at 20.6 MeV. Following this work, using a configuration basis slightly less rich and the bare Kuo-Brown G -matrix elements, Vergados and Kuo (1971) predicted the GDR at 15.6 MeV and two strong analogue states at 19.3 and 23.4 MeV. If one weights the many factors involved, the present results compare with the experimental spectra as well as do the above-mentioned calculations. Vergados and Kuo also calculated the spectrum of ^{90}Zr and, as in the earlier work of Hughes and Fallieros (1969), included proton correlations in the ground state wavefunction. The earlier calculation used a nuclear force with Soper mixture (that proved to be too strong) whereas Vergados and Kuo included core polarizations in the proton-proton part of the interaction used for ^{88}Sr . Though not as detailed in the choice of configuration basis, the present computation shown in Fig. 3 proved relatively successful and so provides some confidence in fixing parameters for the nucleus ^{86}Sr , which is as yet experimentally unexplored.

Radiative widths for states with $E_x \geq 13$ MeV in the two ^{86}Sr calculations are shown in Figs 4a and 4b. Both γ_0 GDR wavefunctions contain strong $1(g_{9/2})1(f_{7/2})^{-1}$ contributions placing them more than 1 MeV above the GDR in the other two

nuclei. In Fig. 4b positions of Γ_{γ_1} widths are shifted relative to Γ_{γ_0} widths by roughly the separation energy (2 MeV) of the ground and first excited 0^+ states. The overall energy positions of γ_0 strength in each calculation match closely, which indicates that the residual interaction was of suitable strength.

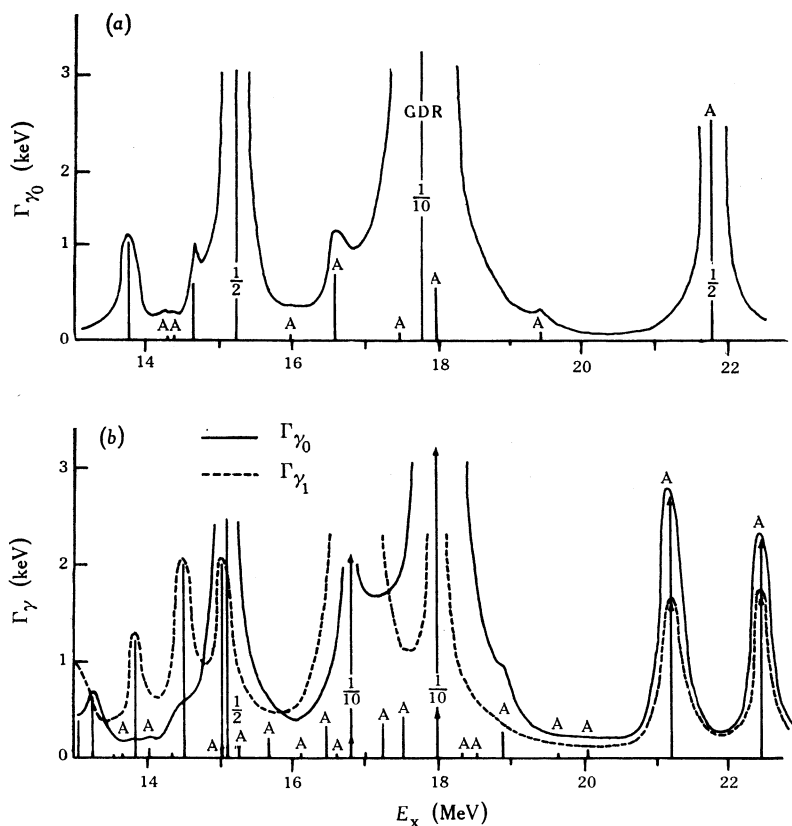


Fig. 4. Radiative width(s) for dipole states with $E_x \geq 13$ MeV in ^{86}Sr . The analogues are denoted by the symbol A and the continuous and dashed curves represent a spreading of widths. Part (a) shows the results of calculation I for $^{86}\text{Sr}(1^- \rightarrow 0^+)$ uncorrelated ground state and part (b) shows the results of calculation II for $^{86}\text{Sr}(1^- \rightarrow 0^+)$ correlated ground state.

Similarities in the calculated results suggest that the ^{86}Sr excitation spectrum, which is still under investigation, will display some of the features noted for the other two nuclei. In the GDR energy region, sharp analogue resonances are expected corresponding to low-lying predominantly single particle excitations in the parent nucleus ^{86}Rb . Dipole states above the neutron threshold (20.7 MeV) concentrate the majority of the analogue transition strength but p_0 emission from these states is expected to be inhibited relative to that from the low-lying analogues. Comparisons between calculated and experimental spectra in ^{88}Sr and ^{90}Zr have been successful for this good isospin model and make future applications to other suitable nuclei of considerable interest.

Acknowledgments

I am indebted to the guidance of Professor C. B. O. Mohr and the assistance of the CPRA scheme.

References

- Berman, B. L., Caldwell, J. J., Harvey, R. R., Kelly, M. A., Bramblett, R. L., and Fultz, S. C. (1967). *Phys. Rev.* **162**, 1098.
- Black, J. L., and Tanner, N. W. (1964). *Phys. Lett.* **11**, 135.
- Eisenberg, J. M. (1963). Lecture Notes on the p - h Model, University of Melbourne.
- Goulard, B., and Fallieros, S. (1967). *Can. J. Phys.* **45**, 3221.
- Goulard, B., Hughes, T. A., and Fallieros, S. (1968). *Phys. Rev.* **176**, 1345.
- Hanna, S. S. (1969). 'Isospin in Nuclear Physics' (Ed. D. H. Wilkinson), p. 658, (North-Holland: Amsterdam).
- Hasinoff, M., Fisher, C. A., Kuan, H. M., and Hanna, S. S. (1969). *Phys. Lett.* **30 B**, 337.
- Hughes, T. A. (1968). *Phys. Rev.* **181**, 1586.
- Hughes, T. A., and Fallieros, S. (1969). Conf. on Nuclear Isospin, Asilomar, California, p. 109, (Academic: New York).
- Kitching, J. E., Morton, J. M., Davies, W. G., Darcey, W. J., and McLatchie, W. (1971). *Nucl. Phys. A* **177**, 433.
- Lepretre, A., Beil, H., Bergiere, R., Carlos, P., and Veyssiere, A. (1971). *Nucl. Phys. A* **175**, 609.
- Mahaux, C., and Weidenmuller, H. A. (1969). 'Shell Model Approach to Nuclear Reactions', p. 280, (North-Holland: Amsterdam).
- Morton, J. M. (1970). Ph.D. Thesis, University of Oxford.
- Paul, P., Amann, J. F., and Snover, K. A. (1971). *Phys. Rev. Lett.* **27**, 1013.
- Petris, L. (1970). *Nucl. Phys. A* **148**, 583.
- Shakin, C. M., Waghmare, Y. R., and Hull, M. H., Jr (1967). *Phys. Rev.* **161**, 1006.
- Shoda, K., Sugawara, M., Saito, T., and Miyase, H. (1969). *Phys. Rev. Lett.* **23**, 800.
- Stautberg, M. M., Kraushaar, J. J., and Ridley, B. W. (1967). *Phys. Rev.* **157**, 977.
- Vergados, J. D., and Kuo, T. T. S. (1971). *Phys. Lett.* **35 B**, 93.
- Zioni, J., Jaffe, A. A., Drory, Ch., and Harchol, M. (1968). *Nucl. Phys. A* **109**, 401.

Manuscript received 10 September 1973