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Tests of Randomness in Astronomical Objects. I Along Single Great Circles

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Abstract

Results are presented for the first of two extensive computer tests for nonrandomness in the inter-object positions of astronomical objects. The aim is to test the two predictions: the geometrical effect that it is impossible to put an arbitrarily large number of objects on the surface of a sphere and have them all randomly equivalent; and the physical effect that any astronomical objects which occur in clumps or clusters (like galaxies) should show the predicted regular-chain effect of Scott *et al.* (1954). Four other physical effects expected to give rise to nonrandomness are also discussed, including the Arp hypothesis. The nonrandomness investigated pertains solely to the difference of latitude or longitude (in a suitable reference system) between astronomical objects. Nonrandomness was detected in the inter-object positions of a large class of bright stars (10σ deviation) and of bright galaxies (15σ deviation). However, peculiar objects from the Arp catalogue were found to be uncorrelated in position at the 1σ level (analogous to those just quoted). These results confirm either or both of the above predictions, and suggest that nonrandomness exists *a priori* in the inter-object positions of astronomical systems generally. Because of controversy surrounding the Arp hypothesis and statistical tests of the type reported here, confirmation has been sought via a different statistical test in Part II (Wesson 1975).

1. Introduction

It is only within the last decade or so that the randomness of the distribution of astronomical objects in the sky has been questioned. The argument of Arp (1970) and others, namely that condensed objects are periodically shot out of peculiar galaxies, has received much criticism. However, the subject has virtually degenerated into a stalemate owing to the difficulty of applying statistical tests to small groups of data and the subjective aspect of selection effects in the original data, which latter are very difficult to delineate.

Stockton (1972) has examined the hypotheses proposed as explanations for the presence of strings of knots projecting radially from elliptical and S0 galaxies. It has been suggested that blobs of matter, shot out of peculiar or condensed objects into an intergalactic medium, experience a ram pressure which results in the formation of objects of the approximate type observed (Mills and Sturrock 1970; Chiao and Wickramasinghe 1972; Wesson 1973). The mechanism suggested by Sturrock and Barnes (1972) is typical of such proposals. Alternatively, Hoyle and Harwit (1962) suggested that intergalactic bridges, considered as cylinders of gas, might break up into ~ 10 blobs due to hydrodynamical instabilities following on the outbursts of supernovae located in the bridge. (A similar well-known hydromagnetic instability can produce the same effect; see e.g. Chandrasekhar 1961, p. 565). Supernovae as a means of producing correlated astronomical objects have also, of course, been implicated in the study of the distribution of pulsars over the sky (Wielebinski *et al.* 1969; Prentice 1970). A statistical investigation of pulsars in space has been carried out by Gold and Newman (1970) using histograms plotted from data derived by Monte Carlo methods.

Another aspect of the problem of nonrandomness involves clusters of galaxies. Firstly, it has been realized for a long time that the lifetime for escape of galaxies from clusters is of the order of the ages of the component galaxies (see e.g. Tuberg 1943). At present, studies are under way to see if the interaction of a small number of dense objects (say three or four) in the nuclei of giant elliptical galaxies or in clusters of galaxies can produce, via Newtonian interaction, bodies which are lost from the system along diametrically opposite directions at roughly the same speed. This problem has obvious connections with the claims of Arp (1970) and with those concerning correlation in the angular momenta of the galaxies comprising a cluster (Opik 1970; Reinhardt 1972). Secondly, as regards clusters in general, it should be noted that a hierarchical cosmology produces spurious 'chains' of galaxies, and these have been claimed as causal chains in the same (erroneous) way that star chains had been previously acclaimed as causal phenomena (Scott *et al.* 1954). Three *physical* chains of galaxies have been reported by Burbidge (1962), Sérsic and Agüero (1972) and Markarian (1961) containing six, seven and eight components respectively.

It must not be thought that only correlations in position are to be expected from extragalactic objects. Fundamentally, it is impossible to put more than a finite number of points on the surface of a sphere (e.g. the celestial sphere) and have them appear uncorrelated (C. Hazard, personal communication), since there exist in Nature only five regular solids. This means that the points of intersection of the vertices of the faces comprising the highest-order solid with the surface of an encompassing sphere are limited in number. Consequently, the presence of any large number of imposed points necessarily means that all points are no longer equivalent, in the sense of being indistinguishable from one another in location. There is thus a compelling fundamental reason to expect that statistical tests on a population of astronomical objects will 'discover' nonrandomness in the given class of objects. No analysis of this problem from a practical point of view (i.e. of testing for such nonrandomness and, if found, of evaluating it) is known to me, and hence I present here such an analysis.

Several methods may be used to test for possible nonrandomness in a class of astronomical objects. The most attractive, that of Fourier analysis, did not seem feasible even though it is theoretically the most elegant method. The reasons for this were practical ones: in recording the positions of $\sim 10^3$ objects for use with a computer program, a human being will almost certainly make mistakes, such as recording the same object twice or misreading the catalogue from which the positions are being abstracted. To avoid incidents of this type, which were found in practice to occur more often than might at first be expected, I decided to use a computer program based on an elementary idea but which could both display and deal with the unavoidable human errors. This program (program I), which works by examining a great circle of the celestial sphere, is essentially a test of ordering of the positions of objects in one dimension. It takes no account of the distances of the objects involved, using only their positions. To underline this point, recently McCrea (1972) gave a historical account of how nine QSOs, on being plotted on a sphere, were all found to fall on a great circle with the exception of only one of them.

I have also developed a second program (program II) which is described in Part II (Wesson 1975, present issue pp. 463–73). Program II works in a plane (or in reality a strip of sky around the equator of a given system of coordinates) and uses the Von Neumann ratio test to detect nonrandomness. The objectives of these programs were (1) to make an explicit test of the expected non-equivalence of a large number of astronomical points on the celestial sphere; (2) to investigate the possible nonrandom distribution of galaxies as a consequence of their being members of a clustered hierarchy. I also attempted (3) to test for ordering in the positions of QSOs (for instance, if all quasars originated by double and symmetrical ejection from a central object this would be immediately apparent) avoiding the old problem of the ineptitude of statistical testing by taking a gross sample of objects and treating it, so to speak, macroscopically; (4) to perform the same analysis on the peculiar objects listed by Arp (1966).

I point out here that the plan just outlined is not dependent on which part of the sky one is looking at, nor does it depend on the presence or absence of obscuring matter along the line of sight. Only the difference in angular measure of any two given sources is employed for, given the coordinates of points on a line (over a great circle, if a sphere is being considered), all the information contained therein is exhausted when the differences between points are tabulated. There is some reason for advocating that the plan I have put forward would be best suited to radio data obtained in a complete survey down to some limiting level, instead of optical data. Such a survey is at present in progress (Gent *et al.* 1973) but the results of it will not be available for some time. After I had completed the present work, a paper by Bogart and Wagoner (1973) appeared which extended earlier unpublished work by Castro (1971). The method of randomness testing used by these authors is the same in basis as the one I have used, except that my technique seems to give better statistical data on the results. Bogart and Wagoner found nonrandomness in the positions of clusters of galaxies, while the present work, which is applied to a much more diverse series of objects, shows that the nonrandomness found by them is part of a more general astronomical phenomenon.

2. Method and Outline of Program I

This section describes the program used to test for correlation in astronomical objects. A flow-chart of the program (based on the method of Forsythe *et al.* (1969) for use with an IBM 370) is available from me on request. The basic data are the coordinates l_i of objects on or near to a great circle encompassing the celestial sphere. For a group of N objects so chosen, the differences δ_j are formed after the l_i have been ordered in ascending magnitude, where

$$\delta_j = l_i - l_{i+1} \qquad \text{for} \qquad i = 1, \ldots, N-1 \quad \text{and} \quad j = 1, \ldots, N-1\,. \tag{1}$$

At every stage of dealing with the δ_j so-formed from the data, the computer carried out an exactly similar treatment of another set of δ_j formed from l_i which were generated by the IBM subroutine RANDU and are random. The two calculations, one using δ_j^{D} (where a superscript D denotes the data set) and one using δ_j^{R} (random set), are carried out in a parallel fashion throughout the entire program, and this is henceforth assumed to be understood.

From the δ_j, the ratios R_k are formed:

$$R_k = \delta_j/\delta_{N-J} \qquad \text{for} \qquad k = 1, ..., N-2, \tag{2}$$

where J is fixed as j varies, giving a sequence of $N-2$ ratios that correspond to division by the smallest interval present (between any two adjacent objects) into all those intervals larger than it. The objective of the test to be applied is to compare R_k^{D} with R_k^{R} and see if there is any significant difference. If the data are in any way ordered at the l_i level, R_k^{D} will fall more nearly around integer or fractional-integer values than will the corresponding R_k^{R}. In order to carry out this comparison, I devised a function H_{I} which can be thought of as operating on the R_k and checking for a possible tendency for the R_k to fall near integral or fractional-integral values. The function H_{I} uses the IBM function AINT, which latter gives the next nearest *lower* integer to its argument (any real number). In the present case it is required to find the nearest integer, whether above or below the actual value of R_k, and also to find the nearest fraction (e.g. if fractions of $\frac{1}{4}$ are used then the operation of H_{I} would seek out the nearest multiple of $\frac{1}{4}$, viz one of 0, $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$). This can be done by operating on any real number x with H_{I}, such that

$$H_{\mathrm{I}}[x] \equiv f * \mathrm{AINT}\big(f^{-1}x + \sin(0{\cdot}5 \sin x)\big), \tag{3}$$

where f is the fraction with respect to which one wishes to categorize and the asterisk denotes computational multiplication. That equation (3) indeed gives what is needed can be checked by working numerically through a few examples. In program I $f = 1$ was used exclusively but in program II (Wesson 1975) I employ $f = 1/4$, $1/8$, $1/16$, ... It should be noted that, while H_{I} gives a hint as to possible order present, it is a closed operator with respect to the unit interval on the real line.

Various operations are carried out by the computer on a quantity defined by

$$P_k = H_{\mathrm{I}}[R_k], \tag{4}$$

details of which can be found by examining the flow diagram referred to above. Execution times on an IBM 370 are about 30 min for a class of 400 objects, and it is not feasible to treat $N > 400$ even on a very fast machine. Order in the data basically means that $|R_k - P_k|$ is systematically smaller than in a random sample. The relevant parameters that emerge are: (1) H_{RMS}, an r.m.s. value of the cumulated sums of $|R_k - P_k|$, suitably confined by a cutoff ratio for the δ_j's of objects lying too near each other (the influence of varying this in the range 20–80 was investigated thoroughly; see also Section 5) and calculated for data and random points; (2) X_{s} and Y_{s}, which measure the summed number of times that the inequality $|P_k^{\mathrm{D}} - R_k^{\mathrm{D}}| \geqslant |P_k^{\mathrm{R}} - R_k^{\mathrm{R}}|$ holds and vice versa (inequality reversed) respectively; (3) ∇, the cumulated sum of the differences $|P_k^{\mathrm{D}} - R_k^{\mathrm{D}}| - |P_k^{\mathrm{R}} - R_k^{\mathrm{R}}|$. The results one would expect if the input data for the program were not random but showed some degree of correlation or ordering are:

$$H_{\mathrm{RMS}}^{\mathrm{D}} < H_{\mathrm{RMS}}^{\mathrm{R}}, \qquad X_{\mathrm{s}} < Y_{\mathrm{s}}, \qquad \nabla < 0, \qquad |\nabla| \gg 1. \tag{5}$$

3. Data

Although the main objective was to test for randomness in those groups of objects for which possible nonrandomness has been suggested (QSOs, objects from the

Arp atlas, and certain classes of galaxies) I eventually chose only six groups, of which three or four were not expected to show any nonrandom effects and have never been suggested as possibly possessing such properties. A synthetic group of ordered objects was also used as a check. The six real groups were (i) QSOs, (ii) pulsars, (iii) bright stars, (iv) Arp atlas peculiar objects, (v) planetary nebulae, (vi) bright galaxies. The results obtained with program I were negative for all except groups (iii) and (vi), and so only these and the synthetic group are considered here.

Synthetic Group

The members of the synthetic group of 35 objects were produced by dividing the intervals 0° to 360° and $-90°$ to $+90°$ into roughly 10° and 3° unit intervals. The programs I and II were checked before and after every run with real astronomical data by running them with the synthetic group. In this way an assurance of reliability was gained from the consistency of the results obtained from this group.

Bright Stars

Bright stars from the Yale catalogue compiled by Hoffleit (1964) were selected in R.A., Dec. and $l^{\mathrm{II}}, b^{\mathrm{II}}$ such that $|\mathrm{Dec.}| < 2°$. This gave a group of 279 objects which were used in program I as groups of 276 objects in $l^{\mathrm{II}}, b^{\mathrm{II}}$ and 241 objects in R.A., Dec. A special subgroup of 51 objects having $|b^{\mathrm{II}}| < 11°$ was also used. The positional accuracies were $\pm 0{\cdot}5$ s and $\pm 1'$ (R.A., Dec.) and $\pm 0{\cdot}5'$ and $\pm 0{\cdot}5'$ ($l^{\mathrm{II}}, b^{\mathrm{II}}$), both ignoring peculiar velocities. Computational restrictions in program I imposed the use of such groups by forcing the deletion of sources which gave $R_k^{\mathrm{D}} = 0$. This bias tended to favour the upholding of the hypothesis of randomness in astronomical objects. It was overcome in program II, and in any case is not a serious biasing effect.

Bright Galaxies

Bright galaxies have been tabulated by de Vaucouleurs and de Vaucouleurs (1964) in supergalactic longitude and latitude (SGL, SGB), i.e. with respect to a proposed coordinate system based on the (still hypothetical) local supercluster. I chose all the objects listed in the de Vaucouleurs catalogue having $|\mathrm{SGB}| < 3°$. These comprised a class of 344 objects, which were listed on cards in SGL, SGB and also in $l^{\mathrm{II}}, b^{\mathrm{II}}$. Of these, some produced zeros in the parameter R_k^{D} of the program as mentioned above and were temporarily discarded, leaving two groups of data comprising 274 objects in SGB, SGL and 337 objects in $l^{\mathrm{II}}, b^{\mathrm{II}}$. The positional accuracies were $\pm 0{\cdot}005°$ and $\pm 0{\cdot}005°$ ($l^{\mathrm{II}}, b^{\mathrm{II}}$) and $\pm 0{\cdot}05°$ and $\pm 0{\cdot}05°$ (SGL, SGB).

The procedure adopted with all the data was: (1) to perform a preliminary manual check of the computer data against the catalogues, (2) to obtain a computer listing of all the data, (3) to re-arrange the data in various ways by computer so that errors could be spotted and corrected. The resulting positions of the members of the various groups of data, after exhaustive checking of this type, were thus assuredly as good as given in the original source catalogue.

4. Results

The values of $H_{\mathrm{RMS}}^{\mathrm{R}}$ were derived from a set of purely random numbers generated by the computer, and so should have a normal distribution when the results of a large number of runs are considered. I decided on 20 program runs having $N = 50$, in each of which *two* lots of random numbers were generated, giving 40 values of H^{R}_{RMS}. As expected, negative values of ∇ were found to occur roughly as often as positive values, and the variance calculated by the computer was approximately constant from run to run. By finding the mean of the 40 values of H^{R}_{RMS}, and calculating the deviations from this mean ($\langle H^{R}_{RMS}\rangle = 0\cdot24585$), as listed in Table 1, a histogram could be drawn. A normal probability curve fits this histogram very

Table 1. Results of program I

(1) Run No.	(2) Data group	(3) No. in sample	(4) Coord-inates	(5) H^{D}_{RMS} H^{R}_{RMS}	(6) X_s	(7) Y_s	(8) ∇
1	Synthetic	35	—	0·0031472 0·2311080	38	522	−127·6580
2	Random	35	—	0·2430285 0·2445498	261	298	−0·2014
3	Bright stars	241	R.A., Dec.	0·1621603 0·2409960	9295	18443	−239·5389
4	Random	241	—	0·2470460 0·2337546	3839	13523	56·8256
5	Bright stars	276	l^{II}, b^{II}	0·2386169 0·2406673	7418	19184	1272·7150
6	Random	276	—	0·2471058 0·2334161	8555	17293	70·6907
7	Low-*b* bright stars	51	l^{II}, b^{II}	0·2254565 0·2428179	538	683	−14·2675
8	Random	51	—	0·2478150 0·2370601	633	582	10·5289
9	Bright galaxies	274	SGL, SGB	0·1352252 0·2221438	1590	24026	−2479·9340
10	Random	274	—	0·2478023 0·2240213	7522	16497	116·8409
11	Bright galaxies	337	l^{II}, b^{II}	0·2306556 0·2334445	5189	28523	−301·1450
12	Random	337	—	0·2481715 0·2372813	7160	27066	105·7891
13	Bright galaxies	274	SGL, SGB	0·1316876 0·2330955	1422	25265	−3045·9800
14	Random	274	—	0·2474730 0·2358766	7127	8055	4·5993
15	Bright galaxies	274	SGL, SGB	0·1328756 0·2416829	7127	0819	−3273·4800
16	Random	274	—	0·2488290 0·2442387	7127	8231	31·4549

well and demonstrates the consistency of the analysis so far. The points of inflexion of the normal curve occur at $\pm\sigma$ on either side of the mean, giving the standard deviation for $N = 50$ as $\sigma \approx 0\cdot007$. This semi-empirical way of deriving the standard deviation is much simpler than working back through the program. The two methods can be shown to be consistent with each other.

The full data for the crucial runs of program I are given in Table 1, in which all values are rounded off to the last decimal place quoted, mostly from an original 15 digit field. Table 1 represents the refinement of a large amount of computational data, with only pertinent results being presented. Columns 2 and 3 specify the class of object being tested for randomness, and column 4 lists the coordinate system in which the objects were tabulated. The remaining column headings are self explanatory. The most valuable program runs used data from narrow strips of width $\sim 2°$ or $3°$ about a specified great circle, the width being taken as narrow as was consistent with obtaining a sample class that was still usefully large. One such subgroup is that listed in run 7, the low-b subgroup of bright stars described in Section 3.

Perhaps the most useful feature of the complete results is the fact that the computational variance is approximately constant over the range of N employed. This implies that σ is roughly a constant independent of N for these data, as expected from the close fit of the histogram of random runs to a normal probability curve with $\sigma \approx 0{\cdot}007$ since (see e.g. Weatherburn 1968) the normal law is obtained from the binomial distribution as the number of discrete data tend to infinity. The probability density can hence be taken as that of a normal distribution

$$\phi(H_{\mathrm{RMS}}^{\mathrm{D}}) = (2\pi)^{-\frac{1}{2}}\sigma^{-1}\exp\{-\tfrac{1}{2}(H_{\mathrm{RMS}}^{\mathrm{D}} - H_{\mathrm{RMS}}^{\mathrm{R}})^2/\sigma^2\}\,, \tag{6}$$

and I assume as usual that the probability of $H_{\mathrm{RMS}}^{\mathrm{D}}$ deviating from $H_{\mathrm{RMS}}^{\mathrm{R}}$ by 1σ and 2σ is $\sim 30\%$ and $\sim 4\%$ respectively.

The quickest way to spot which data of Table 1 contain nonrandom elements is to employ the inequalities (5). Large negative values of ∇ lead one to suspect the presence of some ordering (but this is not in itself sufficient proof of the presence of ordering). Correlation is definitely present if $X_{\mathrm{s}} < Y_{\mathrm{s}}$ to a significant degree (by the random runs present in Table 1, statistical variations in X_{s} or Y_{s} from the random mean are mostly confined to within a multiple discrepancy of $\sim N^{\frac{1}{2}}$) and ordering is definitely present if $H_{\mathrm{RMS}}^{\mathrm{D}} < H_{\mathrm{RMS}}^{\mathrm{R}}$ with regard to the adopted standard deviation σ. One may verify that the purely random runs of Table 1 do not contradict the adoption of $\sigma \approx 0{\cdot}007$ for all the results.

5. Discussion of Nonrandomness

In this section the significance level for those classes of objects which show evidence of nonrandomness is discussed. Examination of columns 5–8 of Table 1 discloses some remarkable results, and these are considered below.

Synthetic Group

The synthetic group of 35 objects examined in runs 1 and 2 show, as designed, an enormous departure from a random distribution. The 100σ deviation from randomness corresponds to the almost-perfect order built into this group of test data.

Bright Stars

Bright stars (listed in R.A., Dec.) forming the 241 member class of runs 3 and 4 show a definite departure from randomness, the extent of which ($\sim 10\sigma$) is considerable. Because the members of this group have far from negligible proper motions, the argument of the Introduction involving the finiteness of the number of equivalent points it is possible to put on a sphere is borne out by this empirical validation of the expected effect. The runs 3 and 4 are with relatively low-declination objects. When the coordinate system is changed to galactic coordinates, the conditions necessary for program I to function as constructed no longer hold. Consequently the 276 member group of bright stars examined in $l^{\mathrm{II}}, b^{\mathrm{II}}$ should not be expected to show the same departure from randomness as those of low declination. This is confirmed by runs 5 and 6. If the $l^{\mathrm{II}}, b^{\mathrm{II}}$ class is now cut down to admit only low-b^{II} stars, as noted in Section 3, the resulting 51 member class might be expected to show some slight nonrandomness, since the objects are now reasonably near to being 'points' on a great circle and program I can examine them as it was designed to do. Runs 7 and 8 confirm this, and the departure from randomness is $\sim 2\sigma$ from the results of run 7.

Bright Galaxies

Bright galaxies, when expressed in the supergalactic coordinates SGL, SGB of de Vaucouleurs, show a definite departure from randomness (in the 274 member class of runs 9 and 10) which is of the relatively large size of 15σ. This inference of the presence of ordering, as expected from the analogous case of bright stars discussed in the previous subsection, is destroyed when the bright galaxies are expressed in $l^{\mathrm{II}}, b^{\mathrm{II}}$ and used as a 337 member group (runs 11 and 12). When the runs using SGL, SGB are repeated with cutoff ratios of 40 (runs 13 and 14) and 80 (runs 15 and 16), as opposed to the original value of 20 (runs 9 and 10), the departure from randomness is found to be still present and still about 15σ.

The above results show that astronomical objects do, in general, show a nonrandom distribution in position. As mentioned in Section 3, four other groups of objects were also tested, but with null results. Arp catalogue objects, comprising a 328 member group, surprisingly showed no significant departure from randomness, although a low-declination 81 member subgroup did show some hint of a departure from randomness but only at the $\sim 1 \cdot 5\sigma$ level. Fuller results for these four groups of objects are presented in Part II.

6. Conclusions

Nonrandomness has indeed been found in bright stars (10σ deviation) and bright galaxies (15σ deviation) by applying the method (program I) described in Section 2 to objects located along a great circle of the celestial sphere. It is obvious from these results that nonrandomness is present generally in galactic and extragalactic objects. However, it is not clear what is responsible for it, i.e. whether it is the geometrical impossibility of putting an arbitrarily large number of equivalent random points on the surface of a sphere or the chain-of-galaxies effect due to clustering in a hierarchical universe. What can be said is that the recognition that the predicted effects *are* significant seriously compromises the claims made by Arp (1970) concerning the distributions of certain types of objects. In view of the low level of correlation found for objects from the Arp catalogue, it would seem that nonrandomness in inter-object positions is the rule rather than the exception and that it does not imply the physical hypotheses discussed by Arp and others concerning the origin of such bodies.

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