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Distinguishing Between Thermalizing and Electrodynamic Coupling For Laser-compressed Thermonuclear Reactions

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Abstract

In the gas-dynamic laser compression of plasmas for exothermal nuclear reactions, certain laser intensities must not be exceeded in order to reach sufficient thermalizing coupling. A criterion for this limitation is developed here, which shows that the published cases of Nuckolls (1974) and Brueckner (1974) are close to this threshold and may not need serious changes due to the coupling process. As well as the long pulse scheme of Afanasyev *et al.* (1975), an alternative method exists in which very short and very high intensity laser pulses are applied, thereby avoiding thermalization but generating fast imploding cold and thick plasma shells due to nonlinear ponderomotive forces of optical explosion. A necessary and sufficient condition for this electrodynamic coupling is derived, and the general nonlinear force equation is integrated to derive ion energies and their exact linear increase with ion charge. Experiments indicating the predominance of the nonlinear force are then analysed. The derived criteria are used to explain examples of nonlinear force compression giving 1000 times greater efficiencies for nuclear reactions than the gas-dynamic case.

1. Introduction

The transfer of laser energy to plasma motion to give the compression and heating required for exothermal nuclear reactions imposes limits to the high laser irradiance necessary, as long as the coupling is based on thermalizing processes. Nevertheless, given these irradiance restrictions, voluminous studies (Brueckner 1974; Kidder 1974; Nuckolls 1974; Bethe and Kidder 1975) have produced some promising results for reasonable power reactors based on thermalizing and gas-dynamic processes. However, at higher laser irradiances, an alternative means of transforming laser radiation into mechanical energy is possible, namely, through an electrodynamic coupling due to the optical (dielectric) properties of the plasma and the nonlinear electrodynamic part of the ponderomotive force, as derived by Hora (1969*a*, 1969*b*). The higher laser irradiances possible with this mechanism can be used to reach much greater nuclear reaction efficiencies by applying simple short laser pulses, as shown by the semi-analytical models of Hora (1975*a*) and proven by Hora *et al.* (1974) with detailed numerical calculations.

The restrictions imposed by the thermalizing interaction of lasers and a plasma are easily understood by considering the collision time of the electrons which, in its nonlinear generalization, increases rapidly at high laser irradiances. A thermokinetic compression, however, does not make sense if the collision time exceeds the time of dynamic interaction, which must necessarily be very short when the Guderley (1942) method of successive shock generation is applied. Alternatively, if the completely different gas-dynamic scheme of Afanasyev *et al.* (1975) is used with longer laser pulses, it may not be affected by the above consideration. But one problem of the gas-dynamic concept is the creation of entropy by shocks. This problem may be totally avoided in principle if the optical scheme of Hora (1975*a*, 1975*b*) is used, as discussed by Hughes (1975*a*, 1975*b*).

The form of the present paper is as follows: We rewrite the criteria for the predominance of the nonthermalizing optical interaction (due to the nonlinear electrodynamic part of the ponderomotive forces) over the thermokinetic forces. An analytic criterion is then derived for the limitation of irradiance with respect to the rise time or pulse length of the laser radiation for the gas-dynamic compression scheme. This criterion is considered for published cases of gas-dynamic compression. Based on the WKB approximation, an exact integration of the equation of motion is possible. The results obtained show a linear increase of the ion energy with the ion charge Z, and a transformation of the dielectrically swollen oscillation energy of the electrons into Z times the translational energy of the ions. A further discussion of experiments which show the predominance of the nonlinear forces is then given on the basis of the derived criteria. Finally, some results of applying nonlinear forces to optical explosion-driven ablation and compression of plasmas for nuclear reactions are discussed to develop an improved compression scheme along the lines of those outlined by Bethe and Kidder (1975).

2. Dielectric Nonlinear Force

The force density $\boldsymbol{f}$ in a plasma is given by the general expression for the ponderomotive force

$$\boldsymbol{f} = -\nabla p + \nabla\cdot\left(\mathbf{U} + \frac{|\tilde{n}|^2-1}{4\pi}\boldsymbol{E}\boldsymbol{E}\right) - \frac{\partial}{\partial t}\left(\frac{\boldsymbol{E}\times\boldsymbol{H}}{4\pi c}\right). \tag{1}$$

In this definition by Landau and Lifshitz (1966), $\boldsymbol{f}$ consists of a gas-dynamic or thermokinetic part $\boldsymbol{f}_{\mathrm{th}} = -\nabla p$ due to the pressure $p = nkT$ (where n is the density of particles, k the Boltzmann constant and T the temperature) and of an electrodynamic part $\boldsymbol{f}_{\mathrm{NL}}$ with terms containing the electric and magnetic field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$, the Maxwellian stress tensor $\mathbf{U}$, the vacuum velocity of light c and the complex refractive index $\tilde{n}$. The electrodynamic part of the ponderomotive force consists therefore of terms with binary products of $\boldsymbol{E}$ and $\boldsymbol{H}$ only, and if both are due to high frequency oscillations of frequency ω (e.g. a laser frequency) then the resulting expressions contain an oscillation frequency of 2ω. The electrodynamic part of the ponderomotive force is then essentially nonlinear and may be called a nonlinear force $\boldsymbol{f}_{\mathrm{NL}}$.

It is the nonlinear force that is of interest in the present analysis, since it creates net motions of laser irradiated plasmas due to the dielectric properties of the plasma. Such a nonlinear dielectric force is mainly determined by the refractive index $\tilde{n}$ of the plasma:

$$\tilde{n}^2 = 1 - \{\omega_{\mathrm{p}}^2/(\omega^2+\nu^2)\}\{1 + \mathrm{i}\nu/\omega\}, \tag{2}$$

where the nonrelativistic plasma frequency ω_{p} is given by

$$\omega_{\mathrm{p}}^2 = 4\,\pi e^2 n_{\mathrm{e}}/m, \tag{3}$$

with e the electron charge, n_{e} the electron density and m the electron mass. The collision frequency ν is given by

$$\nu = \frac{8\cdot 64\times 10^{-7}}{\gamma_E(Z)}\frac{Zn_e}{T^{3/2}}\ln\Lambda, \qquad \text{with} \qquad \Lambda = 1\cdot 55\times 10^{11}\frac{T^{3/2}}{n_e^{1/2}}. \tag{4a, b}$$

The total energy kT of the electrons consists of two components: a component kT_{th} due to the random motion, and a component ε_{osc}^{e} due to the laser-driven coherent oscillation; that is,

$$T = T_{th} + \varepsilon_{osc}^{e}/k. \tag{5}$$

Here

$$\varepsilon_{osc}^{e} = E_v^2/8\pi n_{ec}|\tilde{n}|, \tag{6}$$

where E_v is the amplitude of $\boldsymbol{E}$ for laser radiation in a vacuum, a minor correction for absorption has been neglected, and the cutoff electron density n_{ec} is that for which $\omega_p = \omega$ in equation (3). The relativistic generalization at high laser irradiances has been described by Hora (1975*c*) in connection with the relativistic self-focusing of laser beams in a plasma. Spitzer's (1956) correction $\gamma_E(Z)$ in equation (4a) for electron collisions is effective if T_{th} determines T in equation (5); it has a slowly varying value between $0\cdot 5$ and $1\cdot 0$ depending on the ion charge Z.

The problem of the restriction of equation (1) to nondispersive media has been discussed previously by Hora (1975*b*). The more general validity of equation (1) for plasmas is confirmed by the fact that the equations of the dielectric nonlinear force can be derived (with restrictions of other kinds) from the two-fluid model (Hora 1969*a*) and from the single-particle motion of the electrons in the laser field (Hora 1971). The nonlinear force description could be used in the following circumstances: for an immediate derivation of compensating gas-dynamic pressures in plasmas by stationary magnetic fields; for a derivation of the spontaneously generated magnetic fields in laser produced plasmas (Stamper and Tidman 1973; Stamper 1975); for a general derivation of the various types of instabilities generated in laser produced plasmas (Chen 1972, 1974); for a determination of the resonance-like local increase (Hora 1970) of the laser irradiance (energy flux density) and of resonance absorption (White and Chen 1974); for a treatment of the stimulated Compton scattering of electromagnetic waves in a plasma (Yu *et al.* 1974; Liu and Dawson 1975) and of Brillouin scattering (Palmer 1971); and for a discussion of the switching-on process (Klima and Petrzilka 1972) and of the momentum transfer in laser irradiated plasmas (Lindl and Kaw 1971; Hora 1974), taking into account a net motion (McClure 1974).

A simplification of equation (1) is possible in the case of perpendicular (x direction) incidence of the laser radiation on an inhomogeneous stratified plasma. Neglecting the Poynting vector, as discussed by Hora (1969*a*), the nonlinear force averaged over the time of fast oscillation is

$$\langle f_{NL}\rangle = -\tfrac{1}{8}\boldsymbol{i}_x\,\partial\langle(E_y^2+H_z^2)\rangle/\partial x, \tag{7}$$

where the light is assumed to be linearly polarized with $\boldsymbol{E}$ parallel to the y direction. Now, for $\nu \ll \omega$, the slowly varying fields can be expressed exactly by

$$E_y^2 = E_v^2/2|\tilde{n}| \qquad \text{and} \qquad H_z^2 = |\tilde{n}|^2 E_y^2 = E_v^2|\tilde{n}|/2 \tag{8}$$

following the WKB approximation or (even when the WKB approximation is not applicable) in the Rayleigh case of a special refractive index

$$\tilde{n}_{\mathrm{R}} = (1+\alpha x)^{-1} \qquad \text{with} \qquad \alpha \approx \text{const.} \tag{9}$$

It then follows from equations (7) and (8) that

$$\langle f_{\mathrm{NL}} \rangle = -\boldsymbol{i}_x \frac{E_{\mathrm{v}}^2}{16\pi} \frac{\omega_{\mathrm{p}}^2}{\omega^2} \frac{\partial}{\partial x}\left(\frac{1}{|\tilde{n}|}\right). \tag{10}$$

From the definitions (2) and (8) for the refractive index and, since $\boldsymbol{E} = \boldsymbol{i}_y \boldsymbol{E}_y$, we obtain the result

$$\langle f_{\mathrm{NL}} \rangle = \tfrac{1}{8}\pi^{-1}(|\tilde{n}|^2 - 1)\nabla \boldsymbol{E}^2, \tag{11}$$

used by e.g. Shearer and Eddleman (1973) or Stamper (1975). This result can also be derived immediately from equation (7) using the definitions (8):

$$\begin{aligned}\langle f_{\mathrm{NL}} \rangle &= -\tfrac{1}{8}\pi^{-1}\boldsymbol{i}_x \frac{\partial}{\partial x}\left| E_y^2(1+|\tilde{n}|^2)\right| \\ &= -\tfrac{1}{8}\pi^{-1}\boldsymbol{i}_x(1+|\tilde{n}|^2)\,\partial E_y^2/\partial x - \tfrac{1}{8}\pi^{-1}\boldsymbol{i}_x E_y^2\, 2|\tilde{n}|\,\partial|\tilde{n}|/\partial x \\ &= -\tfrac{1}{8}\pi^{-1}\boldsymbol{i}_x(1-|\tilde{n}|^2)\,\partial E_y^2/\partial x - \tfrac{1}{8}\pi^{-1}\boldsymbol{i}_x\{2|\tilde{n}|^2\,\partial E_y^2/\partial x + 2|\tilde{n}|E_y^2\,\partial|\tilde{n}|/\partial x\}.\end{aligned}$$

The second term of this expression vanishes because it has a factor $\partial(E_y^2|\tilde{n}|)/\partial x$ which is zero because $E_y^2|\tilde{n}| = \tfrac{1}{2}E_{\mathrm{v}}^2$ is constant.

3. Predominance of Nonlinear Force

Hora (1969*a*) examined the conditions under which the nonlinear force f_{NL} is larger than the thermokinetic force f_{th}. These conditions are obtained most directly when the gas-dynamic and electrodynamic pressures are compared before differentiating in equation (1). For a negligible Poynting vector, we then find from equation (1) for plane plasma geometry and perpendicularly incident radiation:

$$nkT_{\mathrm{th}} \leqslant (E^2+H^2)/8\pi - (E_{\mathrm{v}}^2+H_{\mathrm{v}}^2)/16\pi. \tag{12}$$

The last term is an integration constant resulting from the fact that $\langle f_{\mathrm{NL}} \rangle = 0$ in a very thin plasma for time-independent radiation. At sufficiently high irradiances, this correction can be neglected and, in this case, we arrive at the criteria adopted by Marhic (1975) and Wong and Stenzel (1975). Using the temperature T of equation (5) for the electron temperature to calculate the refractive index $\tilde{n}$ we find that, for densities near the cutoff density, we have:

$$f_{\mathrm{NL}} > f_{\mathrm{th}} \qquad \text{holds for} \qquad n_{\mathrm{e}} \lesssim n_{\mathrm{ec}} \quad \text{if} \quad I \gtrsim CT_{\mathrm{th}}^{1/4}, \tag{13}$$

where I is the irradiance in $\mathrm{W\,cm^{-2}}$, T_{th} is measured in electron volts, and the constant $C = 7{\cdot}5\times10^{13}$ or $2{\cdot}37\times10^{11}$ $\mathrm{W\,cm^{-2}\,eV^{-\frac{1}{4}}}$ for neodymium–glass or CO_2 lasers respectively. This formula is approximate but a numerical iteration, which includes an averaged amount of absorption, results in values quite close to these (Hora 1970).

A numerical evaluation by Mulser and Green (1972) of the stationary cases of plasmas with specific density profiles proved the predominance of the nonlinear force over the thermokinetic force for a net acceleration of a relatively thick plasma layer, even for laser irradiances I only slightly above the threshold (13), as shown in Fig. 5 of Hora (1972). However, it was reported qualitatively by Mulser and Green (1972) that, for a general dynamic description just above the threshold, cases could be constructed where the predominance of the nonlinear force was not evident. Such critical cases (if correct) may be accepted as examples which show that (13) is only a necessary but not a sufficient condition for the predominance of the nonlinear force. On the other hand, using laser irradiances of 10–100 times above the threshold (i.e. exceeding 10^{16} W cm^{-2} for neodymium–glass lasers), Shearer *et al.* (1970) detected significant action of the nonlinear force, and Hora (1975*b*) found a transfer of 23% of the laser pulse energy to kinetic energy of motion of a thick ($20\,\lambda$) layer of cold plasma. A similar transfer of energy into shock waves was calculated by Cooper (1973) from an analytical model, though for thin layers only. However, a more serious problem associated with the generation of shock waves than the transfer of energy to too thin a layer is the production of too much entropy. As stressed by Nuckolls (1974) and Kidder (1974), the compression of the plasma must be performed at a minimum of entropy production, and this has to be achieved in the gas-dynamic case by a complicated tailoring of the laser pulses. On the other hand, in the case of compression by the nonlinear force, Hora (1975*a*) has demonstrated that a fast imploding thick and cold plasma shell can be generated with velocity and density profiles that can be designed so that, at the time of collapse, the ideal compression with constant entropy can be reached. This can be achieved, e.g. by starting from a gaussian density profile, a constant temperature and a linear profile for the compressing velocities. Numerical examples deviating from these conditions have been discussed by Goldman (1974) and have demonstrated the bad influence of shocks unavoidably generated as a result of departures from the ideal conditions.

Obviously we can formulate a sufficient condition for the predominance of the nonlinear force for cold plasmas ($T_{\rm th} \ll \varepsilon^{\rm e}_{\rm osc}/k$) and for a laser pulse length $\tau_{\rm L}$ (a measure of the duration of the interaction) which is short compared with the thermalizing collision time of the electrons. We therefore have a sufficient condition for $f_{\rm NL} > f_{\rm th}$ and $n_{\rm e} \lesssim n_{\rm ec}$ with:

$$I \gtrsim C T_{\rm th}^{1/4}, \tag{14a}$$

$$\tau_{\rm L} \ll \tau_{\rm col} = 1/\nu \qquad \text{for} \qquad n_{\rm e} < \tfrac{1}{2} n_{\rm ec}, \tag{14b}$$

$$T_{\rm th} \ll \varepsilon^{\rm e}_{\rm osc}/k\,. \tag{14c}$$

A necessary and sufficient condition has not yet been found because a special classification of the dynamics in a plasma for $\tau_{\rm L} > \tau_{\rm col}$ would be necessary which should apply in the most general cases.

Evaluation of the condition (14b), using the equations (4) and (14c), results in

$$\tau_{\rm col} = 1\cdot 23\times 10^{18}\, I_{\rm v}^{3/2}/(|\tilde{n}|^{3/2} n_{\rm ec}^{3/2} n_{\rm e} \ln \Lambda)\,, \tag{15}$$

where the laser irradiance $I_{\rm v}$ in a vacuum is given in W cm^{-2}, while both the actual electron density $n_{\rm e}$ (selected to provide an upper bound of interaction at $n_{\rm e} = \frac{1}{2} n_{\rm ec}$ which yields $|\tilde{n}| \approx 1$) and the cutoff electron density $n_{\rm ec}$ are given in cm^{-3}. Plots of equation (15) for irradiation by seven different lasers are given in Fig. 1.

4. Minimum Laser Rise-time for Gas-dynamic Compression

The collision time τ_{col} of the electrons can be considered to be the minimum time for thermalization. If the laser is used for a gas-dynamic ablation and compression process (e.g. Nuckolls 1974; Brueckner 1974), its pulse length has to be longer than τ_{col}. On the other hand, the pulses have to be tailored in time to provide the major transfer of the pulse energy in a very short time. In the following discussion, we use the expression (15) for Coulomb collisions under the nonlinear conditions of high laser irradiances. A modification of this expression which allows for instabilities (Chen 1974; Lin and Dawson 1975) and results in a different effective collision time, is not considered here.

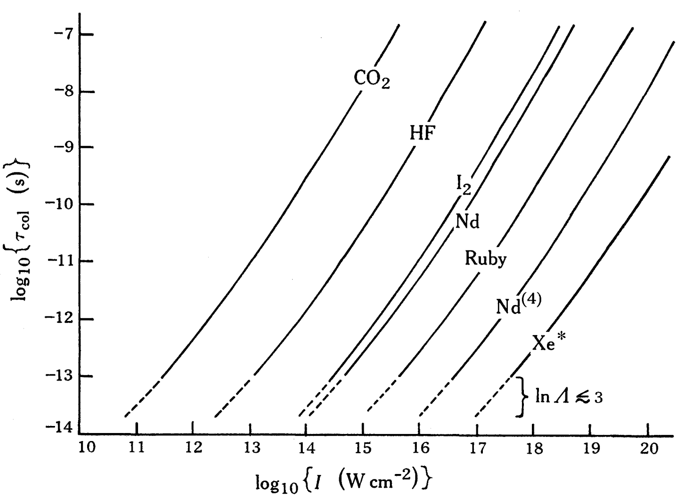

Fig. 1. Plot of the minimum time for thermalization, as given by the collision time τ_{col} of the electrons (equation 15), as a function of the irradiance I by the indicated lasers ($\varepsilon_{osc}^{e} \gg kT_{th}$; $|\tilde{n}| = 1$; $n_e < \frac{1}{2}n_{ec}$). The values of n_{ec} in cm^{-3} are: CO_2, 10^{19}; HF, $8{\cdot}6\times10^{19}$; I_2, $6{\cdot}6\times10^{20}$; Nd–glass, 10^{21}; ruby, $2{\cdot}3\times10^{21}$; 4th-harmonic Nd–glass, $1{\cdot}6\times10^{22}$; Xe*, $3{\cdot}7\times10^{22}$.

We assume for simplicity that the pulse of mechanical power density I_{th} arising from the thermalizing interaction of the radiation with the plasma has the form

$$I_{th} = I_o \sin^2(\pi t/\tau_o) \qquad \text{for} \qquad 0 \leqslant t \leqslant \tau_o, \tag{16}$$

where τ_o is the half-width of the pulse. A generalization to a more complicated pulse shape does not substantially change the following results. The laser pulse has then to arrive earlier (see Fig. 2) by a precursion time which depends on the laser intensity and can be identified with the collision time τ_{col}. This is similar to the observation of Brueckner (1974) from his computer work, that ε_{osc}^{e} has to be less than 1 keV for the plasma to absorb. Thus, for short pulses, the laser behaves in the plasma similar to a light beam in transparent glass and will not produce a thermalizing coupling or a remarkable thermalizing energy transfer (except by nonthermalizing nonlinear forces). The relation between the slope angles α and α' of the pulses (Fig. 2) can be used to find the greatest possible increase of a laser pulse. This is the instantaneous increase corresponding to $\alpha' = \frac{1}{2}\pi$, the highest possible increase I_{th}, which limits the gas-dynamic compression models.

Quantitatively, the maximum increase of I_{th} for a pulse of the shape (16) is given by

$$\partial I_{th}/\partial t = (I_o \pi/\tau_o)\sin(2\pi t/\tau_o) \qquad \text{at} \qquad t = \tfrac{1}{4}\tau_o, \tag{17}$$

or

$$\partial I_{th}/\partial t\,|_{max} = I_o \pi/\tau_o. \tag{18}$$

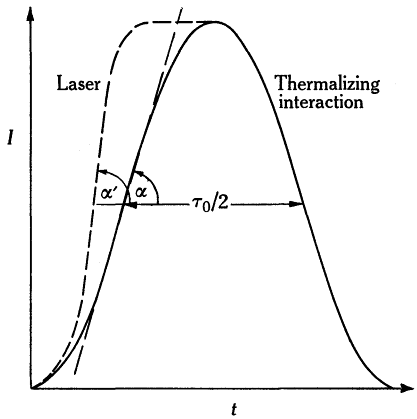

Fig. 2. Illustration showing that the laser (dashed curve) pulse must precede the thermalizing interaction (continuous curve) pulse by an irradiance-dependent precursion time θ (identified with τ_{col} of equation 15) in order to drive a gas-dynamic ablation–compression process. The limitation on the thermalization is reached for $\alpha' = \frac{1}{2}\pi$.

Putting $\theta = \tau_{col}$ we then have from equation (15)

$$\theta = 1{\cdot}23\times 10^{18}\, I_v^{3/2}/(|\tilde{n}|^{3/2} n_{ec}^{3/2} n_e \ln \Lambda), \tag{19}$$

where $I_v = I_o$, as is evident from Fig. 2. From the geometry of Fig. 2, we find the condition corresponding to $\alpha \leqslant \frac{1}{2}\pi$ to be

$$\partial I_{th}/\partial t \leqslant (\partial\theta/\partial I_v)^{-1}. \tag{20}$$

Using equations (18) and (19) and the inequality (20), we obtain

$$I_o/\tau_o \leqslant |\tilde{n}|^{3/2} n_{ec}^{3/2} n_e \ln(\Lambda)/(\tfrac{3}{2}\pi\times 1{\cdot}23\times 10^{18}\, I_v^{1/2}) \tag{21}$$

and, once more identifying I_o with I_v, the restriction for gas-dynamic compression is seen to be

$$I_v^{3/2}/\tau_o \leqslant 1{\cdot}72\times 10^{-19}\, n_e n_{ec}^{3/2} |\tilde{n}|^{3/2} \ln \Lambda, \tag{22}$$

where the units of I_v, n_e and n_{ec} are as given in the previous section, while τ_o is in seconds.

Let us now make a comparison with the computer results of Nuckolls (1974) and Brueckner (1974). They considered a 54 kJ pulse of maximum power 10^{15} W from a neodymium–glass laser interacting with a sphere of a radius $r_o = 400\ \mu$m, with $I_v = 5\times 10^{16}\ \mathrm{W\,cm^{-2}}$ and $\tau_o \approx 100$ ps (sometimes given as 60 ps). From the inequality (22) we find for $|\tilde{n}| = 1$ (needing then $n_e = \frac{1}{2}n_{ec}$) and $\ln\Lambda = 8{\cdot}1$ (for $I = 10^{16}\ \mathrm{W\,cm^{-2}}$):

$$I_v^{3/2}/\tau_o \leqslant 2{\cdot}1\times 10^{34}. \tag{23}$$

Therefore, for $\tau_o = 100$ ps, I_v has to be less than $1 \cdot 7 \times 10^{16}$ W cm^{-2}. The calculation by Nuckolls (1974) has exceeded the limit (23), and this indicates that some revisions of the fusion gain, laser pulse form and other parameters may be required. His later example using shorter wavelengths for the final strong pulse (resulting in higher values for n_{ec} in the inequality 22) relieves the situation. However, the conditions are still very close to the limit. Also, in another case of Brueckner and Jorna (1974), where $r_o = 500\ \mu$m, $\tau_o = 106$ ps and a maximum power of 4×10^{14} W was considered, their value $I_o = 10^{16}$ W cm^{-2} is also very close to the limit (22).

From a more sophisticated point of view, we may agree that there is at least a finite probability for energy transfer to occur at times much less than τ_{col}. However, in the above comparisons we have assumed $|\tilde{n}| = 1$, which favours the gas-dynamic compression. If, as is realistic, $|\tilde{n}| \lesssim 10^{-1}$ then the whole procedure based on the present extensive computer calculations may need to be revised—or to be replaced by a completely different procedure of compression using the nonlinear force (as given by Hora 1975*b*) or the very long pulse process of Afanasyev *et al.* (1975).

5. Integration of Nonlinear Force Equation

In order to discuss some experimental cases in which the conditions for the predominance of the nonlinear force hold and the pulse lengths used are characteristic of nonthermalizing coupling (being insufficient for the gas-dynamic compression of the plasma), we first perform an exact integration of the equation of the nonlinear force. As mentioned in Section 2, the restriction to the WKB approximation for the nonlinear force-dominated equation of motion, with perpendicular incidence of radiation on a stratified plasma, namely

$$f_{NL} = m_i n_i \frac{dv_i}{dt} = -\frac{E_v^2}{16\pi} \frac{n_e}{n_{ec}} \frac{\partial}{\partial x}\left(\frac{1}{|\tilde{n}|}\right), \tag{24}$$

does not apply, at least for the Rayleigh case. The Airy case has been discussed by Lindl and Kaw (1971) beyond the WKB limitation.

Recalling that a velocity v_o gained by a body falling along a distance Δx with an acceleration dv_i/dt is

$$v_o = \{2(dv_i/dt)\Delta x\}^{\frac{1}{2}}, \tag{25}$$

we find for the differential gain of the square of the velocity:

$$\delta v_o^2 = 2(dv_i/dt)\Delta x \tag{26}$$

for varying acceleration. The increase of the kinetic energy of the ions due to the force (24) is then (using $n_e = Zn_i$, with ion charge Z, and avoiding the restriction $\nu \ll \omega$ of the collisionless case)

$$d(\tfrac{1}{2}m_i v_i^2) = m_i \frac{dv_i}{dt} dx = \left|\frac{E_v^2}{16\pi} \frac{Z}{n_{ec}} \frac{\partial}{\partial x}\left(\frac{\exp(-\bar{k}x)}{|\tilde{n}|}\right)\right| dx. \tag{27}$$

Here $\bar{k}$ is the averaged absorption constant for the plasma between a point in the vacuum and the considered point x. The exact integral for the ion energy is then

$$\begin{aligned}\tfrac{1}{2}m_i v_i^2 &= \frac{E_v^2}{16\pi}\frac{Z}{n_{ec}}\int_{x_1}^{x_2}\frac{\partial}{\partial x}\left(\frac{\exp(-\bar{k}x)}{|\tilde{n}|}\right)\mathrm{d}x \\ &= \frac{E_v^2}{16\pi}\frac{Z}{n_{ec}}\left(\frac{\exp(-\bar{k}x_2)}{|\tilde{n}(x_2)|}-\frac{\exp(-\bar{k}x_1)}{|\tilde{n}(x_1)|}\right).\end{aligned} \tag{28}$$

The integration is performed between depth x_1, a point in the vacuum ($|\tilde{n}| = 1$; $\bar{k} = 0$), and x_2, a point in the plasma corresponding to minimum refractive index $|\tilde{n}|$, assuming $\bar{k} \ll 1$ in accordance with the numerical cases of Hora (1975*b*) and Marhic (1975). The maximum energy $\varepsilon_{trs}^{i,abl}$ translational motion towards decreasing electron density which the ablated ions gain due to f_{NL} is then

$$\varepsilon_{trs}^{i,abl} = \frac{E_v^2}{16\pi}\frac{Z}{n_{ec}}\left(\frac{1}{|\tilde{n}|_{max}}-1\right). \tag{29}$$

The ion energy $\varepsilon_{trs}^{i,cpr}$ for the compression of the plasma towards its interior (after selecting the appropriate sign by the same procedure as for equation 26) is given by

$$\varepsilon_{trs}^{i,cpr} = \frac{E_v^2}{16\pi}\frac{Z}{n_{ec}}\left(\frac{1}{|\tilde{n}|_{max}}\right), \tag{30}$$

where the limits of integration in equation (28) are now those for $|\tilde{n}|_{min}$ (with $\bar{k} \ll 1$) and for the interior of the super dense plasma with $\bar{k} \gg 1/x$ and nonvanishing $|\tilde{n}|$. The condition $\bar{k} \gg 1/x$ also holds for laser interaction with a collisionless plasma having a maximum density near n_{ec}. The acceleration of the plasma parallel to the laser beam corresponds to a (dynamical macroscopic) nonlinear absorption with $\bar{k} \gg 1/x$.

The results (29) and (30) can be interpreted very easily by taking into account the fact that the average kinetic energy ε_{osc}^{e} of oscillation of the electrons in the laser field within the plasma is

$$\varepsilon_{osc}^{e} = E_v^2\exp(-\bar{k}x)/16\pi n_{ec}|\tilde{n}|. \tag{31}$$

We assume again that $\bar{k} \ll 1$ holds for the maximum of $1/|\tilde{n}|$, but we include a damping mechanism for the non-WKB case, which prevents the resonance-like increase of $1/|\tilde{n}|$ from reaching unrealistically high values, as pointed out by Hora (1970). This means that the maximum energy of the ablated ions

$$\varepsilon_{trs}^{i,abl} = Z\{(\varepsilon_{osc}^{e})_{max} - (\varepsilon_{osc}^{e})_{v}\} \tag{32}$$

is Z times the difference between the maximum oscillation energy of the electrons in the plasma and in the vacuum. In cases where the swelling of the oscillation energy over its vacuum value reaches a factor $1/|\tilde{n}| \gtrsim 100$, we can neglect the second term in equation (32). This correction, representing the radiation pressure in the vacuum, is then the difference of the ion energy for plasma compression to ablation from equations (30) and (31), namely,

$$\varepsilon_{trs}^{i,cpr} = Z(\varepsilon_{osc}^{e})_{max}. \tag{33}$$

The driving mechanism for the acceleration of a plasma by the nonlinear force is the aggregate of electrons with their quivering motion drifting either toward a lower density (in a mainly collisionless plasma surface) or toward the highly absorbing plasma interior to the rear of an irradiated collisionless plasma. The net motion of the plasma results from the electrostatic drag of all the ions coupled to all the electrons. This process is different from that of Morse and Nielson (1970), in which a few fast electrons produced by the laser radiation near the cutoff density move towards the plasma interior, their space charge being compensated by counter-streaming slow electrons. The driving mechanism for the electrostatic acceleration described by McCall *et al.* (1973) is either electrons subject to a nonlinear force of the kind considered by Hora (1969*a*, 1969*b*), or a similar mechanism for oblique incidence (Chen 1974; Hora 1974*a*). However, such electrostatic mechanisms with an expansion of the electrons in the Debye sheath of the plasma can explain only very small numbers of accelerated ions—of the order of less than 10^{13} per laser pulse under the usual conditions.

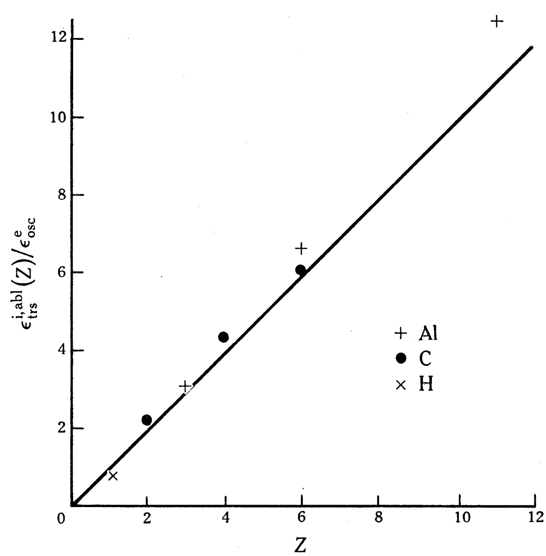

Fig. 3. Comparison of the experimental results of Ehler (1975) with the prediction (straight line) of the nonlinear force theory (equation 32), demonstrating the linear increase in the ion energy $\varepsilon_{trs}^{i,abl}$ with increasing ion charge number Z.

6. Discussion of Experiments

Though several types of experiments for some years now have indicated the presence of the nonlinear force, only the most recent have demonstrated the explicit properties of its action. The earlier experiments consisted of measuring the diameter of self-focusing channels in laser-produced plasmas (Alcock 1972) which were, to the thresholds for self-focusing, in exact agreement with the theory of Hora (1969*a*, 1969*b*). Generation of the complete depletion of the plasma from the beam centre (cavitons) was measured very convincingly by M. C. Richardson and reviewed by Alcock (1972). A very direct diagnostic of the plasma profile generated by the non-linear force and of the depletion region (caviton) due to the nonlinear acceleration has been measured by Marhic (1975).

An excellent example which fulfils the sufficient condition (14) and exceeds the condition (22) for gas-dynamic compression is the experiment of Ehler (1975), who irradiated polyethylene and aluminium with CO_2 laser pulses of $2\times10^{14}\ \mathrm{W\,cm^{-2}}$ irradiance and 1 ns duration, and generated 11-fold ionized aluminum ions of about 2 MeV energy. These results agree with some preliminary measurements by Hughes and Luther-Davies (1976), who irradiated tungsten with 10 ps Nd–glass laser pulses of $10^{16}\ \mathrm{W\,cm^{-2}}$ irradiance, and generated highly charged ions of 3 MeV energy. The linear increase of the ion energy with ion charge Z, as predicted by equation (32), is reconstructed in Fig. 3 from the experimental data of Ehler (1975).

The sufficient condition (14) for the dominating action of the nonlinear force is evidently fulfilled: (1) The irradiance $I = 2\times10^{14}\ \mathrm{W\,cm^{-2}}$ is above the threshold (14a) for $T_{\mathrm{th}} < 100$ eV. (2) The 'random' temperature T_{th} can be assumed to be very low because τ_{col} for electrons in a CO_2 laser field of $2\times10^{14}\ \mathrm{W\,cm^{-2}}$ irradiance is $0{\cdot}92$ ns (equation 15). (3) The mean kinetic energy of oscillation (equation 31) results in $\varepsilon_{\mathrm{osc}}^{\mathrm{e}} = 1{\cdot}1$ keV for areas of low density with $|\tilde{n}| \approx 1$, and is therefore higher than T_{th}. Furthermore, the threshold for thermalizing interaction (equation 23) for CO_2 laser radiation is

$$I_{\mathrm{v}}^{3/2}/\tau_{\mathrm{o}} \leqslant 2{\cdot}1\times10^{29} \qquad \text{for} \qquad \tau_{\mathrm{o}} = 10^{-9}\ \mathrm{s} \quad \text{and} \quad I_{\mathrm{v}} \leqslant 4{\cdot}6\times10^{13}\ \mathrm{W\,cm^{-2}} \tag{34}$$

indicating that, even in the low density regions with $|\tilde{n}| = 1$, the irradiance I_{v} is higher than is necessary for thermalizing energy transfer.

A further experimental result of Ehler (1975) is

$$(\varepsilon_{\mathrm{osc}}^{\mathrm{e}})_{\mathrm{max}} = 200 \quad \mathrm{keV}. \tag{35}$$

This is to be compared with the value $\varepsilon_{\mathrm{osc}}^{\mathrm{e}} = 1{\cdot}1$ keV in the low density region. If no self-focusing were present, this would mean a swelling of

$$1/|\tilde{n}| = (\varepsilon_{\mathrm{osc}}^{\mathrm{e}})_{\mathrm{max}}/\varepsilon_{\mathrm{osc}}^{\mathrm{e}} = 182. \tag{36}$$

However, this is a maximum value only, because self-focusing may have been built up to such an extent that the instant relativistic self-focusing of Hora (1975*c*) may be more appropriate. If the vacuum cross section of the laser beam is ϕ_{o}, the self-focusing decreases this value to $\pi\lambda^2$ with an effective wavelength of $\lambda = \lambda_{\mathrm{o}}(1+|\tilde{n}|)/(2|\tilde{n}|)$, where the vacuum wavelength is λ_{o}. For large swelling ($1/|\tilde{n}| \gg 1$) we have

$$(\varepsilon_{\mathrm{osc}}^{\mathrm{e}})_{\mathrm{max}} = \varepsilon_{\mathrm{osc}}^{\mathrm{e}}\,\phi_{\mathrm{o}}\,4\,|\tilde{n}|/\pi\lambda^2, \tag{37}$$

from which the experimental data of Ehler (1975) give

$$1/|\tilde{n}| = 12{\cdot}7 \tag{38}$$

as a minimum value for the swelling. The real situation will lie between those expressed by equations (37) and (35) but, in any case, the swelling is large enough to generate nonlinear-force translational ion energies (from equation 35) of

$$\varepsilon_{\mathrm{trs}}^{\mathrm{i}} = 200\,Z \quad \mathrm{keV}. \tag{39}$$

Another relevant experiment is that of Wong and Stenzel (1975), who directed microwave pulses of $0{\cdot}4\ \mu$s duration, $I_{\mathrm{v}} = 5{\cdot}3\ \mathrm{W\,cm^{-2}}$ and $\omega = 6{\cdot}28\times10^9\ \mathrm{s^{-1}}$ onto an argon plasma with $T_{th} = 2$ eV and a 4 m deep, monotonic increase of the electron density exceeding the cutoff value n_{ec} of $1 \cdot 36 \times 10^{10}$ cm^{-3}. The criterion (23) then becomes

$$I_v^{3/2}/\tau_o \leqslant 1 \cdot 3 \times 10^7 , \tag{40}$$

which results in $I_v \leqslant 3 \cdot 6$ W cm^{-2}. As $\tau_{col} = 4 \cdot 3 \times 10^{-6}$ s, the conditions of non-thermalization are fulfilled. The observed ion energies of 7 keV can then only be due to the nonlinear ponderomotive force resulting in a swelling, as checked by measuring the profile of the actual irradiance I of

$$1/|\tilde{n}| = 700 . \tag{41}$$

Based on this experimental result, the condition (14a) becomes (for T in electron volts)

$$I \geqslant 5 \cdot 7 \times 10^2 \, T^{1/4} , \tag{42}$$

though the vacuum value $I_v = 5 \cdot 3$ W cm^{-2} is about 100 times less than the minimum value of I. However, the swelling produces $I = 700 \, I_v$, so that condition (14a) is fulfilled in the interior of the plasma. This experiment had the advantage that the time dependence of the density profile at and after the interaction could be measured, showing the net motion of the plasma after its electrodynamically driven explosion (optical explosion) at the cutoff density.

7. Use of Nonlinear Force for Optical Explosion

We have checked (above) that the conditions for the thermalizing laser-compression of plasmas are nearly fulfilled in the published examples of Nuckolls (1974) and Brueckner (1974). We now apply the derived criteria to the nonlinear force compression scheme of Hora (1975*b*). In that paper, the present author used the result that very short laser pulses which generate nonlinear forces with a swelling of $1/|\tilde{n}| = 400$ could transfer more than 23% of the laser energy to kinetic energy of a cold and fast moving plasma (ion energies near 10 keV). Then, using the Rayleigh profile for the refractive index (equation 9) for $\alpha = 4 \times 10^4$ cm^{-1} and neodymium–glass laser radiation, he derived a set of principal conditions for transfering 50% of the energy of a laser pulse of 2×10^{16} W cm^{-2} irradiance within $2 \cdot 83$ ps into a net compressing motion of a plasma having DT (deuterium–tritium) ion energies of 10 keV. The resulting nuclear fusion reactions were found to have about 1000 times higher efficiencies than in cases of thermalizing gas-dynamic compression, so that a total gain of 80 could be reached at laser energies of 328 J for DT, 131 kJ for DD and $1 \cdot 97$ MJ for HB(11). These calculations were based on the ideal adiabatic (isentropic) compression without shocks, which is possible for this scheme, at least in principle. Some more realistic alternative simulations by Goldman (1974) resulted in poorer gains due to shocks but, on adding the reheated α particles, even better gains than the idealized cases were reached.

The condition (23) for neodymium–glass laser radiation for $\tau_o = 2 \cdot 83$ ps is

$$I_v \leqslant 1 \cdot 35 \times 10^{15} \quad \text{W cm}^{-2} , \tag{43}$$

which shows that the cases considered with $I > 10^{16}$ W cm^{-2} are really examples of nonthermalizing coupling. The condition (14a) is immediately seen to hold for $I_v \geqslant 10^{16}\ \mathrm{W\,cm^{-2}}$, and condition (14b) holds because $\tau_{col} \geqslant 38$ ps and $\tau_o = 2{\cdot}83$ ps. Condition (14c) is fulfilled because, in the examples of Hora (1975*b*), T_{th} was a few electron volts or less, while ε_{osc}^{e} exceeded 5 keV.

8. Conclusions

A general criterion for the applicability of the gas-dynamic concept with thermalization has been derived (equation 22), which is fulfilled quite closely by calculations in the literature, but needs correction if the dielectric swelling is strong. For shorter laser pulses of higher irradiance, coupling of the laser radiation by the nonlinear ponderomotive force of optical (dielectric) explosion can be used, and a necessary criterion for this has been derived. For sufficiently short laser pulses and low electron temperatures, a sufficient criterion for the nonlinear force is also given. Comparison with the experimental measurements of Ehler (1975) confirms the prediction that the nonlinear force results in ion energies which increase linearly with the charge on the ions. The strong deviation from the thermalizing gas-dynamic conditions may well be the reason why thermonuclear reactions of the gas-dynamic type could not be observed. It can be concluded therefore that, from the beginning, the nonlinear force compression scheme has to be used for thermonuclear reactions, as it is described by a case of Rayleigh-like initial density, for which all the criteria of nonthermalization are fulfilled.

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