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Tests of Randomness in Astronomical Objects. II* Along Two Orthogonal Great Circles

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Abstract

A refinement in computing technique and the use of the von Neumann ratio test are shown to improve the detectability of nonrandomness in astronomical object positions. Correlation is confirmed for those objects (bright galaxies and bright stars) shown to be nonrandomly distributed in Part I. The presence of correlation is also demonstrated, with $\sim 10:1$ likelihood ratio at the 98% confidence level, for the other groups of objects considered in Part I (QSOs, pulsars, planetary nebulae and Arp peculiar objects). The level of nonrandomness in the inter-object positions of the latter groups is considerably smaller than that of the former. Five physical explanations are considered of the nonrandomness, in addition to the geometrical one that it is impossible to put an arbitrarily large number of points on the surface of a sphere and have them all equivalent. It is concluded that the geometrical explanation probably accounts for the low-level (but finite) correlations, implying that Arp's (1970) claims are statistically valid but physically baseless. Confirmation of this view is found in the fact that Arp peculiar objects are only correlated with each other at a low level compared with that of galaxies and stars. The high-level correlations for the latter groups are identified as being the expected consequence of the clustering of astronomical objects.

1. Introduction

In Part I (Wesson 1975, present issue pp. 453–61) I described a computer program (program I) which took the coordinates of astronomical objects on or near a great circle of the celestial sphere, treated them as points on a line and tested them for nonrandomness. The specific objectives of this one-dimensional procedure were: (1) to test for the expected nonrandomness that results from the fundamental limit to the number of equivalent points it is possible to put on the surface of a sphere (located at infinity in this case); (2) to perform similar tests on QSOs and the peculiar objects listed by Arp (1966); and (3) to look for a possible identification of the predicted chain-of-galaxies effect of hierarchical universes (Scott *et al.* 1954). Objectives (1) and (3) were successfully achieved, while (2) proved indeterminate with the computer program used.

The objectives of using the computer program (program II) described in the present paper are: (1) to confirm the results obtained by program I; (2) to refine those results by employing a different method of data analysis, and then using the von Neumann ratio test to establish statistical significance; and, most importantly, (3) to extend the search for unexpected correlations between astronomical objects by using two great circles, which intersect at 90°, to test for nonrandomness in a plane on the celestial sphere (at infinity). Objective (3) is achieved by using a method

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basically similar to that of program I, but now employing data in longitude l and latitude b covering a strip around the equator of the coordinate system used. This strip is kept down to about $\pm 15°$ from the equator, so that distortion from an essentially Euclidean plane is not introduced.

2. Outline of Program II

The primary data for program II, after reduction from R.A., Dec. or $l^{\mathrm{II}}, b^{\mathrm{II}}$ or SGL, SGB, are the coordinates l, b of the N objects under consideration. The program forms the differences δ_j^l and δ_j^b (cf. Section 2 of Part I) in the two orthogonal directions of the coordinates but with no contemporaneously formed random group of data. Instead, the program is first run with real data to form the ratios R_k^{D} (equation (2) of Part I) and then run with N random data, generated by the IBM subroutine RANDU in two substages (for the l_i and the b_i), so enabling corresponding ratios R_k^{R} to be formed. Since two orthogonal directions are involved, there are four relevant ratios:

$$R_k^{\mathrm{D}}(l), \qquad R_k^{\mathrm{R}}(l), \qquad R_k^{\mathrm{D}}(b), \qquad R_k^{\mathrm{R}}(b). \tag{1}$$

With the understanding that the program is first run with real data and then with random data, I henceforth drop the qualifiers D (data) and R (random). A cutoff value of 10, 20, 40, 80 or 160 was used in forming the R_k, and ratios which exceeded the cutoff were put equal to zero. A 'bookkeeping' device in the program kept these ratios separate from those that were really zero to the accuracy of the source data.

Having formed the $N-2$ ratios, the program then uses the function H_{I} (see Section 2 of Part I) to put them into appropriate bins. This function was most often used with a fractional argument $f = 8$, that is,

$$H_{\mathrm{I}}[x] = \tfrac{1}{8} * \mathrm{AINT}\big(8x + \sin(0\cdot 5 \sin x)\big), \tag{2}$$

which had the affect of splitting up the ratios into eight different bins according as they fell nearest to any of the eight values:

$$0, \quad \tfrac{1}{8}, \quad \tfrac{1}{4}, \quad \tfrac{3}{8}, \quad \tfrac{1}{2}, \quad \tfrac{5}{8}, \quad \tfrac{3}{4}, \quad \tfrac{7}{8}. \tag{3}$$

The computer then used two subroutines, one for the l data and one for the b data, to count the corresponding number of ratios which have been eight-fold categorized, and puts these counts into eight bins.

All the preceding is performed for a given value of J (cf. Section 2 of Part I) and the whole procedure is repeated for the sequence of J values ranging from 1 to W, where $W = N-2$ or $\frac{1}{2}(N-2)$. At the end of every J loop, the bins are recalled and the correspondingly eight-fold categorized ratios $R_k(l)$ and $R_k(b)$ are dumped into them. After the final J loop, the contents of the bins are written out. These values are given in columns 4 and 5 of Table 1 (see Section 4 below), each of which is subdivided into eight subcolumns, as dictated by the sequence (3). The program then proceeds to work out various other parameters, but results for these calculations are not presented here.

The net output of program II, as far as testing for correlation is concerned, comprises the number contents of the bins $0, \frac{1}{8}, \ldots, \frac{7}{8}$ for original data in l and b. Each of the latter were obtained for two values of W (those noted above) in order to check that no spurious results were being produced by a hidden sensitivity to the number of

J loops that had been run through. If the original data were ordered in a way similar to the synthetic group described in Section 3 below, the program would detect this as a relative excess in the contents of the first bin, which would correspond to near-integer values for the ratios $R_k(l)$ and $R_k(b)$. For this purpose, the integers $0, 1, \ldots$ are, as for program I, identified with each other, and this corresponds to mapping the whole range of the rationals Q into the unit interval, i.e. for bins in both l and b,

$$Q \rightarrow [0,1] \qquad \text{in units of } \tfrac{1}{8}. \tag{4}$$

The presence of correlations in the data is thus expected to result in an irregular histogram for the contents of the eight bins, the notable expectation being an excess of counts in the first (i.e. integral) bin if order is present in the original data. The significance of the 'spikiness' of the data histograms relative to the random-run histograms was tested carefully with the aid of the von Neumann ratio test (see Sections 4 and 5 below).

3. Data

A notable improvement of program II over program I was its ability to treat values of R_k which were numerically zero, either because they exceeded the chosen cutoff ratio or because some of the data had coincident coordinates in either l or b to the accuracy available in the source lists. The latter was found to happen in a substantial fraction of all the cases in which $R_k = 0$ was involved. This results from the inclusion of sources over an extended range in b as well as in l, so that objects with different l might have coincident b, and vice versa. Owing to the capability of program II to handle such zeros, the full classes of data could be used, as had not been possible with program I. The data employed, labelled according to the abbreviations adopted in column 2 of Table 1, were:

QSO, 53 quasars listed in R.A. (± 1 s) and Dec. ($\pm 1'$) from the Parkes catalogue (Ekers 1969).

qso, 32 member subgroup of the above with $|\text{Dec.}| < 11°$.

PUL($\alpha\delta$), 61 pulsars listed in R.A. (± 1 s) and Dec. ($\pm 1'$) by Manchester and Tayler (1972).

PUL(lb), same group as above listed in $l^{\rm II}$ ($\pm 0{\cdot}05°$) and $b^{\rm II}$ ($\pm 0{\cdot}05°$).

pul, 37 member subgroup of the above with $|b^{\rm II}| < 11°$.

Synth, synthetic group, which (as for program I) was a highly ordered contrived set of data that was used as a check after every run of program II and as a test of sensitiveness.

STAR($\alpha\delta$), 279 bright stars with $|\text{Dec.}| < 2°$ listed in R.A. ($\pm 0{\cdot}5$ s) and Dec. ($\pm 1'$) from the Yale catalogue of Hoffleit (1964).

STAR(lb), same group as above listed in $l^{\rm II}$ ($\pm 0{\cdot}5'$) and $b^{\rm II}$ ($\pm 0{\cdot}5'$).

star, 51 member subgroup of the above with $|b^{\rm II}| < 11°$.

ARP, 335 peculiar objects listed in R.A. ($\pm 0{\cdot}05$ min) and Dec. ($\pm 0{\cdot}05'$) from the Arp (1966) catalogue.

arp, 82 member subgroup of the above with $|\text{Dec.}| \leqslant 10°$.

PLAN, 201 planetary nebulae with $|b^{\rm II}| < 2°$ listed in $l^{\rm II}$ ($\pm 0{\cdot}005°$) and $b^{\rm II}$ ($\pm 0{\cdot}005°$) from the catalogue of Perek and Kohoutek (1967). This group formed a somewhat restricted class of data from the point of view of the capability of program II.

Table 1. Results of program II

(1) Run No.	(2) Data group	(3) Cut-off	(4) l bins 0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	(5) b bins 0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
1	QSO	20	142	160	157	138	132	121	128	116	200	152	134	155	139	106	120	110
2	Rand	20	177	203	180	163	143	142	124	126	136	179	163	149	132	122	108	123
3	qso	20	50	46	40	47	44	49	49	46	72	53	45	59	46	36	44	37
4	Rand	20	56	67	53	60	55	42	42	42	54	73	68	52	59	47	33	47
5	qso	10	47	39	41	42	35	43	44	38	59	49	41	51	40	31	37	34
6	Rand	10	46	44	44	65	39	51	36	38	53	51	31	48	52	36	37	40
7	qso	40	56	51	46	54	50	52	55	54	86	53	48	60	48	38	46	37
8	Rand	40	49	76	65	52	57	49	46	47	55	63	59	48	58	47	52	41
9	qso	20	104		82		93		92		107		97		104		86	
10	Rand	20	105		124		94		96		88		115		85		72	
11	qso	20	20	35	23	18	23	19	26	26	17	22	28	25	22	23	24	20
			10	21	14	13	15	12	15	16	11	16	23	19	18	15	17	16
12	Rand	20	30	31	36	30	28	35	29	27	16	22	26	14	22	18	18	22
			23	20	25	18	20	15	17	18	11	16	17	15	20	11	13	18
13	PUL(lb)	20	267	225	231	236	174	172	178	140	428	70	133	170	179	123	77	50
14	PUL($\alpha\delta$)	20	194	229	236	192	164	178	204	157	220	233	220	208	191	202	167	168
15	Rand	20	205	248	245	194	183	186	162	155	225	260	229	221	190	198	189	157
16	PUL(lb)	10	225	220	210	214	154	155	163	128	362	69	117	165	148	108	64	39
17	Rand	10	188	201	234	166	168	132	155	159	176	206	156	201	182	155	148	142
18	PUL(lb)	20	439		482		378		324		492		243		273		182	
19	Rand	20	414		447		381		318		373		421		365		349	
20	PUL(lb)	20	191	95	117	113	123	101	145	91	94	92	75	92	92	72	66	64
			47	62	81	72	89	68	97	64	76	66	55	60	66	56	51	48
21	Rand	20	98	109	112	121	109	88	106	95	101	101	95	86	69	79	82	79
			70	75	77	85	74	58	68	68	67	73	68	59	67	60	56	60
22	PUL(lb)	40	297	239	231	240	177	178	191	150	478	74	135	140	198	150	84	59

23	Rand	40	208	254	230	196	215	183	180	174	201	267	264	198	214	196	189	156
24	pul	20	80	96	78	81	66	50	68	53	228	18	26	53	68	32	16	5
25	Rand	20	72	102	87	66	72	71	57	58	74	99	81	77	71	69	77	46
26	Rand	20	70	102	91	81	61	78	69	60	72	95	63	68	70	74	61	71
27	Synth	20	561	0	0	0	0	0	0	0	397	2	7	52	52	36	2	13
28	Rand	20	58	87	88	63	65	53	55	59	62	80	76	53	59	64	59	62
29	star	20	211	158	129	168	132	105	109	97	196	119	119	142	125	107	65	49
30	Rand	20	154	147	116	111	113	90	45	102	128	157	136	128	125	124	113	101
31	arp	20	392	425	390	361	330	320	292	289	736	309	342	239	485	225	230	200
32	Rand	20	349	431	361	361	340	313	309	302	739	419	430	363	357	335	300	297
33	ARP	20	9754	5507	6513	5683	6456	5436	4479	3561	12825	5182	5093	5717	6647	4844	3928	2872
34	Rand	20	6587	7470	6821	6435	6007	5621	5165	5199	6621	7809	7059	6680	6130	5524	5463	5030
35	STAR($\alpha\delta$)	20	15991	809	1943	2417	4407	1936	1207	365								
36	STAR(*lb*)	20	5497	4546	4163	4236	4159	3762	3443	3009	7224	3776	4178	4020	4649	3609	3122	2297
37	Rand	20	4723	5432	5002	4675	4389	4142	3849	3691	4363	5219	4817	4616	4268	3904	3740	3518
38	PLAN	20	2392	2274	2076	2122	1862	1838	1662	1592								
39	Rand	20	2353	2740	2559	2335	2201	2022	1912	1812	2364	2734	2619	2341	2134	2006	1913	1808
40	PLAN	10	1971	1971	1771	1777	1601	1576	1400	1331								
41	Rand	10	2150	2598	2457	2169	2025	1920	1798	1614	2241	2571	2352	2213	2021	1805	1678	1669
42	PLAN	40	3685	2451	2266	2336	2039	2010	1826	1762								
43	Rand	40	2477	3061	2723	2517	2381	2304	2081	1941	2498	2947	2632	2537	2251	2229	2168	1997
44	PLAN	20	1510	1054	1182	1054	1049	1094	1065	857								
			1172	659	771	638	706	758	697	664								
45	Rand	20	1154	1470	1424	1279	1301	1235	1164	1136	1042	1124	1032	974	930	982	925	887
			782	955	940	873	894	895	859	807	783	870	765	722	714	744	697	709
46	PLAN	20	4319		4304		3752		3403									
47	Rand	20	4629		5089		4322		3856		3295		3451		3112		2988	
48	GAL(*lb*)	20	9670	5242	5527	5949	5768	5641	4292	3529	12447	4182	5020	5588	6493	4727	4068	2747
49	Rand	20	6865	8001	7446	6947	6686	6081	5679	5265	6937	8158	7753	7045	6478	6211	5657	5473
50	GAL(SG)	20	23097	907	1410	2464	4209	2159	997	507								
51	Rand	20	7061	8249	7584	7029	6584	6194	5976	5704	6712	7886	7178	6769	6345	5945	5708	5291

GAL(SG), 344 bright galaxies with $|\mathrm{SGB}| < 3°$ listed in SGL ($\pm 0{\cdot}05°$) and SGB ($\pm 0{\cdot}05°$) from the de Vaucouleurs (1964) catalogue.

GAL(*lb*), same group as above listed in l^{II} ($\pm 0{\cdot}005°$) and b^{II} ($\pm 0{\cdot}005°$).

Rand, group of random data in l, b generated by the computer.

The positional accuracies are given in the parentheses. The same checking procedure as for program I was used to ensure exactitude, as far as was practically possible, using the computer. The planetary nebulae data group was of practical use only in so far as l was employed, the restricted range in b making the application of program II in this case spuriously invalid. This makes column 5, rows 35, 38, 40, 42, 44 and 46 of Table 1 of negligible significance except as an upheld (negative) test of the expected behaviour of program II in this situation. The same comment applies to the GAL data group and row 50, column 5, of Table 1. The corresponding entries in Table 1 are left vacant.

4. Results

The contents of the bins, as described in Section 2, are presented in Table 1. Column 1 of the table lists the program run number; column 2 lists the abbreviated name of the object class being tested (see Section 3); column 3 lists the cutoff ratio used in the program (see Section 2); column 4 lists the contents of the (usually 8 but sometimes 4 or 16) l bins in order of increasing size, the first bin (the zero or integer bin) having been adjusted to remove those cases where $R_k(l) \equiv 0$ was counted, i.e. the content of the first bin is a true count; and column 5 lists the contents of the b bins in order of increasing size similarly corrected. Results are only presented for $W = N-2$ whereas runs were actually made with two values of W.

Random-data runs were always made immediately after the preceding run with a set of real data, and comparison of the two sets of results enables one to pick out significant deviations easily. Runs with the synthetic data (row 27) were made repeatedly, as a check on consistency throughout the program-running operation times. To test objectively for deviations from randomness, let it be supposed that there exist n measurements of a given set of parameters y_μ ($\mu = 1, ..., n$). The mean of the distribution is, as usual, defined by

$$\bar{y} \equiv n^{-1} \sum_{\mu=1}^{n} y_\mu, \tag{5}$$

the variance is similarly defined by

$$p^2 \equiv n^{-1} \sum_{\mu=1}^{n} (y_\mu - \bar{y})^2, \tag{6}$$

and the successive differences squared is defined by

$$q^2 \equiv (n-1)^{-1} \sum_{\mu=1}^{n-1} (y_{\mu+1} - y_\mu)^2. \tag{7}$$

Then the ratio q^2/p^2 is an intuitive test of spike formations in a set of data plotted in histogram form, and would seem to be the best way of getting an insight into the von Neumann ratio test. The latter consists in forming the ratio

$$\mathscr{R} \equiv q^2/p^2 = L(L-1)^{-1}\left(\sum_{\nu=1}^{n-1} (n_{\nu+1}-n_\nu)^2\right) \Big/ \left(\sum_{\nu=1}^{n-1} (n_\nu-\bar{n})^2\right), \tag{8}$$

where there are L bins in the histogram, and n_ν is the number content of bin ν.

The von Neumann ratio test was used by Karlsson (1971) in a study of quasar redshifts. This work, while now known to have reached an invalid conclusion owing to the clustering of redshifts having turned out to be due to a subjective systematic error (Hazard 1973), is yet a good example of the application and sensitivity of the von Neumann test. Values for the probability of the ratio (8) have been tabulated by Hart (1942) at the 98% significance level, and Hart showed that, as $L \to \infty$, the distribution defined by the bins tends to that of the normal distribution.

Fig. 1 shows a comparison between the results of running program II on the almost perfectly ordered set of synthetic data (run 27) and a random data group (run 28). The distribution of the synthetic group is so striking that its probability of having been randomly caused is infinitesimal, being too small to have been tabulated by Hart (1942).

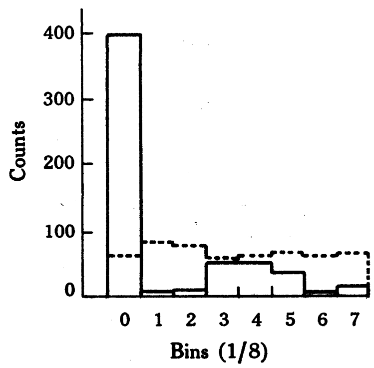

Fig. 1. Comparison of results of program II for synthetic data (solid histogram) and random data (dashed histogram). The values are taken from rows 27 and 28, column 5, of Table 1. Similar results hold for the l data.

5. Discussion of Nonrandomness

There are several preliminary points one should note about the results to be examined. Firstly, the numbers of identically zero R_k values in Table 1 are generally larger for the data runs than for the random runs. This effect arises from the inherent nonuniformity in the positions of astronomical objects compared with the (purely random) statistical fluctuations present in the random runs. The effect of this difference is to make the average level of the random run histograms (dashed lines) higher than the data run histograms (solid lines) since the latter have had more identically zero elements removed than the former. Consequently the *shapes* of the histograms should be given more attention than the absolute heights. Secondly, as is apparent from an examination of the data histograms of Figs 2*a*–2*f* there is a tendency for the bins to be filled in the following manner: the first bin is always higher than all succeeding bins, usually considerably so, tending *a priori* to support the hypothesis of correlation in the original data; there is then a notable slump in the succeeding few bins, gradually recovering to build up to a smaller secondary peak around the middle bin. This is also seen in Fig. 1, and is evident in Fig. 2*c*

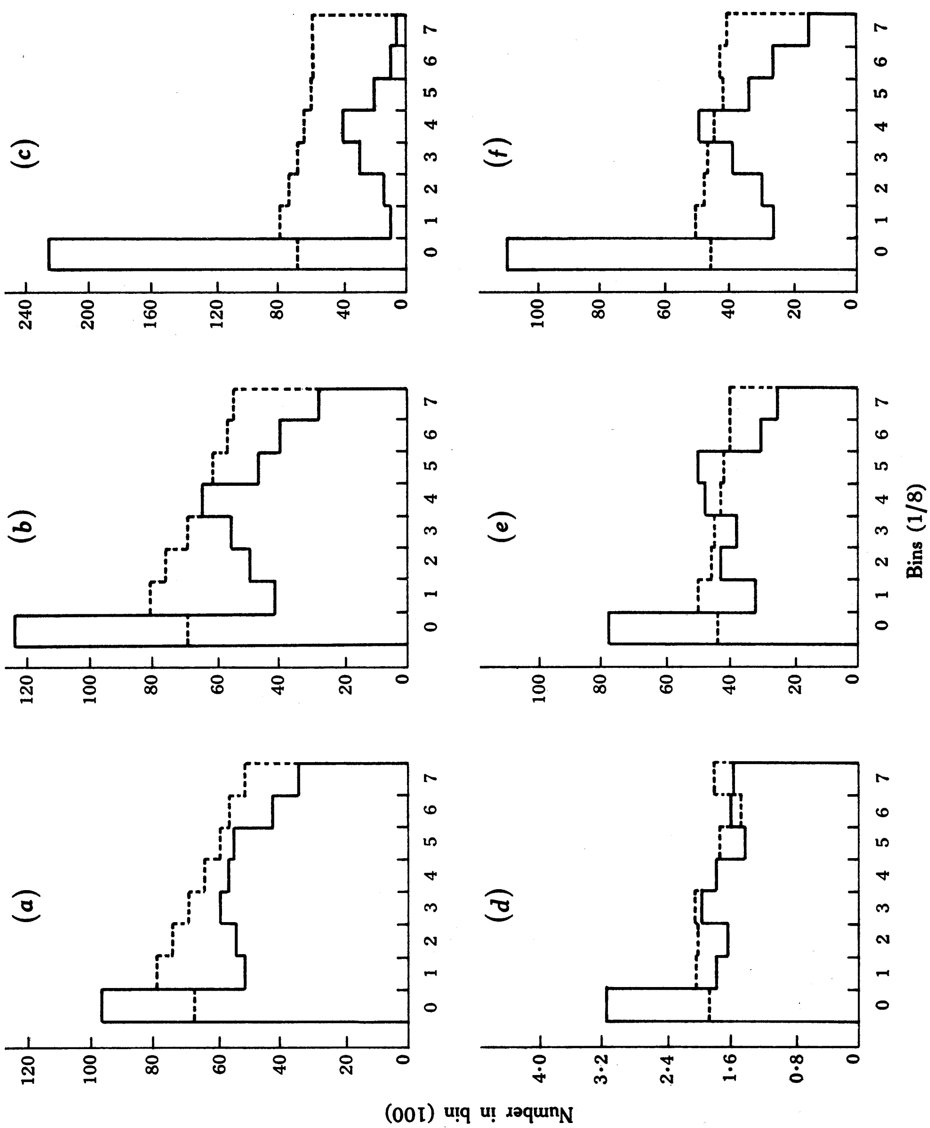

Fig. 2 *(see caption on facing page).*

especially, and in Figs 2*e* and 2*f*. This phenomenon is *not* an artifact produced by the computer but arises inevitably if there is order present in the data, because the presence of a rough ill-defined 'unit' interval in the distances between objects (which is equivalent, approximately, to the hypothesis of nonrandomness and correlation) is bound to show up eventually as a second, more nebulously-defined unit twice as large. Hence the presence of a secondary peak at the bin-$\frac{1}{2}$ stage.

Program I discovered nonrandomness or a hidden ordering in bright galaxies and bright stars, and tentatively ($1 \cdot 5$–2σ deviation only) in Arp peculiar objects and planetary nebulae. The first two firm results are dramatically confirmed by program II, as can be seen from runs 48–51 (bright galaxies with random data for comparison), and runs 29, 30, 35, 36 and 37 (bright stars). In particular, runs 48 and 50 (Figs 2*a*–2*c*) show a distinct nonrandomness in the distance distribution of bright galaxies, and columns 4 and 5, run 35, of Table 1 are spectacular confirmations of a similar result for bright stars. The positive conclusions of program I are therefore upheld.

The other four classes of data may be examined numerically in rows 1 and 3 (QSOs); 13, 16, 18 and 24 (pulsars); 31 and 33 (Arp objects); 42 and 44 (planetary nebulae) of Table 1 or graphically in Fig. 2*d* (QSOs) and in Figs 2*e* and 2*f* (Arp objects).

Program II is able to give a sufficiently sensitive indication of whether nonrandomness is present in the interesting cases of QSOs and Arp peculiar objects, in which correlation was most expected to be present. For the latter group, Fig. 2*f* does suggest the presence of nonrandom structure. The data of row 33 of Table 1 yield a value for $\mathscr{R}$ of $1 \cdot 42$, which from the tabulation of Hart (1942) indicates that the structure shown in Fig. 2*f* has at most a 1 in 10 chance of being due to random fluctuations. Similar calculations for QSOs, pulsars and planetary nebulae show that the chances of the results being random are about 1 in 8, 1 in 6 and 1 in 5 respectively. The hypothesis that *all* groups of data show a significant degree of nonrandom distribution in inter-object positions can thus be seen to be upheld.

6. Conclusions

The results of program II can be summed up succintly in this way: they confirm the results of program I on bright galaxies and bright stars, and extend the identification of correlation to the other groups of data (QSOs, Arp catalogue objects, pulsars and planetary nebulae) chosen for analysis. The likelihood status of the latter classes of objects are of the order of 10 : 1 in favour of there existing a correlation in the positions of astronomical objects (98% significance level of Hart 1942), in the sense of an ordering of inter-object positions with respect to test by comparison with the integers.

The above conclusion rests entirely on the statistics of large numbers ($\sim$50–400) of objects, and cannot be impeached by any appeal to particular cases. The reason that the nonrandomness has come to light appears to rest on the fact that both program I and program II operate on an effective sample of N^2 objects ($N \approx$ 50–400),

Fig. 2. Comparison of results of program II for real data (solid histograms) and random data (dashed histograms). The data are: (*a*) bright galaxies in l^{II} (rows 48 and 49 of Table 1); (*b*) bright galaxies in b^{II} (rows 48 and 49); (*c*) bright galaxies in SGL (rows 50 and 51); (*d*) QSOs in Dec. (rows 1 and 2); (*e*) Arp catalogue objects in R.A. (rows 33 and 34); (*f*) Arp catalogue objects in Dec. (rows 33 and 34).

as can be seen by studying the relevant programs. This does not mean that any more information is being extracted from the sample, but rather that the data are being used more effectively. Of course, both programs use only angular intervals as prime data. Linear intervals depend on the distances of the objects, which are only defined as means for a given class when data are used as above. Consequently, one could consider that the nonrandomness discovered is purely an indication of the fact that it is impossible to put an indefinitely large number of points on a sphere and have them all randomly equivalent. If this is the only explanation, i.e. if this geometrical effect is not only present but dominant in astronomical samples, it would have important consequences in astrophysics. It would imply, for example, that Arp's (1970) contentions are statistically true but physically baseless. However, the restriction of the data employed to well-defined groups also allows one to say that the nonrandomness discovered could be partly due to a physical phenomenon affecting the inter-object distances. Peebles (1974), for example, has carried out a different type of analysis using angular intervals. From these data, via the covariance function, he has inferred results concerning the linear scales of clustering of galaxies (his paper summarizes this and previous work). While the basic geometrical reason for expecting nonrandomness is undoubtedly present at some level, it is therefore also necessary to consider physical reasons for an ordering of linear inter-object positions.

As to cosmological reasons for the nonrandomness, I offer these suggestions: (1) All astronomical objects in equilibrium situations (stars in the Galaxy, or galaxies in a cluster) tend to be at roughly the same mean distance from their nearest neighbours due to gravitational and dynamic interaction tendencies. (2) All astronomical objects are created in multiple-unit processes which result in approximate correlations in distances between co-epoch objects. In this class of hypotheses falls the ejection mechanism of Arp (1970) and others. (3) All astronomical objects are born out of plasma columns supporting azimuthal magnetic fields, which columns break up into equally spaced subcondensations ('sausage' instability). (4) The nonrandomness in galaxies, etc. is empirical validation of the predicted chain-of-galaxies phenomenon peculiar to hierarchical cosmologies. (5) If very long wavelength gravitational radiation is present in the universe, this could lead to multiple images of astronomical objects, as pointed out by Wheeler (1962) and Landau and Lifshitz (1971).

Which, if any, of these explanations are correct, it is of course impossible to say. The results presented seem to point to some effect acting mainly on the members of ensembles which have attained some degree of kinematic equilibrium, namely, galaxies in clusters and stars within a given galaxy. However, hypothesis (4) can also be expected to figure prominently. The chain-of-galaxies effect in a hierarchical universe is predicted to arise because of the superposition of remote and nearby clusters on a photographic plate, although the galaxies forming such a chain belong to different clusters. This effect is therefore an essential property of the clustering of objects and of their observation, and it might be expected to have some connection with the star chains visible on actual photographs (Scott *et al.* 1954). What is certain is that the notable nonrandomness detected for galaxies in clusters and stars in galaxies has some physical cause peculiar to their own groups. The equally large group of objects from the Arp catalogue shows only a very low level of nonrandomness by comparison.

The speculative nature of reasons (2), (3) and (5) suggests the following consistent interpretation of the low level of nonrandomness found in QSOs, Arp peculiar objects, pulsars and planetary nebulae, and of the high level of nonrandomness found in bright galaxies and bright stars. Because it is impossible to place an arbitrarily large number of randomly equivalent points on a sphere, finite nonrandomness of a low level is present in all classes of astronomical objects examined. This means that the claims of Arp (1970) are statistically sound but physically baseless. Confirmation of this view comes from the Arp catalogue objects, which show that peculiar objects are nonrandomly separated from each other, but only to a very small degree compared with the nonrandomness in the inter-object positions of bright stars and bright galaxies. The latter cases are very noticeable, and could be explained by hypotheses (1) or (4). Insofar as hypothesis (4) is a predicted effect of observations in a hierarchy composed of clusters (of stars, galaxies, etc.) while (1) is speculative, it seems likely that the geometrical nonrandomness is here swamped by the effects of the hierarchical distribution of astronomical objects.

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