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Dynamical Consequences of a Photon Regge Trajectory

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Abstract

It is argued that the newly discovered narrow resonances are Regge recurrences of the photon Regge trajectory. The widths and masses are calculated using the Anselm–Gribov technique, taking the data on the $3\cdot 1$ GeV resonance as an input.

1. Introduction

In the last few months several speculations (De Rújula and Glashow 1975; Goldhaber and Goldhaber 1975; Nieh *et al.* 1975) have been made in an attempt to understand the recently discovered narrow and broad resonances (Abrams *et al.* 1974; Aubut *et al.* 1974; Augustin *et al.* 1974; Bacci *et al.* 1974). Before this discovery the $e^+e^- \to$ hadrons cross section was presumed to go through the *elementary* photon intermediate state with $J^{PC} = 1^{--}$. With this discovery, however, one is tempted to argue that the photon is not an elementary particle. A few years ago Blankenbecler *et al.* (1962*a*, 1962*b*) proposed the interesting possibility that the photon too is a Regge trajectory. In the present paper, an attempt is made to describe the newly observed resonances as Regge recurrences of the photon and its even signature partner.

In Section 2 we consider the photon Regge-pole analysis of the $e^+e^- \to \pi\pi$ reaction. In Section 3, the form of the photon trajectory is calculated and, in Section 4, the slope and the imaginary part of the trajectory are evaluated. Finally, in Section 5 we consider some implications of these results.

2. Invariant Amplitudes

Before descending into technicalities it may be useful to run through an outline of the idea of a Regge photon and some of its consequences. For simplicity we consider the $e^+e^- \to \pi\pi$ reaction with two pions in the $I = 1$ state. Following the notations of Frazer and Fulco (1960), the matrix element can be written as

$$T = u(p_2)\{a - \tfrac{1}{2}r(q_1 - q_2)b\}u(p_1)\,, \tag{1}$$

and the corresponding unitarity equations for the auxiliary partial-wave amplitudes can be written as

$$\operatorname{Im}\{f(J,s)\} = q^{2J+1}s^{-\frac{1}{2}}\,h^*(J,s)\,f(J,s)\,\theta(s-4m_\pi^2)\,, \tag{2}$$

$$\operatorname{Im}\{h(J,s)\} = q^{2J+1}s^{-\frac{1}{2}}\,h^*(J,s)\,h(J,s)\,\theta(s-4m_\pi^2)\,, \tag{3}$$

where f is the $e^+e^- \to \pi\pi$ partial-wave amplitude and h is the $\pi\pi \to \pi\pi$ partial-wave amplitude. The signature factors in equations (2) and (3) are implied. The solutions of these unitarity equations were investigated by Blankenbecler *et al.* (1962*a*, 1962*b*) and can be summarized as follows. If we have

$$h(J,s) = N(J,s)/D(J,s) \qquad \text{with} \qquad D(J,s) = d(J,s)\{\alpha_1(s)-J\}\{\alpha_\rho(s)-J\} \tag{4}$$

then we also have

$$f(J,s) = r(J,s)/D(J,s) \qquad \text{with} \qquad r(J,s) = \pi^{-1}\int_{-\infty}^{s_0} ds' \frac{\mathrm{Im}\{f(J,s')\}}{s'-s} D(J,s'), \tag{5}$$

where $\alpha_1(s)$ and $\alpha_\rho(s)$ are the photon and the ρ trajectories. Given these singularities one can now evaluate the invariant amplitudes using the Sommerfeld–Watson transform, that is,

$$a(s,t) = \frac{4\pi\{2\alpha_1(s)+1\}(pq)^{\alpha_1(s)}\beta(\alpha_1(s))}{p^2\sin(\pi\alpha_1(s))\{\alpha_1(s)-\alpha_\rho(s)\}}$$

$$\times\left(P_{\alpha_1(s)}(-Z_s) - P_{\alpha_1(s)}(Z_s) + \beta'(s)\,mZ_s\frac{(P'_{\alpha_1(s)}(-Z_s) - P'_{\alpha_1(s)}(Z_s))}{[\alpha_1(s)\{\alpha_1(s)+1\}]^{\frac{1}{2}}}\right) + \alpha_1(s)$$

$$\to \alpha_\rho(s) + \text{background integral}, \tag{6}$$

$$b(s,t) = \frac{4\pi^2\{2\alpha_1(s)+1\}(pq)^{\alpha_1(s)-1}\beta(\alpha_1(s))\,\beta'(s)}{\sin(\pi\alpha_1(s))\{\alpha_1(s)-\alpha_\rho(s)\}}\left(\frac{P'_{\alpha_1(s)}(-Z_s) + P'_{\alpha_1(s)}(Z_s)}{[\alpha_1(s)\{\alpha_1(s)+1\}]^{\frac{1}{2}}}\right) + \alpha_1(s)$$

$$\to \alpha_\rho(s) + \text{background integral}. \tag{7}$$

Some interesting features of the above representations are as follows:

(1) When $s \to 0$, $a(s,t) \to 0$. This result is consistent with the first-order perturbation theory of quantum electrodynamics.

(2) For small s we have

$$b(s,t) \approx P(s)/s, \tag{8}$$

where $P(s)$ is the pion form factor. In equation (7) the β's are defined in such a way that $P(s) \approx 2e^2$ as $s \to 0$.

(3) The idea of generalized vector dominance remains valid in this representation. We expect the slope of the photon trajectory to be small, so that as $s \to m\rho^2$ we have

$$P(s) \approx \{s - m\rho^2 + \alpha'_1(m\rho^2/\alpha'_\rho)\}^{-1} \approx (s - m\rho^2)^{-1}. \tag{9}$$

However, at higher energies the term containing the slope of the photon trajectory also contributes and the hypothesis of vector dominance breaks down. This is also consistent with present experimental data (Schildknecht 1974).

(4) We also expect very narrow resonances associated with the photon trajectory (i.e. vanishing of $\sin(\pi\alpha_1(s))$ in equations (6) and (7) when $\alpha_1(s) = 3, 5, \ldots$) which are the Regge recurrences of the photon.

(5) Finally, the second term in equations (6) and (7) associated with the propagator $\sin(\pi\alpha_\rho(t))$ is due to the direct coupling of vector mesons to the leptons. Blankenbecler *et al.* (1962*a*, 1962*b*) have suggested that this coupling will be small at low energies and that broad high spin resonances will appear at high energies.

3. Photon Regge Trajectory

To calculate the photon trajectory we consider e^+e^- scattering. In this reaction both the photon and its even signature partner contribute and both trajectories can be evaluated. The amplitude $A(s,t)$ for this process can be obtained by the Sommerfeld–Watson transform

$$A^{\pm}(s,t) = \tfrac{1}{4}\mathrm{i}\int_{L-\mathrm{i}\infty}^{L+\mathrm{i}\infty} \mathrm{d}j\,\{(2j+1)/\sin(\pi j)\}\{-d^j_{1-1}(-Z) \pm d^j_{11}(Z)\,f_j^{\pm}(s)\}, \qquad (10)$$

where $A(s,t) = A^+(s,t)+A^-(s,t)$ and other symbols carry their usual meaning. The partial-wave amplitude $f_j^{\pm}(s)$ can be continued in the complex j-plane through the Froissart–Gribov representation, and the elastic unitarity condition for $f_j(s)$ is given by

$$a^{\pm}(j,s+\mathrm{i}\varepsilon) - a^{\pm}(j,s-\mathrm{i}\varepsilon) = 2\mathrm{i}\,(s-4m_e^2)^{j+\frac{1}{2}}s^{-\frac{1}{2}}\,a^{\pm}(j,s+\mathrm{i}\varepsilon)\,a^{\pm}(j,s-\mathrm{i}\varepsilon), \qquad (11)$$

where $a(j,s) = f_j(s)/(s-4m_e^2)^j$. Anselm and Gribov (1972) have studied the analytic continuation of $a(j,s)$ in the complex j-plane in the limit $4m_e^2 \to 0$. They obtained the following Regge trajectories for $\alpha(0) > 0$

$$\alpha_1(s) = 1+\alpha_1' s - \pi^{-1}Ks\log(-s), \qquad \text{for} \qquad \alpha(0) = 1, \qquad (12)$$

$$\alpha(s) = \alpha(0)+\alpha' s - a(-s)^{\alpha(0)}/\sin(\pi\alpha(0)) \qquad 0 < \alpha(0) < 1. \qquad (13)$$

We associate $\alpha_1(s)$ with odd signature amplitude because a zero mass particle, the photon, occurs at $J = 1$ on this trajectory. For the even signature amplitude there are two possible cases: (I) exchange degeneracy of even and odd signature trajectories or (II) the even signature trajectory is given by $\alpha(s)$.

4. Slope Parameters

We first consider the case of exchange degeneracy. To calculate the slope of $\alpha_1(s)$ we assume that the 3·1 GeV resonance is the first Regge recurrence on the exchange degenerate photon trajectory with $J^P = 2^+$. Then using a linear approximation we get

$$\alpha_1' = 0{\cdot}104 \quad (\mathrm{GeV})^{-2}. \qquad (14)$$

With this slope we can now calculate the trajectory, and the corresponding masses are given in column 2 of Table 1.

To calculate the width of all the particles on this trajectory we use the experimental data on the 3·1 GeV resonance and predict the widths of the other resonances. For the spin-two particle of mass 3·1 GeV we obtain

$$\Gamma_{e\bar{e}}\Gamma_h/\Gamma = \{(3{\cdot}1\ \mathrm{GeV})^2/10\pi^2\}\int \sigma(e\bar{e} \to h)\,\mathrm{d}E, \qquad (15)$$

$$\Gamma_{e\bar{e}}\Gamma_{\mu\bar{\mu}}/\Gamma = \{(3{\cdot}1\ \mathrm{GeV})^2/10\pi^2\}\int \sigma(e\bar{e} \to \mu\bar{\mu})\,\mathrm{d}E. \qquad (16)$$

This gives

$$\Gamma_{e\bar{e}}\Gamma_h/\Gamma = 2\cdot884\times10^{-3}\ \text{MeV} \quad \text{and} \quad \Gamma_{e\bar{e}}\Gamma_{\mu\bar{\mu}}/\Gamma = 1\cdot96\times10^{-4}\ \text{MeV}, \tag{17}$$

and assuming $\Gamma_{e\bar{e}} = \Gamma_{\mu\bar{\mu}}$ we also have

$$\Gamma_{\mu\bar{\mu}}/\Gamma_h = 0\cdot0679 \quad \text{and} \quad \Gamma = 2\Gamma_{e\bar{e}}+\Gamma_h. \tag{18}$$

The solution of equations (17) and (18) gives

$$\Gamma = 49\cdot4\ \ \text{keV}. \tag{19}$$

Using now the photon trajectory given by equation (12), we get

$$K/\alpha_1' = 15\cdot93\times10^{-6}, \tag{20}$$

and the width of a resonance at $s = s_R$ is given by

$$15\cdot93\times10^{-6}(s_R^{\frac{1}{2}}\ \text{GeV}) = \Gamma_R. \tag{21}$$

We can now calculate widths of all resonances on this trajectory and they are given in column 3 of Table 1.

Table 1. Predicted masses and widths

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\alpha_1' = 0\cdot104\ (\text{GeV})^{-2}$			$\alpha_1' = 0\cdot208\ (\text{GeV})^{-2}$			$\alpha_1(0) \neq \alpha(0)$		
J^P	M(GeV)	Γ(keV)	J^P	M(GeV)	Γ(keV)	J^P	M(GeV)	Γ(keV)
1^-	0	0	1^-	0	0	2^+	2·68	33·81
2^+	3·1	49·4	2^+	2·19	27·63	4^+	4·1	51·73
3^-	4·3	68·5	3^-	3·1	39·11	6^+	5·14	64·85
4^+	5·3	84·4	4^+	3·79	47·81	8^+	6	75·76
5^-	6·2	98·8	5^-	4·38	55·25			

In the case of exchange degeneracy, a further interesting possibility concerns the assignment of spin three to the 3·1 GeV resonance. This leads to a larger slope $\alpha_1' = 0\cdot208\ (\text{GeV})^{-2}$. In this case the width of a resonance on this trajectory is given by

$$12\cdot61\times10^{-6}(s_R^{\frac{1}{2}}\ \text{GeV}) = \Gamma_R. \tag{22}$$

The corresponding mass–width spectrum is given in columns 4–6 of Table 1.

If the photon trajectory is not exchange degenerate with the even signature trajectory, the situation is rather complicated. One can take the attitude that the photon trajectory is like the pomeranchuk trajectory and that the even signature trajectory is like the ρ–f^0 trajectory, having a lower intercept. In such a situation the even signature trajectory will be given by $\alpha(s)$ of equation (13). However, without additional information we have no way of calculating either $\alpha(0)$ or α'. One can only say that $a \approx K$, as both arise from the same interaction. We now choose $\alpha(0) = \frac{1}{2}$ by analogy with the ρ–f^0 trajectory and keep the scale of interaction the same, that is, $\alpha' \approx \alpha_1'$. The predicted masses and widths are given in columns 8 and 9 of Table 1.

5. Conclusions

The interesting feature of the proposed model is that it explains the narrow resonances by a conventional Regge theory. If there is an exchange degeneracy, the model predictions are straightforward. However, if there is no exchange degeneracy, we have two free parameters which cannot be calculated without more resonances in the spectrum. As in the Regge theory of strong interactions, we also have daughter trajectories. The possibility that the new particles belong to the daughter trajectories cannot be ruled out. However, in this case they would have lower spins but their widths would be of the same order of magnitude as those given in Table 1.

Experimentally, it is possible to test exchange degeneracy. If one assumes Gell-Mann's ghost killing mechanism for both even and odd signature amplitudes, then in case I the differential cross section for $e^-e^- \to e^-e^-$ (or $e^+\mu^- \to e^+\mu^-$) would not vanish at $t = -4 \cdot 81$ (GeV)^2 where $\alpha_1(-4 \cdot 81) = 0$. However, in case II there would be a dip at $t = -4 \cdot 81$ (GeV)^2 for these processes.

Finally, we examine whether a slope of the order of $0 \cdot 2$ to $0 \cdot 1$ (GeV)$^{-2}$ for the photon trajectory can be tolerated by present experiments. There are deviations from the one-photon exchange contribution of conventional quantum electrodynamics (e.g. the Rosenbluth formula) owing to the exchange of more than one photon and to radiative corrections. These can be calculated to some extent, and it should be possible to separate them from the non-elementary nature of the photon. The deviations from the Rosenbluth formula using a Regge photon were first considered by Freund (1962). In that calculation the detailed structure of the matrix elements was not taken into account and one obtained

$$\rho = \frac{R_{\mathrm{Reg}}}{R_{\mathrm{Ros}}} = \left(\frac{s}{s_0}\right)^{2\{\alpha_1(t)-1\}}, \qquad \text{where} \qquad R = \frac{d\sigma}{d\Omega}\Big/\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \tag{23}$$

and the subscipts Reg and Ros refer to Regge-photon and Rosenbluth evaluations.

Using $s_0 = 0 \cdot 4$ (GeV)^2 Freund obtained $\alpha' \leqslant 0 \cdot 2$ (GeV)$^{-2}$. With the recent accurate data of Kirk *et al.* (1973) we obtain a smaller slope than this. However, as shown by Blankenbecler *et al.* (1962*a*, 1962*b*) the real situation is rather complicated. This is because in e–p scattering all six invariant amplitudes contribute for a Regge photon in place of the two form factors which appear for the elementary photon. We thus obtain

$$\begin{aligned} R_{\mathrm{Reg}} = {} & \beta_0(t)(s/s_0)^{2\{\alpha_1(t)-1\}} + \beta_1(t)(s/s_0)^{\alpha_1(t)+\alpha_\rho(t)-2} + \beta_2(t)(s/s_0)^{\alpha_1(t)+\alpha_\omega(t)-2} \\ & + \beta_3(t)(s/s_0)^{2\{\alpha_\omega(t)-1\}} + \beta_4(t)(s/s_0)^{2\{\alpha_\rho(t)-1\}} + \beta_5(t)(s/s_0)^{\alpha_\rho(t)+\alpha_\omega(t)-2}. \end{aligned} \tag{24}$$

At extremely high energies only, the first term dominates. At energies ($s \approx 34$ (GeV)^2) for which accurate experimental data (Coward *et al.* 1968) are presently available one expects contributions from the last five terms. This is because we have five free parameters $\beta_1(t)$, ...$\beta_5(t)$. Furthermore, the scale of the interaction s_0, which is usually taken as $1/\alpha'$, is not completely determined as we have three different slopes α'_ρ, α'_ω and α'_1. Thus s_0 can also be taken as a parameter such that $1/\alpha'_\rho \leqslant s_0 \leqslant 1/\alpha'_1$. Even if ρ–ω exchange degeneracy is assumed, we have three free parameters and we can obviously make a reasonable fit to the data (Kirk *et al.*), although no accurate limit on the slope of the photon trajectory can be calculated. In order to place a limit on the slope of the photon trajectory, we thus require data on high energy electron scattering from *spin zero* targets. Such data are either available only at low energies or are not accurate enough to rule out a slope of $0 \cdot 2$ to $0 \cdot 1$ $(\mathrm{GeV})^{-2}$.

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