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Massive Scalar Field: Source of the Graviton and 'Strong Gravity'

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Abstract

An exact class of nonstatic spherically symmetric solutions is obtained for the Einstein field equations with a massive scalar field as source. The solutions are found to characterize the 'strong gravity' associated with elementary particles, and it is shown that Ivanenko's (1965) massive graviton possesses zero spin.

1. Introduction

The study of scalar fields without a mass parameter in general relativity has led many authors to some significant conclusions regarding both the singularities which emerge and Mach's principle, but only a scalar field with a mass parameter (a massive scalar field) that satisfies the Klein–Gordon equation can be physically meaningful. For example, it is well known in quantum physics that a massive scalar field is associated with zero-spin chargeless particles like π and K mesons, and the study of such a field in general relativity has been initiated to provide an understanding of the nature of the space–time and the gravitational field associated with neutral elementary particles of zero spin.

Candelas (1974) has recently proposed that, by admitting a massive scalar field into general relativity, one could avoid the emergence of singularities. Earlier, Roy and Rao (1972), by considering a massive scalar field as a source of gravitation, arrived at the conclusion that an axially symmetric gravitational field was not possible without a vanishing mass parameter. In the present paper, we re-examine this conclusion in terms of Einstein's field equations in their most general form, with the extra term Λg_{ij},

$$G_{ij} \equiv R_{ij} - \tfrac{1}{2}Rg_{ij} + \Lambda g_{ij} = -kT_{ij}. \tag{1}$$

When these equations are taken to represent large-scale phenomena in cosmology, Λ plays the role of the physically controversial cosmological constant.

The stress energy tensor corresponding to a massive scalar field is given by

$$T_{ij} = (1/4\pi)\{V_{,i}V_{,j} - \tfrac{1}{2}g_{ij}(V_{,k}V^{,k} - M^2V^2)\}, \tag{2}$$

where the scalar V satisfies the Klein–Gordon equation

$$g^{ij}V_{;ij} + M^2V = 0 \tag{3}$$

and M is related to the mass m of the zero-spin particle by

$$M = m/\hbar .$$

Here a comma or a semicolon followed by a subscript or superscript denotes partial differentiation or covariant differentiation respectively. To solve the field equations we assume the space–time associated with this distribution to be spherically symmetric with maximally symmetric three-dimensional subspaces whose metrics have positive eigenvalues and arbitrary curvature. This assumption leads to a metric of the form (Weinberg 1972, p. 403)

$$ds^2 = dt^2 - Q^2(t)\{dr^2/(1-Kr^2) + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2\}, \tag{4}$$

where K is the curvature index which can take the values -1, 0 or 1. This metric is identical with the familiar Robertson–Walker metric of cosmology. In Section 2 we write the field equations explicitly and in the following section we obtain a class of exact solutions, while in Section 4 some interesting physical consequences of these solutions are considered.

2. Field Equations

For a stress energy tensor of the form (2), the field equations (1) become

$$G_{ij} \equiv R_{ij} - \tfrac{1}{2}Rg_{ij} + \Lambda g_{ij} = -(k/4\pi)\{V_{,i}V_{,j} - \tfrac{1}{2}g_{ij}(V_{,k}V^{,k} - M^2V^2)\}. \tag{5}$$

For the metric (4), this set of field equations becomes

$$G_{11} \equiv 2QQ_{,44} + Q_{;4}^2 + K - \Lambda Q^2 = -(k/8\pi)\{(1-Kr^2)V_{,1}^2 + Q^2(V_{,4}^2 - M^2V^2)\}, \tag{6}$$

$$G_{22} \equiv 2QQ_{,44} + Q_{;4}^2 + K - \Lambda Q^2 = -(k/8\pi)\{-(1-Kr^2)V_{,1}^2 + Q^2(V_{,4}^2 - M^2V^2)\}, \tag{7}$$

$$G_{33} \equiv G_{22}, \tag{8}$$

$$G_{44} \equiv 3(Q_{;4}^2 + K) - \Lambda Q^2 = (k/8\pi)\{(1-Kr^2)V_{,1}^2 + Q^2(V_{,4}^2 + M^2V^2)\}, \tag{9}$$

where

$$V_{,2} = V_{,3} = 0, \qquad Q_{,4} = dQ/dt, \qquad V_{,1} = \partial V/\partial r \qquad \text{and} \qquad V_{,4} = \partial V/\partial t.$$

The velocity of light c is chosen to be unity throughout our discussion.

From equations (6) and (7), we have

$$V_{,1} = 0.$$

Hence equations (6)–(9) reduce to

$$2QQ_{,44} + Q_{;4}^2 + K - \Lambda Q^2 = -(k/8\pi)\{Q^2(V_{,4}^2 - M^2V^2)\} \tag{10}$$

and

$$3(Q_{4,}^2 + K) - \Lambda Q^2 = (k/8\pi)\{Q^2(V_{,4}^2 + M^2V^2)\}, \tag{11}$$

while the Klein–Gordon equation (3) becomes

$$V_{,44}+3(Q_{,4}/Q)\,V_{,4}+M^2V=0. \tag{12}$$

3. Solutions of Field Equations

We now proceed to solve the field equations (10)–(12). On eliminating the $Q_{;4}^2+K$ term from equations (10) and (11) we obtain

$$Q(Q_{,44}-\tfrac{1}{3}\Lambda Q) = -(k/24\pi)\{Q^2(2V_{;4}^2-M^2V^2)\}.$$

Since $Q \neq 0$, we have

$$Q_{,44}-\tfrac{1}{3}\Lambda Q = -(k/24\pi)\{Q(2V_{;4}^2-M^2V^2)\}. \tag{13}$$

To solve the highly nonlinear field equations we make the assumption

$$Q_{,44}-\tfrac{1}{3}\Lambda Q = 0, \tag{14a}$$

which from equation (13) leads to

$$2V_{;4}^2-M^2V^2 = 0. \tag{14b}$$

On integrating equation (14a) we have either

$$Q = A\exp(\alpha t) \qquad \text{or} \qquad Q = B\exp(-\alpha t), \tag{15a, b}$$

where $\alpha^2 = \frac{1}{3}\Lambda$, while A and B are arbitrary constants of integration. Similarly from equation (14b) we have either

$$V = C\exp\{-(M/\sqrt{2})t\} \qquad \text{or} \qquad V = D\exp\{(M/\sqrt{2})t\}, \tag{16a, b}$$

where C and D are arbitrary constants of integration. It may be mentioned that a linear combination of equations (15a) and (15b), although a solution of (14a), does not satisfy the other field equations. This is also the case with a linear combination of the solutions (16a) and (16b). Thus the following four combinations

$$Q = A\exp(\alpha t), \qquad V = C\exp\{-(M/\sqrt{2})t\}; \tag{17a}$$

$$Q = A\exp(\alpha t), \qquad V = D\exp\{(M/\sqrt{2})t\}; \tag{17b}$$

$$Q = B\exp(-\alpha t), \qquad V = C\exp\{-(M/\sqrt{2})t\}; \tag{17c}$$

$$Q = B\exp(-\alpha t), \qquad V = D\exp\{(M/\sqrt{2})t\} \tag{17d}$$

comprise the possible solutions of the field equations (10)–(12).

From equations (11), (12) and (17a) we find that

$$\alpha = M/\sqrt{2} \qquad \text{and} \qquad C = \frac{4}{AM}\left(\frac{\pi K}{k}\right)^{\frac{1}{2}}.$$

Hence the solution (17a) is

$$Q = A\exp\{(M/\sqrt{2})t\} \qquad \text{and} \qquad V = \frac{4}{AM}\left(\frac{\pi K}{k}\right)^{\frac{1}{2}}\exp\{-(M/\sqrt{2})t\}. \tag{18}$$

Similarly the solution (17b) is

$$Q = B\exp\{-(M/\sqrt{2})t\} \qquad \text{and} \qquad V = \frac{4}{BM}\left(\frac{\pi K}{k}\right)^{\frac{1}{2}} \exp\{(M/\sqrt{2})t\}. \tag{19}$$

The remaining two solutions (17c) and (17d) can be identified with either equation (18) or (19). Thus, in general, we have the set of two solutions (18) and (19), and for both of them the value of Λ is $\frac{3}{2}M^2$. The metrics corresponding to the solutions (18) and (19) are

$$ds^2 = dt^2 - A^2 \exp(\sqrt{2}\,Mt)\{dr^2/(1-Kr^2) + r^2\,d\theta^2 + r^2 \sin^2(\theta)\,d\phi^2\} \tag{20}$$

and

$$ds^2 = dt^2 - B^2 \exp(-\sqrt{2}\,Mt)\{dr^2/(1-Kr^2) + r^2\,d\theta^2 + r^2 \sin^2(\theta)\,d\phi^2\} \tag{21}$$

respectively.

4. Conclusions

The solutions we have obtained characterize the interaction between the gravitational field and the massive scalar field. In discussing these solutions it is sufficient to consider the solution (18), since the analysis of the metric is identical for both solutions except that the solution (18) represents an expanding model while (19) represents a contracting model. However, it can be clearly observed that in the expanding space–time (20), the scalar field V decreases exponentially with time. The energy density associated with the scalar field is given as (Anderson 1967, p. 289)

$$\rho = \tfrac{1}{2}(V_{,4}^2 + M^2 V^2).$$

From the equations (18) we have

$$\rho = (12\pi K/A^2 k)\exp(-\sqrt{2}\,Mt)$$

so that, in the expanding space–time (20), the energy density also decreases with time but at a faster rate than the scalar V.

Owing to the formal identity of our metric with the Robertson–Walker metric of cosmology, the analysis of the form (20) can be pursued along familiar lines. The value of the Hubble constant corresponding to the metric (20) is

$$H = Q_{,4}/Q = M/\sqrt{2}, \tag{22}$$

and the value of the deceleration parameter q is

$$q = -Q_{,44}\,Q_{,4}/Q^2 = -1.$$

These values for H and q may lead one to feel that the space–time represents a steady state model, but this is not so because in a steady state model the curvature index K is necessarily zero (Weinberg 1972, p. 459). For $K = 0$, it can be easily verified that the energy density $\rho = 0$ and the metric (20) corresponds to a de Sitter model. It is also interesting to note that the space–time does not have a singularity at any finite epoch.

When we take the massive scalar field to correspond to a neutral π meson, the value of H from equation (22) is given by

$$H = M/\surd 2 = m_{\pi}/\hbar\surd 2 \approx 2\times 10^{22} \quad \mathrm{s}^{-1},$$

where m_{π} is the mass of a neutral π meson. This value suggests a characteristic time

$$T = H^{-1} \approx 5\times 10^{-23} \quad \mathrm{s},$$

which is of the order of strong interaction times, thus apparently corroborating the recently developed idea of the 'strong gravity' (Isham *et al.* 1971; Tennakone 1974) associated with elementary particles.

If we were to take the space–time (20) to represent a cosmological universe, the relationship (22) would suggest that a characteristic mass can be obtained from the observed value of Hubble's constant. Thus, by comparing the relationship (22) with that due to Ivanenko (1965, quoted from Horák 1970), namely,

$$H \approx m_g/\hbar$$

in the units $c = 1$, we conclude that the mass parameter in the Klein–Gordon equation is the mass of the graviton. This leads us to conclude that the graviton suggested by Ivanenko has zero spin. In other words, the spin-zero graviton is responsible for the cosmological effects. It may be interesting to note in this connection the results obtained by Gursey (1963) who, through a different interpretation of general relativity, showed that the spin-zero graviton is responsible for cosmological effects, while the spin-two graviton is responsible for gravitational attraction.

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