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Energy Spectrum of ^{51}V by the Intermediate Coupling Approach

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Abstract

Level energies of the low-lying excited states in ^{51}V have been calculated using the intermediate coupling approach of the unified model.

In recent years the nucleus ^{51}V has been extensively studied, both experimentally by Horoshko *et al.* (1970), using the ^{48}Ti$(\alpha, p\gamma)^{51}$V reaction, and theoretically in terms of shell model calculations by many authors (McCullen *et al.* 1964; Horoshko *et al.* 1970; Lips and McEllistrem 1970). Mixed configuration shell model calculations by Lips and McEllistrem, in particular, have successfully reproduced the low-lying energy levels of ^{51}V. However, strong coupling rotational model calculations by Malik and Scholz (1966) did not give satisfactory results. In the present work, the intermediate coupling unified model (Bohr and Mottelson 1953; Choudhury 1954) is applied to ^{51}V. The purpose of this work is to show that the intermediate coupling model can successfully predict the low-lying energy levels of ^{51}V.

In the intermediate coupling model, an odd-mass nucleus is described as a coupled system consisting of an odd numbered shell of valence nucleons which can undergo excitations of the form $(lj)^n_J \rightarrow (lj)^n_{J'}$ coupled to a neighbouring even–even vibrating core (Choudhury and Clemens 1969). The last three protons in the $1f_{7/2}$ shell in ^{51}V were assumed to be coupled to the quadrupole vibrations of the surface of the ^{48}Ca core. The relevant three proton states are $1f_{7/2}$, $1f_{5/2}$ and $2p_{3/2}$ and the core vibrates with up to three phonons. The total Hamiltonian for the core–particle system is of the form

$$H = H_c + H_p + H_{int},$$

where H_c is the Hamiltonian associated with the collective core, H_p the Hamiltonian associated with the last odd nucleons, and H_{int} the core–particle interaction Hamiltonian (Choudhury and O'dwyer 1967; Choudhury and Clemens 1969). The interaction Hamiltonian is given by

$$H_{int} = -\xi \hbar\omega (\tfrac{1}{5}\pi)^{\frac{1}{2}} \sum_{i,\mu} \{b_\mu + (-)^\mu b^\dagger_{-\mu}\} Y_{2\mu}(\hat{r}_i),$$

where ξ is the dimensionless coupling parameter, $\hbar\omega$ is the phonon excitation energy of the core and b_μ and $b^\dagger_\mu$ are the annihilation and creation operators. The basis eigenvectors of the coupled system are denoted by

$$|JNRIM\rangle = \sum_{m,m'} (JRmm' | IM) |Jm\rangle |NR\,m'\rangle,$$

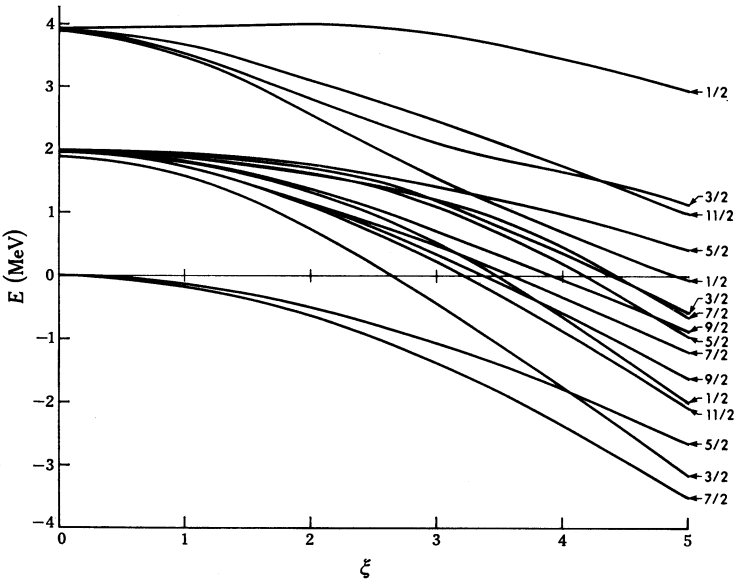

Fig. 1. Plots of energy eigenvalues E as a function of the coupling parameter ξ in ^{51}V.

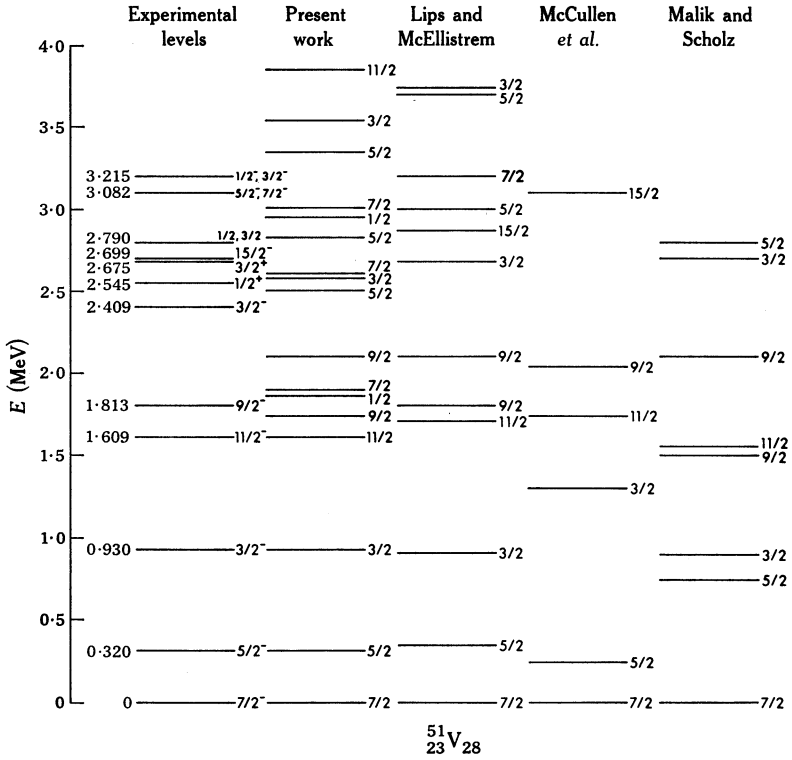

Fig. 2. Comparison of the predicted energy levels of ^{51}V from the present work with the experimental levels and with the results of other calculations.

where J is the total angular momentum of the last odd nucleons, N the number of phonons, R the core angular momentum and I the total angular momentum of the nucleus with its z component M. The eigenvalues and the expansion coefficients of the eigenvectors are obtained by diagonalizing the Hamiltonian H. The eigenvectors of H are of the form

$$|EIM\rangle = \sum_{J,NR} A^{I}_{J,NR}(E)\,|JNRIM\rangle .$$

Table 1. Expansion coefficients corresponding to lowest five levels of ^{51}V

Basis states	Levels $\|E(\text{MeV}); I^{\pi}\rangle$				
$\|NR;j\rangle$	$\|0;7/2^-\rangle$	$\|0{\cdot}320;5/2^-\rangle$	$\|0{\cdot}930;3/2^-\rangle$	$\|1{\cdot}609;11/2^-\rangle$	$\|1{\cdot}813;9/2^-\rangle$
$\|00;7/2\rangle$	0·7679				
$\|12;7/2\rangle$	−0·5100	0·2363	0·7169	0·7760	0·7560
$\|20;7/2\rangle$	0·1700				
$\|22;7/2\rangle$	−0·0190	0·0040	−0·4056	−0·2549	0·3103
$\|24;7/2\rangle$	0·1603	−0·1466	−0·1802	−0·4229	−0·4610
$\|30;7/2\rangle$	−0·0155	0·0107			
$\|32;7/2\rangle$	−0·0734	0·0298	0·1387	0·1303	0·1206
$\|33;7/2\rangle$	−0·0049	−0·0020	−0·0796	0·0337	−0·0420
$\|34;7/2\rangle$	−0·0204	0·0136	0·0720	0·0351	−0·0860
$\|36;7/2\rangle$	−0·0272	0·0317		0·1418	
$\|00;5/2\rangle$		0·7950			
$\|12;5/2\rangle$	−0·1559	0·4898	0·2282		−0·0746
$\|20;5/2\rangle$		0·1380			
$\|22;5/2\rangle$		−0·0874	−0·1002		0·1510
$\|24;5/2\rangle$	0·1000	0·0824	−0·0928	−0·1695	−0·0982
$\|30;5/2\rangle$		0·0107			
$\|32;5/2\rangle$	−0·0204	−0·0525	0·0435		−0·0120
$\|33;5/2\rangle$	−0·0078	0·0001	−0·0411	0·0066	−0·0287
$\|34;5/2\rangle$	−0·0099	0·0148	0·0437	0·0179	−0·0595
$\|36;5/2\rangle$	−0·0201			0·1147	0·0694
$\|00;3/2\rangle$			0·3340		
$\|12;3/2\rangle$	0·1881	−0·0916	−0·1689		
$\|20;3/2\rangle$			0·1769		
$\|22;3/2\rangle$	−0·1142	0·0425	0·0541		
$\|24;3/2\rangle$	−0·0479	0·0349		0·2395	0·1303
$\|30;3/2\rangle$			−0·0681		
$\|32;3/2\rangle$	0·0376	−0·0156	−0·0570		
$\|33;3/2\rangle$	0·0251	−0·0136	0·0019		−0·1050
$\|34;3/2\rangle$	0·0194	−0·0157		−0·1113	−0·0317
$\|36;3/2\rangle$				−0·0503	−0·0230

The effective relative proton configuration energy spacings ε_1 and ε_2,

$$\varepsilon_1 = E[(1f_{7/2})^3{}_{5/2}] - E[(1f_{7/2})^3{}_{7/2}], \qquad \varepsilon_2 = E[(1f_{7/2})^3{}_{3/2}] - E[(1f_{7/2})^3{}_{7/2}],$$

the phonon energy $\hbar\omega$ and the coupling parameter ξ were considered to be adjustable parameters. The best fit to the experimental levels (Horoshko *et al.* 1970) was for $\xi = 3{\cdot}072$, $\varepsilon_1 = 0{\cdot}020$ MeV, $\varepsilon_2 = 5{\cdot}370$ MeV and $\hbar\omega = 1{\cdot}950$ MeV. The energy eigenvalues are plotted as a function of the coupling parameter in Fig. 1, while the predicted energy levels are compared with the experimental spectrum and other theoretical results in Fig. 2. The wavefunctions for the lowest five levels in ^{51}V are

listed in Table 1. As can be seen from the table, the squares of the expansion coefficients of the basis states corresponding to the highest phonon states are nearly negligible compared to unity, indicating the validity of including states of up to three phonons only. Fig. 2 shows that agreement with the experimental levels is as good as that obtained with the mixed configuration shell model by Lips and McEllistrem (1970) and is better than those with the pure configuration shell model (McCullen *et al.* 1964) and the strong Coriolis coupling model (Malik and Scholz 1966). The r.m.s. deformation (Brink 1960) obtained in terms of the present intermediate coupling parameters, $\beta_{rms} = 0{\cdot}265$, is consistent with the experimental value of $0{\cdot}26$ for ^{51}V (Ritter *et al.* 1962), suggesting that the parameters determined in these calculations fall in the correct range.

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