

On Stationary Distributions of Charged Dust

A. Banerjee,^A N. Chakravarty^B and S. B. Dutta Choudhury^B

^A Department of Physics, Jadavpur University, Calcutta 700032, India.

^B St Anthony's College, Shillong 793003, India.

Abstract

The correct value of the ratio of charge density to mass density is obtained for a stationary distribution of charged dust with vanishing Lorentz force. It is found that the ratio is truly an arbitrary constant, in disagreement with the apparently incorrect result obtained by Misra *et al.* (1972).

In recent communications Misra *et al.* (1972, 1973) have investigated the Einstein-Maxwell equations for a stationary distribution of charged dust in order to find the relation between mass and charge density. Unfortunately their calculations appear to be in error owing to an incorrect choice for the sign of ε in their use of the field equations given by Harrison (1968). We show in this note that with a correct formulation of the field equations the value of $|\sigma|/m$, the ratio of the charge density to the mass density, turns out to be truly an arbitrary constant, allowing both extremes of a massless charge distribution and an uncharged dust distribution. This is in contrast to the ratio obtained by Misra *et al.* which had an upper limit that was less than unity.

For stationary charged incoherent matter without any spatial symmetry, we use the line element

$$ds^2 = e^{2u}(dt + f_a dx^a)^2 + e^{-2u} \gamma_{ab} dx^a dx^b, \quad (1)$$

where Latin indices assume values from 1 to 3 and the metric is independent of time t . In what follows Greek indices may have values from 1 to 4.

The field equations are now

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi T_{\mu\nu}, \quad (2)$$

where

$$T_{\mu\nu} = mv_\mu v_\nu - (F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta})/4\pi, \quad (3)$$

which is a combination of the energy tensors due to the matter and the electromagnetic field. Assuming that the Lorentz force $F_{\mu\nu}v^\nu$ vanishes everywhere and using comoving coordinates, one gets from the Bianchi identity

$$v_{\mu;\nu}v^\nu = -(\sigma/m)F_{\mu\nu}v^\nu = 0. \quad (4)$$

This relation yields $g_{00} = \text{const.}$ and without loss of generality we may assume

the constant to be equal to unity. Also from equation (4) we have that the component $F_{4a} = 0$.

With now the magnetic potential A in the form (Harrison 1968)

$$F^{ab} = (-g)^{\frac{1}{2}} \varepsilon^{abc} A_{,c} \quad (5)$$

and the electric potential $B = 0$ (since $F_{4a} = 0$), the correct formulation of the relevant Einstein-Maxwell equations are (see Harrison 1968)

$$\Delta_1(\phi, A) = -4\pi\sigma, \quad \Delta_2(A) = 0, \quad (6a, b)$$

$$-\Delta_1(A) + \frac{1}{2} \Delta_1(\phi) = -4\pi m, \quad \Delta_2(\phi) = 0. \quad (6c, d)$$

Here

$$\phi_{,a} = Z_a \quad \text{with} \quad f_{[a,b]} = \frac{1}{2} \varepsilon_{abc} \gamma^{cd} Z_d (-\gamma)^{\frac{1}{2}}$$

and

$$\Delta_1(A) = \gamma^{ab} A_{,a} A_{,b}, \quad \Delta_1(\phi, A) = \gamma^{ab} \phi_{,a} A_{,b}, \quad \Delta_2(A) = \gamma^{ab} A_{;ab}.$$

The above field equations were obtained from those given by Harrison by putting $\varepsilon = -1$. From his expression for the electromagnetic energy tensor it is clear that this is the correct assignment, and it is justified when one considers a time-like interval and chooses x^4 as the time coordinate (see Example 3 in Section 4 of Harrison's paper). One should also be careful to note that the Ricci tensors and the field equations are written by Harrison in the same form as that of Landau and Lifshitz (1962). The present equation (6c) differs from the corresponding one of Misra *et al.* (1972) only in that they have taken $\varepsilon = +1$, but this makes a significant difference to the result obtained for the ratio $|\sigma|/m$.

Assuming now that the magnetic potential A and the twist potential ϕ are functionally related, one gets from equations (6b) and (6d) $\phi = \alpha A + \beta$, where α and β are constants. Thus from equations (6a) and (6c) the charge to mass ratio is found to be given by

$$|\sigma|/m = \alpha/(\frac{1}{2}\alpha^2 - 1). \quad (7)$$

Equation (7) shows that the ratio is truly an arbitrary constant, while the relation obtained by Misra *et al.* (1972), namely $|\sigma|/m = \alpha/(\frac{1}{2}\alpha^2 + 1)$, possesses an upper limit of $1/\sqrt{2}$ in magnitude and leads to the incorrect conclusion that the charge density is always less than the mass density. It is worthwhile to note here that a similar mistake has been made by Misra *et al.* (1973) in their separate treatment of stationary axially symmetric charged dust with rigid rotation and vanishing Lorentz force, in which an error in sign occurs in their expression for the electromagnetic energy-momentum tensor. The correct formulation leads to the same expression (7) for the ratio of charge to mass density.

The relation (7) above for the ratio $|\sigma|/m$ was first given by Som and Raychaudhuri (1968) for a cylindrically symmetric system of charged dust with rigid rotation and vanishing Lorentz force. The importance of the present work lies in noting that the conclusion reached by Som and Raychaudhuri, that the extreme case $m = 0$ (massless charge) may be permitted, is also valid under more general conditions with less or no restriction on spatial symmetry. This extreme occurs when $\alpha = \sqrt{2}$, i.e. when the gravitational effect of the magnetic field balances the centrifugal force due to rotation.

References

- Harrison, B. K. (1968). *J. Math. Phys. (New York)* **9**, 1744.
Landau, L., and Lifshitz, E. (1962). 'Classical Theory of Fields' (Addison-Wesley: Reading, Mass.).
Misra, R. M., Pandey, D. B., and Srivastava, D. C. (1972). *Nuovo Cimento B* **9**, 64.
Misra, R. M., Pandey, D. B., and Srivastava, D. C. (1973). *Proc. Indian Natl Sci. Acad. A* **39**, 132.
Som, M. M., and Raychaudhuri, A. K. (1968). *Proc. R. Soc. London A* **304**, 81.

Manuscript received 22 August 1975

