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Overdriven Supersonic Heat Waves

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Abstract

If a supersonic heat wave is overtaken by a shock, either generated externally or induced by the sudden cooling of the wave front, an overdriven heat wave is formed which is similar to an overdriven detonation. The Mach number of the overdriven heat wave is proportional to the product of the Mach numbers of the pursuing shock and the heat wave.

Introduction

Decaying heat waves are common phenomena (e.g. T tubes, laser sparks, HII regions). An interesting feature of these flows is the transition from the supersonic to subsonic mode which occurs as the input power decays. This transition involves the ejection of a preceding adiabatic shock from the heat wave, and it presents the one occasion on which all parameters of the heat wave may be determined from a single measurement of the front velocity. In the present paper we describe the consequences of a rapidly decaying supersonic heat wave in which a forward-going shock is expected to be created in the hot gas behind the heat wave. If this shock catches up with the heat wave, it can induce a transition to a flow which is similar to an overcompressed detonation.

Overdriven supersonic heat waves are formed whenever a shock overtakes a supersonic heat wave. Such shocks may arise as a consequence of a sudden drop in the absorbed power leading to a quick and drastic drop in the temperature of the heat wave. Due to the continued supersonic propagation of the heat wave, the associated drop in pressure in the front leaves a pressure step behind in the hot gas. This pressure step is like that in a shock tube immediately after the diaphragm is broken, and it necessarily generates a rearward-facing rarefaction wave and a forward-facing shock, which can catch up with the heat wave. In addition one may create a shock behind a supersonic heat wave by external means, e.g. forward-facing shocks can be generated in T tubes by the secondary current maxima of a ringing capacitor bank or by badly tailored laser pulses in supercompression experiments.

Heat Waves

The properties of heat waves have been described in detail by Ahlborn and Strachan (1973), and we review here only those properties relevant to the effect under consideration. The term heat wave is used to describe a zone of strong change in enthalpy h, which is produced whenever an energy flux of intensity W ($\mathrm{W\,m^{-2}}$) is locally deposited into a gas of density ρ. This heat wave travels with some characteristic velocity $V \propto W/\rho h$ through the medium and generates a pressure wave, or zone of strong momentum change, travelling with a velocity U. Depending on the thermodynamic response of the medium and on the absorbed power W/ρ, one of three distinct propagation modes is attained: the subsonic mode, if $V < U$; the Chapman–Jouguet detonation mode, if $V = U$; or the supersonic mode, if $V > U$.

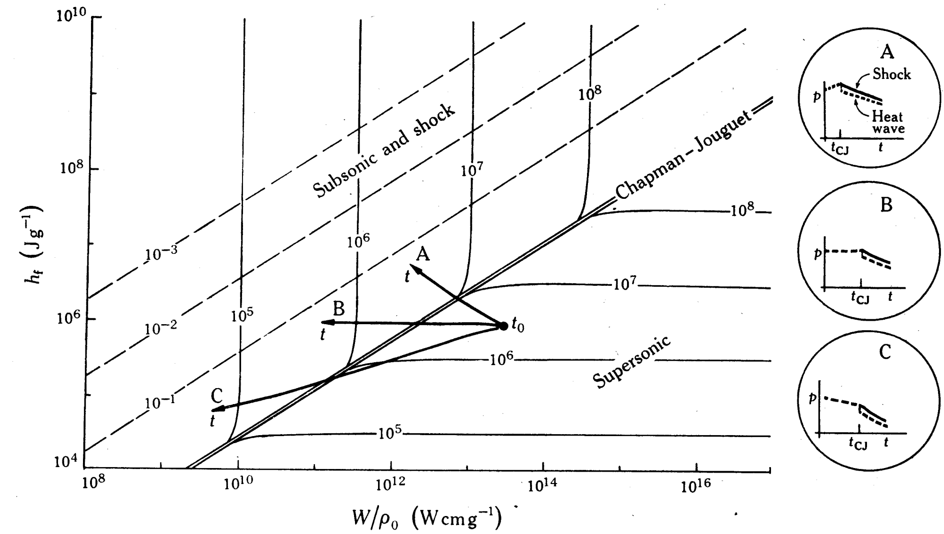

Fig. 1. Paths of three different decaying heat waves A, B and C shown in the response plane, or plot of the enthalpy h_f behind the heat wave as a function of the absorbed power W/ρ_0. The pressure–time histories of the three paths, from t_0 through the Chapman–Jouguet condition at t_{CJ} to time t, are shown in the inserts at the right of the diagram. The other curves denote paths for the indicated values of the following ratios: p_f/ρ_0 (atm cm^3 g^{-1}), horizontal curves; p_i/ρ_0 (atm cm^3 g^{-1}), vertical curves; ρ_f/ρ_0, diagonal curves. Note the division of the response plane by the Chapman–Jouguet line into regions corresponding to the three modes of propagation of heat waves.

It is convenient to approximate the detailed structure of heat waves by a plane step model, in which only the equilibrium conditions ahead of and behind the heating zone are considered and details of the transition zones are disregarded. In this case the flow can be described by the integral conservation equations for mass, momentum and energy. This assumption is adequate only if the widths of both the enthalpy and the momentum transition regions are narrow compared with the dimensions of the entire experiment (which is often the case). The one-dimensional analysis usually contains all the relevant physical effects, so that the gas flow with a locally defined energy source can be described by the following one-dimensional jump equations in the frame of the step heat wave:

$$\rho_i v_i = \rho_f v_f, \qquad p_i + \rho_i v_i^2 = p_f + \rho_f v_f^2, \qquad \tfrac{1}{2}v_i^2 + h_i + W/\rho_i v_i = \tfrac{1}{2}v_f^2 + h_f. \tag{1}$$

Here v is the flow velocity; p, ρ and h are the pressure, density and enthalpy; W is the absorbed intensity; and the indices i and f refer to the initial and final state respectively.

The solution of the equations (1) is best understood in terms of two parameters: the Mach number $M = v_i/c_i$ of the step heat wave, and the absorbed power W/ρ_i, often conveniently rewritten in terms of the energy parameter β, where

$$\beta = 1 \pm \{1-2(g_f^2-1)W/\rho_i c_i^3 M^3\}^{\frac{1}{2}}. \tag{2}$$

In this equaton, g is the adiabatic constant (Ahlborn and Salvat 1967) and c is the speed of sound. It is found that $\beta = 2$ for a shock, $\beta = 1$ for a Chapman–Jouguet detonation, $1 < \beta < 2$ holds for an overcompressed detonation, and $0 < \beta < 1$ holds for a supersonic heat wave.

Solutions to the equations (1) can be presented on a response plane (see Fig. 1) where it can be seen that the wave type is uniquely determined once the absorbed power W/ρ_i and the enthalpy h_f behind the heat wave are known. If a high power input leads to relatively low final enthalpy, the wave attains the supersonic mode so that, although the front velocity may be quite high, the density ratio ρ_f/ρ_i is close to unity and the final pressure p_f is a function of h_f only. The momentum wave in this case is a rarefaction wave trailing behind the heat wave, and both the input and exhaust velocities are supersonic.

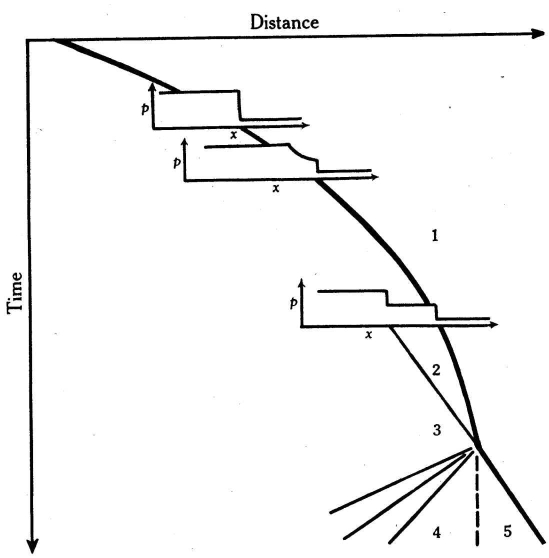

Fig. 2. Distance–time plot (with time increasing downwards) of the rapid decay of a supersonic heat wave $\langle 1,2\rangle$ leading to a sudden drop in pressure (see superimposed pressure–distance plots) and the formation of a shock $\langle 2,3\rangle$ on the rear. Other conditions are: contact surface $\langle 4,5\rangle$, overcompressed detonation $\langle 1,5\rangle$, and reflected rarefaction wave $\langle 3,4\rangle$.

A heat wave driven by a constant flux of energy can be represented by a point in the response plane. If the power input is reduced the front will be described by a different point and, for a gradual change of power input, the evolution of the heat wave is determined by the time history of the solution on the response plane. Schematic examples of the time evolution of heat waves are given (a) in Fig. 1, in which the time evolution of the pressure behind the heat waves is shown in the inserts, and (b) in a distance–time diagram in Fig. 2. The change in the mode of propagation occurs at the instant that the evolution of the heat wave intersects the

CJ (Chapman–Jouguet) line. Physical examples of such decaying heat waves include the radial expansion of HII regions around hot young stars, as discussed in the astrophysical literature (Axford 1961); expanding laser sparks (Daiber and Thompson 1967); the expanding fire ball of a nuclear explosion (Zeldovich and Raizer 1966); the plasma produced by exploding wires, and the plasma which escapes out the end of a pulsed capillary arc (Cross *et al.* 1971). These decaying heat waves exhibit the same general fluid behaviour (treating the flows by arguments similar to the one-dimensional) although the scale lengths vary from a few millimetres and tens of nanoseconds for decaying laser sparks to light year distances and 10^6 yr time intervals for interstellar radiation fronts.

Induced Shocks Behind Supersonic Heat Waves

The situation under consideration here corresponds to case C of Fig. 1, in which the final enthalpy and pressure decrease as W decreases. If the pressure falls quickly behind the supersonic heat wave, a pressure step is created and a forward-going shock will be generated in the hot gas behind the front (see Fig. 2). The induced shock may catch up with the (still supersonic) heat wave if it has more than a given threshold strength. If the shock is too weak, the supersonic heat wave will go through the transition to the subsonic mode independently, and the shock will catch up and perturb the flow at some later time. The shock will catch up with the supersonic heat wave if its Mach number M_s exceeds the exhaust Mach number M^1 of the supersonic heat wave, that is,

$$M_s > M^1 = \{(g_f+1-\beta)/g_f\beta\}^{\frac{1}{2}}. \tag{3}$$

In order to illustrate the formation of a shock behind the decaying supersonic heat wave, we consider a fast supersonic heat wave, such that $M^2 \gg 1$ and all the adiabatic constants are equal to $\gamma = \text{const.}$ Consequently we have

$$p_f/p_i = Z \approx \gamma\beta M^2/(\gamma+1) \qquad \text{and} \qquad v_f/v_i = \rho_i/\rho_f \approx (\gamma+1-\beta)/(\gamma+1)\,.$$

In the limit of the exhaust Mach number also being large, we have $\beta \ll 1$, and therefore

$$\beta \approx (\gamma^2-1)W/\rho_i c_i^3 M^3\,, \qquad \rho_i \approx \rho_f\,, \qquad Z \approx \gamma(\gamma-1)W/\rho_i c_i^3 M\,,$$

$$c_f \approx \{\gamma(\gamma-1)W/\rho_i c_i M\}^{\frac{1}{2}} \qquad \text{and} \qquad u_f \approx (\gamma-1)W/\rho_i c_i^2 M^2\,,$$

where u is the flow velocity in the laboratory frame and v is the flow velocity in the frame moving with the heat wave.

Since we are interested in case C, we have Z and W decreasing in time and for clarity of illustration we take $M = \text{const.}$ during the decay. If W decays linearly by an order of magnitude in a time $\Delta t = t_2-t_1$ then a sound speed differential of $\sqrt{10}$, a pressure differential of 10 and a flow velocity differential of 10 have been created in the hot gas left behind the supersonic heat wave. These gradients exist over a distance $\Delta x \lesssim M\Delta t$. A forward-going shock is formed when the positive characteristics at the tail and head of this gradient intersect, i.e. in a time τ, where

$$\tau \lesssim M\,\Delta t\bigg/\left\{\frac{9\,W(t_2)(\gamma-1)}{\rho_i c_i^2 M^2}+\left(\frac{\gamma(\gamma-1)\,W(t_2)}{\rho_i c_i M}\right)^{\frac{1}{2}}(\sqrt{10}-1)\right\},$$

and the shock trails the heat wave by a distance $M\tau$. The time τ is the time required for the pressure gradient to steepen and form a shock, i.e. it is the time required for the forward-going pressure waves generated along the pressure gradient to all meet. This shock is driven by the gas dynamic expansion as, for instance, in a simple pressure-driven shock tube. The reason that such shocks can form at all is because the heat wave is supersonic with respect to ordinary gas dynamic phenomena and can thus leave behind the appropriate conditions (under the special circumstances considered here) for the formation of a positive going shock.

Overdriven Supersonic Heat Waves

If the shock is strong enough to catch up to the supersonic heat wave then the energy release zone receives additional push and forms an overcompressed detonation. This overcompressed detonation will, with further decay in W, convert into a subsonic heat wave, ejecting a shock, or it may remain an overcompressed detonation if the power stays high enough. In this section we calculate how much the supersonic heat wave is overdriven when it is reached by the shock from the rear. The parameter β_{15} spans the range from 1 for a plane and steady detonation to 2 for a strong shock and thus characterizes the resulting wave. In this calculation we make use of the interaction scheme illustrated in Fig. 2 which holds whether the succeeding shock is induced by the decaying heat wave or by some other external perturbation (e.g. a capacitor discharge in the hot gas).

The solution for the final state 5 is obtained by using the appropriate step equations across the various waves and closing the set at the contact surface in the usual manner so that we obtain one equation for β_{15} and M_{15} in terms of known parameters as follows

$$\begin{aligned}\beta_{15}M_{15}/M_{12} = {} & \beta_{12}+g\beta_{12}(g+1-\beta_{12})^{\frac{1}{2}}2(g+1)^{-1}(M_{23}-1/M_{23}) \\ & +2(g^2+1)^{-1}g\beta_{12}(g+1-\beta_{12})(g-1+2/M_{23}^2)(2gM_{23}^2-g+1)^{\frac{1}{2}} \\ & \times\left\{1-\left(\frac{\beta_{12}M_{12}(g+1)}{\beta_{15}M_{15}(2gM_{23}^2-g+1)}\right)^{(g-1)/2g}\right\}. \qquad (4)\end{aligned}$$

Equation (4) is solved numerically using adiabatic coefficients g for the hot regions and γ for the cold regions. If we know the properties of the initial supersonic heat wave, represented by β_{12}, and the strength of the catching-up shock, represented by M_{23}, then the properties of the resultant overcompressed detonation β_{15} can be calculated provided we know something about the energy absorption mechanism. In general, we do not have enough information about the detailed microscopic nature and macroscopic consequences of the energy absorption, and typically this lack of information causes all the difficulties in the fluid description of these flow fields. Therefore, in order to have these calculations cover actual experiments, we do not wish to make mathematical assumptions about how the energy is absorbed but rather we make three assumptions which are of most benefit to the experimenter:

(1) We take the energy absorbed in the initial front to be a known quantity, that is, β_{12} is a known parameter. This requires the measurement of at least one other fluid parameter (p, ρ or v) besides the phase velocity of the front.

(2) The time variation of the energy absorption will determine the size of the induced shock and thus our expected lack of information about the absorption mechanism means that we cannot calculate this quantity. Hopefully, we can measure the velocity of this shock relatively easily and thus we can take M_{23} as the independent parameter.

(3) We must know something about how the absorbed energy changes as a result of the collision with the catching-up shock which changes the thermodynamic state of the gas. We consider two cases based upon the supposed behaviour of some absorption mechanisms. In the first case, we assume absorption of a constant power $W_{12} = W_{15}$, so that the absorbed power is continuous across the collision and

$$M_{15} = M_{12}\{\beta_{12}(2-\beta_{12})/\beta_{15}(2-\beta_{15})\}^{1/3}, \tag{5a}$$

which completes the set of equations. Solutions to these equations are shown in Figs 3*a* and 3*c*, and should apply to externally powered heat waves. In the second case, we assume that the energy per particle remains constant, which would be typical of a heat wave powered by a chemical reaction or of an ionizing radiation front. Constant power per particle implies that

$$W_{12}/c_i \rho_i M_{12} = W_{15}/c_i \rho_i M_{15},$$

so that

$$M_{15} = M_{12}\{\beta_{12}(2-\beta_{12})/\beta_{15}(2-\beta_{15})\}^{\frac{1}{2}}. \tag{5b}$$

Solutions of this type are shown in Figs 3*b* and 3*d*.

It may be seen from Figs 3*a* and 3*b* that rather weak shocks ($M_{23} \gtrsim 4$) which catch up with supersonic heat waves overdrive the heat wave practically to the saturation value $\beta = 2$ where the front takes on all the features of a strong shock. This observation suggests the possibility of creating really strong shocks by first launching a fast supersonic heat wave and then setting a shock of moderate strength in pursuit. When this weak shock catches up to the supersonic heat wave a strong shock of high velocity will result. The Mach number of this new wave can be obtained from the interaction scheme given in Fig. 2. In the approximation of $\beta_{15} \approx 2$ and for large Mach numbers (M_{15}, M_{23} and M_{12}), we can solve for M_{15} using the interaction scheme shown in Fig. 2 to obtain

$$M_{15} = \tfrac{1}{2}\beta_{12} M_{12} + \{g\beta_{12}(g+1-\beta_{12})\}^{\frac{1}{2}}\{M_{12}M_{23}/(g+1)\}$$
$$\times\left\{1+\left(\frac{2g}{g-1}\right)^{\frac{1}{2}} - \left(\frac{2g}{g-1}\right)^{\frac{1}{2}}\left(\frac{(g+1)M_{15}^2}{g\beta_{12}M_{12}^2M_{23}^2}\right)^{(g-1)/2g}\right\}, \tag{6}$$

or

$$M_{15} \approx \{g\beta_{12}(g+1-\beta_{12})\}^{\frac{1}{2}} M_{12}M_{23}/(g+1).$$

Equations (4) and (6) hold if the reflected wave $\langle 3,4\rangle$ is a rarefaction wave, while a similar equation holds if $\langle 3,4\rangle$ is a shock. The solution to equation (6) is shown in Figs 3*c* and 3*d*. Equation (6) gives the surprising result that the new Mach number M_{15} is proportional to the product of the initial heat-wave Mach number M_{12} and the shock Mach number M_{23}. For instance, if a Mach-4 shock catches up from the

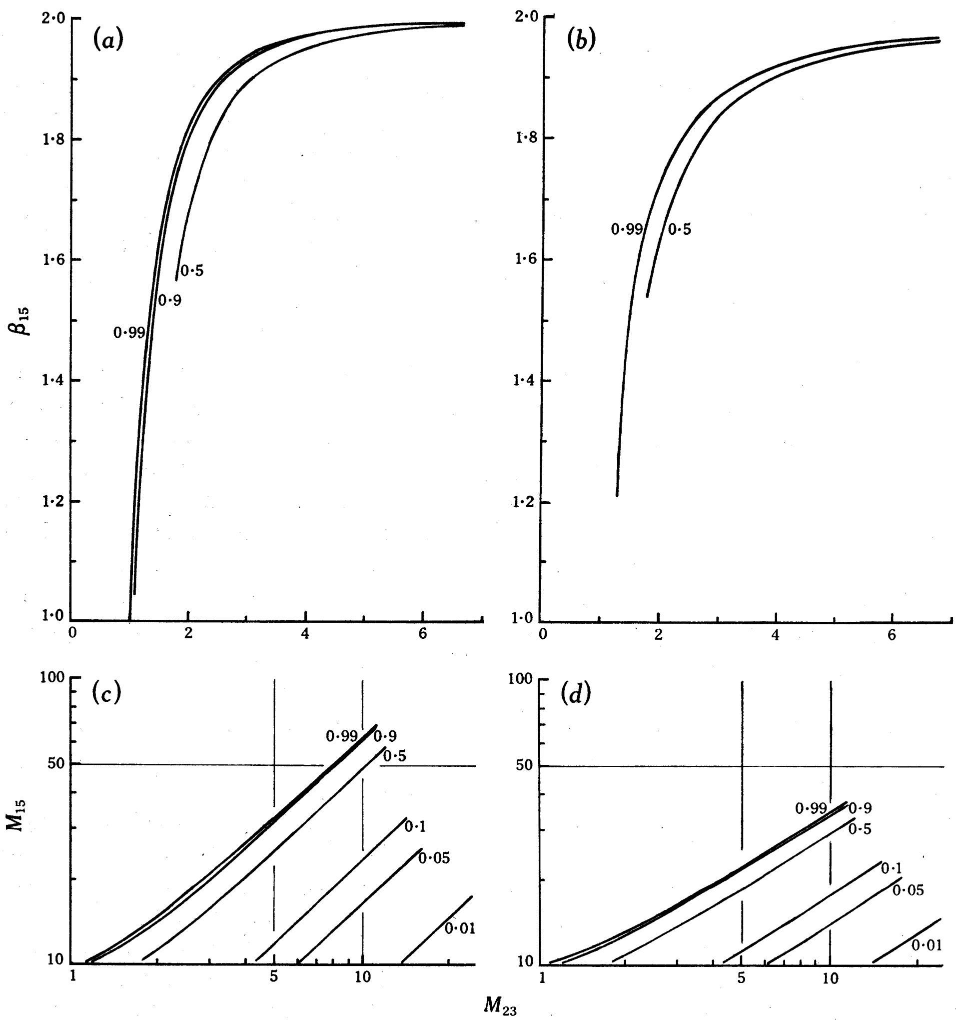

Fig. 3. Variation as a function of the Mach number M_{23} of the pursuing shock (for the indicated values of the power absorption coefficient β_{12} of the initial supersonic heat wave) of the following properties of the overdriven supersonic heat wave:

(*a*) β_{15} for constant power absorption $W_{12} = W_{15}$, $g = 1\cdot2$ and $\gamma = 1\cdot67$;

(*b*) β_{15} for constant energy per particle, $g = 1\cdot2$ and $\gamma = 1\cdot67$;

(*c*) M_{15} (for strongly overdriven heat waves) for constant power absorption $W_{12} = W_{15}$, $M_{12} = 10$, $g = 1\cdot2$ and $\gamma = 1\cdot67$;

(*d*) M_{15} (for strongly overdriven heat waves) for constant energy per particle, $M_{12} = 10$, $g = 1\cdot2$ and $\gamma = 1\cdot67$.

rear with a Mach-10 supersonic heat wave with $\beta = 0\cdot5$, a shock-wave-like discontinuity will result with a Mach number of 20 in the first case (constant power absorption) and a Mach number of 16 in the second case (constant energy release per particle). On first sight this Mach number amplification effect appears attractive although we note that a detailed energy balance may reveal that it is easier to produce a Mach 20 shock directly.

Conclusions

This work has described the manner in which heat waves decay in time when the absorbed power is decreased rapidly. Supersonic heat waves generally undergo a gradual decrease in front velocity and approach the Chapman–Jouguet conditions at which time a shock wave separates from the heat wave. If the energy absorption behaves such that the enthalpy immediately behind the supersonic heat wave decreases severely as the absorbed power decays then a forward-going shock can be formed and can catch up with the supersonic heat wave. In this case, a transition to an over-compressed detonation-type wave will occur and, depending upon details of the energy absorption, this type of wave may either be maintained or may quickly decay into the subsonic mode.

In actual physical examples, it is believed that expanding HII regions decay with h_f constant so that interstellar radiation fronts will pass through the Chapman–Jouguet condition. However, some experiments have revealed transitions which appear to be induced by shocks formed in the heated gas behind the front, and one result of this work is to encourage more detailed measurements of the fluid parameters in these experiments. Offenberger and Burnett (1972) produced laser sparks from CO_2 lasers that were 'adequately described by assuming a breakdown wave [i.e. a supersonic heat wave] during the fast-rising portion of the laser pulse followed by radiation-driven detonation'. As the input power decayed, the expansion of the already heated plasma in their experiment affected the propagation of the laser spark front.

The general features of decaying heat waves are also observed on Brinkschulte's (1967) interferograms for the flow field produced by a T tube. The luminous front (which can be interpreted as a heat wave; Strachan *et al.* 1972) is the leading front up to $t_1 = 4\,\mu$s. A shock wave develops at t_1, when a perturbation has caught up with the heat wave. The change of modes is due to the steady decay in the absorbed power W/ρ_1. This decay occurs since the intensity drops with distance from the source for any transfer mechanism and since, in addition, the current pulse decays. Although the theory presented here can account for the formation of a shock wave in the heated gas behind the luminous front, we point out that extra heating at or near the electrodes can also produce shocks. Whatever the source of the secondary shocks may be, the transition is that described by the interaction scheme shown in Fig. 2.

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