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A Particle–Hole–Rotator Coupling Model for the Giant Resonance of Carbon-12

Anne McDougall and B. M. Spicer

School of Physics, University of Melbourne, Parkville, Vic. 3052.

Abstract

A collective correlations calculation has been made for the giant resonance of ^{12}C. The low-lying states are treated as members of two rotational bands, and higher energy low-lying states are included in the coupling procedure in an attempt to examine the connection of these states with structure in the 30–35 MeV region, and to examine a proposed rotational band of states built on the 7·65 MeV (0^+) level. The calculation fails to transfer strength to the extent expected.

Models for the Giant Resonance of ^{12}C

The earliest theoretical accounts of the giant resonance in ^{12}C were based on the particle–hole model. Vinh-Mau and Brown (1962) correctly predicted the main peak in the giant resonance region to be at $\sim$22 MeV. Although Gillet and Vinh-Mau (1964) used a more realistic finite-range force for the particle–hole interaction, their results were not very different from those of Vinh-Mau and Brown. More recently a calculation by Seaborn and Cooper (1971), using interaction matrix elements deduced directly from two-nucleon scattering phase shift experiments, gave states with energies and wavefunctions differing slightly from those given by the earlier calculations. However, these calculations could not account for the finer structure seen when photonuclear absorption by ^{12}C is observed with good resolution.

The approach which has been most successful in accounting for the structure in the giant resonance has been the collective correlations philosophy adopted by Greiner (Drechsel *et al.* 1967). This contends that, in a calculation of all the 1^- states of a total nuclear system, states produced by the coupling of the basic one particle–one hole excitations to the low-lying even parity levels of the nuclear 'core' must be included. In the collective correlations calculation for ^{12}C, the low-lying levels were described by Drechsel *et al.* as those of a harmonic vibrator. The coupling resulted in the transfer of some of the dipole absorption strength from the original particle–hole states to 1^- states constructed on the excited states of the core. More dipole states with significant absorption strength appeared, and the result bore considerably more resemblance to the experimental picture than did the particle–hole calculations.

Two other calculations have been performed for ^{12}C. Kamimura *et al.* (1967) coupled the basic particle–hole states to the first excited state at 4·43 MeV, described in terms of its particle–hole configurations, and accounted well for the peak in the giant resonance at 25·5 MeV. Ward-Smith (1971), by coupling the basic particle–hole states to the first two excited states at 4·43 and 7·65 MeV, predicted in addition the peak around 29·5 MeV. Owen and Spicer (1969) provided some experimental evidence for this latter state.

All the calculations on the ^{12}C nucleus noted above are bound-state calculations, i.e. they take no account of the fact that the states they discuss are states in the continuum. The present work continues in this philosophy. The neglect of the continuum nature of the dipole states does not affect their calculated energies significantly, but only prevents the calculation of level widths.

In the case of ^{16}O the low-energy spectrum is better described in terms of rotational bands. This description was used in a collective correlations calculation for the giant resonance of ^{16}O by Dracoulis (1970). Morinaga (1966) suggested that a rotational description might be appropriate for ^{12}C also and, in particular, that the state at $\sim$10 MeV was the 2^+ state of a rotational band built on the 7·65 MeV level. However, Ajzenberg-Selove and Lauritsen (1968) recommended a spin assignment of 0^+ for the 10 MeV state in ^{12}C.

In the present work, Dracoulis's (1970) method is used to calculate the giant resonance states of ^{12}C. The low-lying states of ^{12}C are treated as members of two rotational bands, one based on the ground state and the other based on the 7·65 MeV (0^+) level. Higher energy low-lying states are included in the coupling procedure in an effort to examine the connection of these states with structure in the 30–35 MeV region of the absorption cross section. Various energies are tried for that of the 2^+ state, which might be built on the 7·65 MeV (0^+) state as base, in order to test whether the photonuclear cross section can give any information concerning the proposal that a rotational band is built on this base state.

Calculation

The overall Hamiltonian for our particle–hole–rotator model of the giant resonance of ^{12}C may be written as

$$H = H_{\mathrm{ph}} + H_{\mathrm{r}} + H_{\mathrm{Q}}, \tag{1}$$

where H_{ph} is the particle–hole Hamiltonian, H_{r} is the Hamiltonian describing the low-lying rotational states and H_{Q} is the Hamiltonian describing the interaction between the two excitations. The Hamiltonians H_{ph} and H_{r} are assumed to be prediagonalized. The rotational states are assumed to have the wavefunctions of a spherical rotator

$$|Lm_L\rangle = \chi_{\mathrm{i}}\, y_L^{m_L}(\theta_{\mathrm{c}}, \phi_{\mathrm{c}}), \tag{2}$$

where L is the total angular momentum of the low-lying rotational states, and χ_{i} is the intrinsic nucleon wavefunction. Coupling the 4 one particle–one hole states to each of the 4 low-lying rotational states being considered results in the formation of 16 final states. The coupling procedure is carried out independently for each of two sets of one particle–one hole dipole states: the first being those calculated by Gillet and Vinh-Mau (1964) and the second being those calculated by Seaborn and Cooper (1971).

The interaction matrix elements are

$$\begin{aligned}
&\langle jLJM|H_Q|j'L'J'M'\rangle \\
&= f_2(-1)^{J'+j'}\begin{Bmatrix} j & L & J \\ L' & j' & 2\end{Bmatrix}\hat{L}\hat{L}'\begin{pmatrix} L & 2 & L' \\ 0 & 0 & 0\end{pmatrix}\delta_{JJ'}\delta_{MM'} \\
&\quad\times\langle L|\alpha(R_c)|L'\rangle\sum_{\substack{ls\\l's'}} a^j_{ls}a^{j'}_{l's'}\hat{j}\hat{j}'(-1)^{l+j'+s}\begin{Bmatrix} l & j & s \\ j' & l' & 2\end{Bmatrix}\langle j|\alpha(r_{\rm ph})|j'\rangle\hat{l}\hat{l}' \\
&\quad\times\Bigg[(-1)^{l_2+l'}\hat{l}'\hat{l}\begin{pmatrix} l_1 & 2 & l_1' \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} l_1 & l & l_2 \\ l' & l_1' & 2\end{Bmatrix}\delta_{l_2 l_2'} \\
&\qquad+(-1)^{l_1+l_2'+l+l_2}\hat{l}_2\hat{l}_2'\begin{pmatrix} l_2 & 2 & l_2' \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} l_2 & l & l_1' \\ l' & l_2' & 2\end{Bmatrix}\delta_{l_1 l_1'}\Bigg]\delta_{ss'}\,,
\end{aligned} \tag{3}$$

where the angular momentum coupling coefficients are written as $3j$- and $6j$-symbols, l_1 and l_2 are the orbital angular momenta of the hole and the particle of the original unperturbed particle–hole excitation, and f_2 is a constant whose value is unknown. The factors $\langle L|\alpha(R_c)|L'\rangle$ and $\langle j|\alpha(r_{\rm ph})|j'\rangle$ specify the radial dependence of the interaction. The matrix elements are evaluated in terms of a parameter Q given by

$$Q = f_2\langle L|\alpha(R_c)|L'\rangle\langle j|\alpha(r_{\rm ph})|j'\rangle\,, \tag{4}$$

which is regarded as a measure of the strength of the interaction. Each particle–hole state has been expanded as

$$|j\rangle = \sum_{ls} a^j_{ls}\,|lsj\rangle\,, \tag{5}$$

where the coefficients a^j_{ls} are the elements of eigenvectors of each particle–hole state, and the $|lsj\rangle$ are the unperturbed particle–hole configurations in LS coupling.

Each matrix is diagonalized, for various values of Q, to give eigenvalues and eigenvectors. The dipole strength of each eigenstate is then found for comparison with the experimental giant resonance data. The energy range 20–26 MeV is concentrated upon here in the determination of the best value of the parameter Q, as in this region the structure of the cross section is well determined. Examination of cross sections for ^{12}C(γ, n) reactions and for proton capture by ^{11}B suggests that the main giant resonance peak for ^{12}C can be regarded as containing two component levels, separated by $\sim$1 MeV and with the larger component at the lower energy. A second main peak of the dipole strength should be found at an energy of 25·5 MeV.

Results

Using the particle–hole data of Seaborn and Cooper (1971), we find that, for $Q = 12$ MeV, the dipole strength distribution has three peaks with the required energy spacing and in reasonably good agreement with the relative strengths required. After an energy adjustment of almost 2 MeV for all the peaks, the structure predicted for $Q = 12$ MeV is that shown in Fig. 1. The (γ, n) cross sections as given by Cook *et al.* (1966) and Firk *et al.* (1963) are also shown. The distribution of dipole strengths resulting from the calculation fails to account for the strengths of the peaks seen experimentally at 27·5 MeV and 29·5 MeV.

In the 30–35 MeV region the effect of the major peaks in the giant resonance on the energies and wavefunctions of states appears to be marginal, with very little transfer of strength into the higher energy region occurring. Thus the calculation is unable to predict structure in this region to allow us to distinguish between the different energy assignments tried for the higher 2^+ rotational level. Finally, we note that the particle–hole data of Gillet and Vinh-Mau (1964) give dipole strength distributions of quality inferior to those of Seaborn and Cooper (1971).

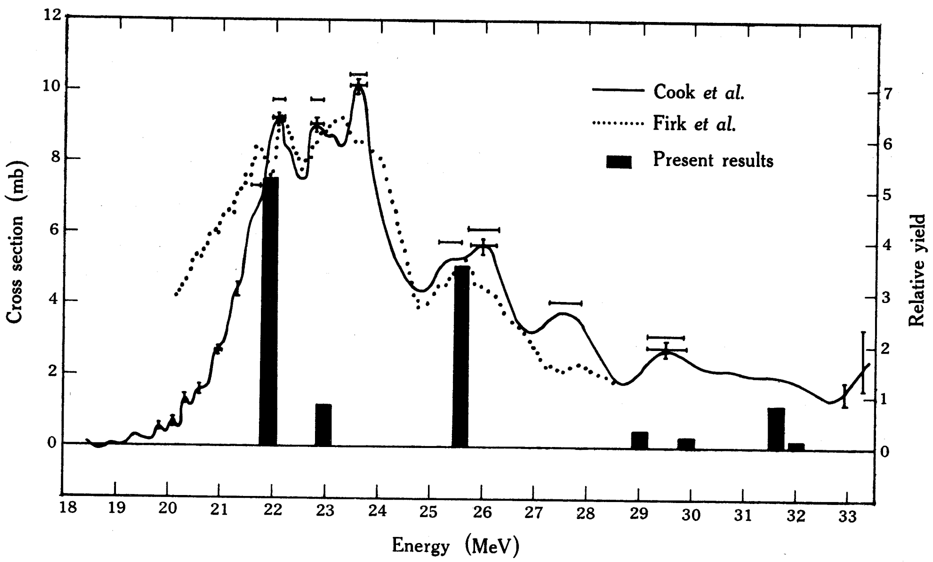

Fig. 1. Comparison of our predicted structure for the giant resonance of ^{12}C with experimental data. The present collective correlations calculation is based on the particle–hole data of Seaborn and Cooper (1971), using 10·3 MeV energy assignment for the higher 2^+ rotational level, for $Q = 12$ MeV. The data of Cook *et al.* (1966) are the CLSR output for the reaction $^{12}C(\gamma, n)^{11}C$ with $\Delta E = 125$ keV, while those of Firk *et al.* (1963) are neutron time-of-flight relative yields.

Discussion

The use of a rotator description for the low-lying levels of the nuclear core in a collective correlations calculation for ^{12}C does predict more dipole states in the giant resonance than do the particle–hole calculations. As expected, these states are at least slightly shifted in energy by the coupling process, the extent to which this occurs in a particular case being related to the strength of the coupling. The model does produce some redistribution of the dipole strength from the original particle–hole states. It is interesting to note that the value of 12 MeV for the strength parameter Q which gives the most satisfactory results for this calculation is very close to the value $Q = 10$ MeV selected by Dracoulis (1970) for best results from his calculation for ^{16}O.

However, the calculation fails to transfer strength to the extent expected. Dracoulis (1970) noted that, while he could produce good agreement in position and strength for the two main peaks of the ^{16}O giant resonance, the calculation did not transfer enough strength to the other peaks. This failure is even more marked in the case of ^{12}C, suggesting a limitation inherent in the model as such. To achieve larger transfer of absorption strength to the higher energy dipole states would require a much larger value of Q than was used, but the use of such a value would destroy the agreement in the energies of the dipole states.

It may be noted that a constant value of Q is assumed in this calculation, but this is a simplification, as the definition of Q given in equation (4) contains a factor $\langle j|\alpha(r_{\mathrm{ph}})|j'\rangle$. Correct accounting for such a factor would require a different value for each particle–hole state, and consequently different values of Q within the energy matrix. However, the transfer of absorption strength from the main peak (22 MeV) to the satellite peak at 25·5 MeV suggests that the value of Q used is not too far wrong. It would appear therefore that this calculation makes too weak a coupling between the ground state of ^{12}C and its 7·65 MeV state, thus inhibiting the transfer of absorption strength, or else it suffers (at higher energies) from neglect of the one particle–one hole $3\hbar\omega$ excitations. This unsatisfactory redistribution of absorption strength means that the present method is not satisfactory for examining the proposed rotational band of states built on the 7·65 MeV level in ^{12}C.

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