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# The World as Quarks, Leptons and Bosons*

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*Abstract*

A descriptive review is given of gauge theories of weak, electromagnetic and strong interactions. The strong interactions are interpreted in terms of an unbroken Yang–Mills gauge theory based on SU(3) colour symmetry of quarks and gluons. The confinement mechanism of quarks, gluons and other nonsinglets is discussed. The unification of the weak and electromagnetic interactions through a broken Yang–Mills gauge theory is described. In total the basic constituents are then the quarks, leptons and gauge bosons.

## Introduction

Particle physics is an attempt to find the fundamental laws of natural science at the most basic level. Cosmology is a part of that effort also. We hope to achieve a grand synthesis which ends in summarizing practically everything that is known about the world in one or two simple laws. The last time that happened was in the 1920s when quantum mechanics was discovered and made relativistic. Dirac said that his equation explained virtually all of physics and the whole of chemistry. What we have been trying to do ever since is to correct for treatment of the atomic nucleus as a point and to deal with the enormous complexity at the nuclear and subnuclear level. But now it appears that we shall have shortly another synthesis that will include these complex phenomena. Curiously this synthesis closely resembles the original quantum electrodynamics (QED). In QED the coupling of the photon to electrons is written as

$$-\mathrm{i}eA_\alpha(\bar{e}_\mathrm{L}\gamma_\alpha e_\mathrm{L}+\bar{e}_\mathrm{R}\gamma_\alpha e_\mathrm{R})\,. \tag{1}$$

The electron field $e_\mathrm{L}, e_\mathrm{R}$ is left-handed and right-handed only at very high speeds. As well as the electron $\mathrm{e}^-$ the lepton family includes $\mathrm{e}^+$, $\mu^+$, $\mu^-$, $\nu_\mathrm{e}$, $\bar{\nu}_\mathrm{e}$, $\nu_\mu$ and $\bar{\nu}_\mu$. The leptons feel the electromagnetic and the weak and new associated interactions; but they do not feel the strong interaction.

The strongly interacting particles are completely different; these are the baryons $\mathrm{n}^0$, $\mathrm{p}^+$, $\Lambda^0$ ..., the mesons $\pi^-$, $\pi^0$, $\pi^+$, ... and the antibaryons $\bar{\mathrm{n}}^0$, $\bar{\mathrm{p}}^-$, $\bar{\Lambda}^0$, .... They feel the strong, electromagnetic, weak and associated interactions. But unlike the leptons the hadrons constitute a large spectrum similar to the atomic, molecular or nuclear spectrum.

* Notes by R. T. Cahill based on lectures given by M. Gell-Mann at the Australian Institute of Physics NUPP Group Summer School held at Goolwa, S.A., 1975.

The hadrons are made of quarks ($J^{\pi} = \frac{1}{2}^{+}$) which come in various flavours; the $u$ ($Q = 2/3$) and $d$ ($Q = -1/3$) flavoured quarks form an isotopic doublet. The strange quark $s$ ($Q = -1/3$) is in some sense more massive. A new hypothetical flavour is charm $c$ ($Q = +2/3$) which is an isotopic singlet and even more massive. Quarks are also coloured; red, white and blue, with charge independent of colour. This gives us 12 quarks and 12 antiquarks.

The observed non-exotic baryon is principally a colour SU(3) singlet

$$q_{\mathrm{R}}\, q_{\mathrm{B}}\, q_{\mathrm{W}} + q_{\mathrm{B}}\, q_{\mathrm{W}}\, q_{\mathrm{R}} + q_{\mathrm{W}}\, q_{\mathrm{R}}\, q_{\mathrm{B}}$$
$$- q_{\mathrm{B}}\, q_{\mathrm{R}}\, q_{\mathrm{W}} - q_{\mathrm{W}}\, q_{\mathrm{B}}\, q_{\mathrm{R}} - q_{\mathrm{R}}\, q_{\mathrm{W}}\, q_{\mathrm{B}},$$

so that colour is averaged out in this totally antisymmetric pattern. Here flavour $(u, d, s)$ is indicated by order. Then in space and spin and flavour everything is symmetrical. There is also an admixture of $qqqq\bar{q}$ etc. and the glue which is introduced to hold the quarks together.

The known non-exotic mesons have as their basic configuration $q\bar{q}$ along with an admixture of $q\bar{q}q\bar{q}$ etc. Here we get a colour $\mathrm{SU}_3$ singlet through $\bar{q}_{\mathrm{R}}\, q_{\mathrm{R}} + \bar{q}_{\mathrm{W}}\, q_{\mathrm{W}} + \bar{q}_{\mathrm{B}}\, q_{\mathrm{B}}$.

In this standard quark picture there are only colourless states in nature. The coloured triplets, octets, etc. are assumed to be not physically separable. It does not mean that the theory is content-free, however, because all the observed particles have to act as if they were made up of quarks and glue in the manner described. The glue is completely inert with respect to electromagnetism and weak interactions. The confinement of colour nonsinglets is supposed to be associated with the existence of the gluons which are themselves colour octets. The gluons are confined also by the same mechanism. When you try to separate things you can bring them out only in batches which are colour singlets.

Now why do we think the quark picture is good? Well it has had spectacular success in certain domains of particle physics and no failures.

Further, why do we believe in the existence of the colour variable, especially since in the standard quark picture we think it is a true hidden variable? It was invented in order to make the ground state wavefunction of the baryons come out symmetrical in space, spin and flavour—so it had to be antisymmetric in colour. However, there is a confirmation in the decay $\pi^0 \to 2\gamma$ which can be calculated very accurately in the approximation $m_{\pi}^2$ small compared with other $m^2$ values. The lifetime then comes out in terms of the sum of the charges squared of the quarks times the isotopic spin $z$ component. The decay rate is wrong by a factor 9 if you don't put in colour.

Another nice thing about the quarks is that they are true Dirac particles. They couple to the photons and to the weak interaction just as the leptons do; they don't have anomalous magnetic moments. If the electron has coupling to the photon of the form (1) then the quarks have the analogous coupling

$$\sum_{\text{colour}} \sum_{\text{hand}} (\tfrac{2}{3}\mathrm{i}\,\bar{u}\,\gamma_{\alpha} u - \tfrac{1}{3}\mathrm{i}\,\bar{d}\,\gamma_{\alpha} d - \tfrac{1}{3}\mathrm{i}\,\bar{s}\,\gamma_{\alpha} s) A_{\alpha}. \tag{2}$$

The weak interaction can be represented as an interaction of pairs which differ in charge by one: the $\mathrm{e}^{\pm}$ and $\nu_{\mathrm{e}}$; $\mu^{\pm}$ and $\nu_{\mu}$; $q_u$ and something like $q_d$; $q_c$ and something like $q_s$. The carrier of the weak interaction is presumably a charged axial vector particle $X^{\pm}$, the intermediate boson. Unlike the photon, the weak interaction is coupled only to the left. It is thought that the weak interaction strength parameter should be roughly the same as for electromagnetism and so, to get the Fermi coupling constant correct, $X$ is required to have a mass in the range 50–100 GeV.

A satisfactory theory of electromagnetic and weak interactions can be made only if they are unified in a particular way, and that requires a neutral interaction carried by a neutral intermediate boson $Z^0$. Further intermediate bosons can also be incorporated—and these are called associated interactions. The associated interactions with boson masses 50–100 GeV would give forces that could be seen now in the same way as we see the usual weak interaction. One such example is the neutral current interaction which has recently been seen. The neutral current interaction must also be found in other phenomena, such as atomic physics where it will give a parity-violating force between the electron and the proton. In nuclear physics it will give two neutrino decays. The $\gamma$, $X^{\pm}$ and $Z^0$ then form a unified gauge theory, the broken Yang–Mills theory, of weak, electromagnetic and associated interactions. However, one needs charm to accomplish the unification. In the weak interaction $X_\alpha$ is coupled to $e'$ (which is similar to $e$) then $ie'X_\alpha$ is coupled to

$$\bar{\nu}_{eL}\gamma_\alpha e_L + \bar{\nu}_{\mu L}\gamma_\alpha \mu_L + \sum_{\text{colour}} (\bar{u}_L \gamma_\alpha d'_L + \bar{c}_L \gamma_\alpha s'_L), \tag{3}$$

where

$$d' \equiv d\cos\theta + s\sin\theta, \qquad s' \equiv s\cos\theta - d\sin\theta. \tag{4}$$

Nature seems to have chosen linear combinations of the $s$ and $d$ quarks, with $\theta \approx 15°$. If the charm quark $c$ is not used then it is not known how to explain the smallness of the $K_0^1 - K_0^2$ mass difference or the virtual absence of any neutral current with strangeness change among the hadrons. From the mass difference you conclude that the effective mass of the charmed quark is approximately 2 GeV. The absence of the $\Delta S = 1$ neutral current does not depend on the masses.

Another example of how the quark picture is succeeding is in the deep inelastic experiments with electrons and neutrinos, for example

$$e^- + p \to e^- + \text{ anything}.$$

Then in the Bjorken limit, where lepton energy and momentum transfer are large but with constant ratio, one finds Bjorken scaling. That is, the inclusive cross sections can be described in terms of one or two Bjorken functions $F(\xi)$ that are nearly independent of energy. Here $\xi = -q^2/2p.q$, where $p$ is the four momentum of the struck nucleon and $q$ is the lepton momentum transfer, and we always have $0 < \xi < 1$. It is found that the results agree well with the quark picture and also that the interpretation is simple for $\xi \gtrsim \frac{1}{4}$. In that range the nucleon appears to be made essentially of three quarks (plus glue, which is inert in these interactions). For $0 < \xi < \frac{1}{4}$, $q\bar{q}$ pairs show. Hence hadrons just look like three quarks imprisoned inside and almost free except in the far infrared, which is that region of large distances in which confinement takes place and interactions apparently become strong.

In the unified gauge theory of the weak, electromagnetic and associated interactions the gauge symmetry is badly broken. This breaking gives rise to the photon mass remaining zero while the mass of the intermediate boson becomes 50–100 GeV. This theory is the only theory of the weak interactions that seems to hold any promise.

It needs the charmed quark, and also the symmetry breaking has to be done by introducing spin-zero bosons. It is not known if these auxiliary bosons have to be introduced as fundamental objects or whether they might be produced dynamically.

A similar theory to the above can now be given of the strong interactions: namely an unbroken Yang–Mills gauge theory built on $SU_3$ colour, in which the gauge bosons are the gluons. Because the symmetry is unbroken there is no need for auxiliary spin-zero bosons. The gluons are confined so that it is only some kind of effective mechanical mass that is zero. If this theory works, one would have a strong interaction theory built on almost the same principles as the theory of weak, electromagnetic and associated interactions. The three last interactions operate on left and right and flavour and the strong interactions operate on colour, and these are orthogonal variables.

This brief description gives an outline of a possible theory which is finite and so far in agreement with observation and is a relatively simple generalization of the QED of 45 years ago. Yang–Mills gauge theory constitutes a nontrivial generalization of QED in which the intermediate bosons ($\gamma$, $X^{\pm}$, $Z^0$) couple directly with one another.

Here is a list of some basic unanswered questions.

1. Does the unmodified colour-octet Yang–Mills gluon theory give confinement?
2. Is this picture correct of confined fractionally charged quarks and confined gluons or must we go over to a variant picture with real, probably integrally charged quarks with real colour excitation and real gluons?
3. Does charm work? Do we have to look for some new explanation of the absence of $\Delta S = 1$ neutral currents and the smallness of the $K_0^1 - K_0^2$ mass difference? Is the number of flavours restricted to four? Charm may already have been found in the new narrow resonances in $e^+ e^-$ annihilation, which may be vector mesons made of the form $\bar{c}c$.
4. What is the final form of the weak, electromagnetic and associated interaction system? Do we really need the spin-zero $\phi$ bosons to break the symmetry?
5. Is there actually an overall master theory that unites the weak, electromagnetic and associated interactions with the strong interaction? If so why does it split up in this way? How does the coupling constant 1/137 emerge for the flavour part, whereas some kind of variable coupling constant occurs in the colour part?
6. How can we explain the $CP$ or time reversal violation? We could explain it if the $\phi$s that break the Yang–Mills theory were pseudo scalar.
7. How are we going to deal with quantum gravity?

### Yang–Mills Theory and Colour Confinement

Yang–Mills theory is a generalization of QED in which the charges do not have to commute. It was done by Yang and Mills (1954) for three charges commuting like the generators of $SU_2$, but it can be generalized. The canonical form of the commutation relations for hermitian charges is

$$[Q_i, Q_j] = \mathrm{i}\,\varepsilon_{ijk}\,Q_k. \tag{5}$$

Consider the trivial group $U_1$. There is one charge and we get QED. Then for a spinor field $\psi(x)$

$$[Q, \psi] = -Q_\psi \psi, \tag{6}$$

where $eQ_\psi$ is the charge of the particle—electron, quark etc. The electromagnetic field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and the gauge invariant Lagrangian density is

$$\mathscr{L} = -\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi}\gamma_\alpha(\partial_\alpha - ieQ_\psi A_\alpha)\psi - [(\partial_\alpha - ieQ_\phi A_\alpha)\phi]^\dagger [(\partial_\alpha - ieQ_\phi A_\alpha)\phi]$$

if we add a spinless field $\phi$. Note the gauge derivative $(\partial_\alpha - ieQ_\phi A_\alpha)$. Now this Lagrangian not only conserves electric charge (it is gauge invariant in the first sense) but it is also gauge invariant under space–time dependent local gauge transformations; this was known by Maxwell:

$$A_\alpha(x) \to A_\alpha(x) + \partial_\alpha \Lambda(x), \qquad \psi(x) \to \exp\big(ieQ_\psi \Lambda(x)\big)\, \psi(x),$$

or infinitesimally

$$\psi(x) \to \big(1 + ieQ_\psi \Lambda(x)\big)\, \psi(x), \qquad \phi(x) \to \big(1 + ieQ_\phi \Lambda(x)\big)\, \phi(x).$$

Now consider the Yang–Mills generalization by considering a simple compact Lie group such as $SU_2$ or $SU_3$ with equation (5). Then equation (6) generalizes to

$$[Q_i, \psi_a] = -T^i_{ab}\psi_b,$$

and for consistency we have

$$[T^i, T^j]_{ab} = i\, C_{ijk}\, T^k_{ab}.$$

The matrices $T^i$ thus form a representation of the algebra of the charge operators $Q_i$, and so the field now has many components (e.g. the quark colour triplet). The same is true for the $\phi$s and we use the indices $A, B$ and the matrices $R^i_{AB}$ instead of $T^i_{ab}$,

$$[Q_i, \phi_A] = -R^i_{AB}\phi_B.$$

The gauge transformation is now ($\Lambda(x)$ infinitesimal)

$$\psi_a(x) \to \big(\delta_{ab} + igT^i_{ab}\Lambda_i(x)\big)\psi_b(x), \qquad \phi_A(x) \to \big(\delta_{AB} + igR^i_{AB}\Lambda_i(x)\big)\phi_B(x).$$

For the whole system to be gauge invariant then you have to introduce a massless intermediate boson field $B_{i\alpha}(x)$ for each $Q_i$. In this case

$$\begin{aligned} \mathscr{L} = &-\tfrac{1}{4} G_{i\mu\nu} G_{i\mu\nu} - \bar{\psi}_a \gamma_\alpha(\partial_\alpha \delta_{ab} - igT^i_{ab} B_{i\alpha})\psi_b \\ &- [(\partial_\alpha \delta_{AB} - igR^i_{AB} B_{i\alpha})\phi_B]^\dagger [(\partial_\alpha \delta_{AB} - igR^i_{AB} B_{i\alpha})\phi_B] \end{aligned}$$

is completely gauge invariant if

$$B_{i\alpha} \to B_{i\alpha} + \partial_\alpha \Lambda_i + gC_{irs} B_{r\alpha} \Lambda_s$$

and

$$G_{i\mu\nu} = \partial_\mu B_{i\nu} - \partial_\nu B_{i\mu} + gC_{ijk} B_{j\mu} B_{k\nu}.$$

We see that the gauge invariant field strength is nonlinear. Then the intermediate bosons have triple and quadruple vertices. That is, in this theory the intermediate bosons carry charges for one another, whereas in QED the photon is neutral and nothing nonlinear happens except through the coupling with $\psi$s and $\phi$s.

To make a theory of the strong interaction we take colour SU(3) which has eight charges and therefore eight gluon fields $B_{i\alpha}$. We do not need any $\phi$s because we are not going to break the symmetry. The $\psi$s are the quark fields with three colours. There are as many such triplets as flavours; there are probably four flavours, but we do not have to worry about the flavour much any more. With 4 flavours there are 12 quark fields, 1 universal constant $g$ for the quark–gluon coupling and 4 masses $m_u$, $m_d$, $m_s$ and $m_c$ with $m_c > m_s > m_u \approx m_d$. Now, what these are is a little peculiar:

In QED the corresponding charge $e$ and mass $m_e$ are well defined. The renormalized mass $m_e$ is the mass you measure when you create an electron and measure its inertia. However, confinements stop this for quarks. If you started out with no mass for the quarks then you might have difficulty in getting any because of chiral invariance.

In QED you renormalize the charge at zero frequency, and that is the charge in Coulomb's law $e^2/r$. Likewise you renormalize the electron mass $m_e$ to the real electron mass. Neither process makes sense for the quark case. Here one renormalizes at an arbitrary and variable mass point $M$. Then you have $g_M$ and $m_u(M)$, $m_d(M)$, $m_s(M)$ and $m_c(M)$.

Occasionally we will look at the case where we forget the quark masses and we have a formally scale-free theory with the arbitrary mass scale $M$ the only mass left.

Consider charge renormalization in QED. Here the vacuum polarization corrects the photon propagator and this modifies Coulomb's law. Then the effective potential between two test charges is

$$e_1^2\left(\frac{1}{(2\pi)^3}\int \mathrm{d}^3k\, \mathrm{e}^{ikx}\left[\frac{1}{k^2} + \int \frac{f(\mu^2)\,\mathrm{d}\mu^2}{(k^2+\mu^2)\mu^2}\right]\right),$$

with $f(\mu^2) > 0$, which is

$$e_1^2\left(r^{-1} + \int f(\lambda)\, r^{-1}\mathrm{e}^{-\lambda r}\,\mathrm{d}\lambda\right).$$

So the effective coupling constant $e^2(k^2)$, given by

$$e^2(k^2) = e_1^2\left(1 + k^2\int \frac{f(\mu^2)\,\mathrm{d}\mu^2}{(k^2+\mu^2)\mu^2}\right),$$

goes from $e_1^2$ at $k = 0$ and increases monotonically with $k^2$. The bare charge is $e(\infty)$ and may be infinite. You arrive at the bare charge when you get to zero distance or infinite frequency. This behaviour in QED is perfectly understood for $k^2 \approx 0$ where corrections are small because $1/137$ is very small. But in the far ultraviolet the radiative corrections are very large. Now Yang–Mills theory works in exactly the opposite way.

In Yang–Mills theory the behaviour is understood for $M^2 \to \infty$ and is unknown as $M^2 \to 0$. The coupling constant has the opposite behaviour from QED, that is, it goes to zero in the ultraviolet but its behaviour in the infrared is unknown, but appears to increase. In other words, from zero distance, if you look at distance in ordinary space, the coupling constant goes to zero at very small distances and as you go out in distance it increases. If the coupling constant increases faster than $r$, then multiplied by $r^{-1}$ it gives a potential that increases at large distances and you have quark confinement. The fact that it vanishes at small distances is called 'asymptotic freedom', and the conjecture that it increases faster than $r$ at large distances is called 'infrared slavery'. This is the potential between any two coloured objects and the conjectured behaviour gives confinement of all coloured objects. Then colour becomes a hidden variable, which was the idea of the standard quark picture.

Asymptotic freedom is the closest you can come to Bjorken scaling in any field theory with nonzero coupling. It looks as if there is no field theory in which you can get exact Bjorken scaling. However, if the coupling constant is small, as suspected, then you get very good approximate Bjorken scaling.

Now consider some aspects of the renormalization group. For the moment forget the quark masses, so that the only scale in the problem is the arbitrary scale $M$. The theory is now a one-parameter family of theories since there must be a connection between what happens when you vary $g$ and when you vary $M$. Hence there are curves in the two dimensions of $g$ and $M$ along which the theory is the same. The transformations from one point to another on that curve form the renormalization group.

Consider the effective potential between two coloured objects, which involves just the gluon propagator $f(p/M, g_M)$. Then we have

$$\left(M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+2\gamma(g)\right)f = 0,$$

which expresses the fact that $f$ remains unchanged if we make compensating changes in $M$ and $g$. The function $\beta(g)$ is the key to the whole problem as $\gamma(g)$ is relatively unimportant. $\beta(g)$ is evaluated by doing second- to lowest-order perturbation theory, i.e. first-order radiative corrections will tell you $\beta(g)$ for small $g$. One finds $\beta = -bg^3$ where $b = -(16\pi^2)^{-1}\times(11-\frac{2}{3}N_f)$, where $N_f$ is the number of flavours. The sign is crucial, and it makes the theory one in which you understand the ultraviolet while the infrared is mysterious. With more than 16 flavours the sign changes and we no longer get asymptotic freedom, but a case like QED.

Now consider what happens to the renormalization theory when you put in quark masses, which are renormalized at an arbitrary point. Their ratios are physical, however, because using PCAC the masses of the pseudoscalar mesons $\pi$, K and $\eta$ can be determined and we find

$$(m_u+m_d)/2m_s \approx \tfrac{1}{20}-\tfrac{1}{25} \approx m_\pi^2/2m_{K^2}.$$

We note that the strange quark is very much more massive than the up–down quark. For the charm quark a similar argument gives

$$m_s/m_c \approx m_K^2/m_{K_c}^2,$$

where $K_c$ is the charmed analogue of K. This ratio is suspected to be $\frac{1}{15}-\frac{1}{20}$ if $m_{K_c}$ is of order 1–2 GeV.

Is there a natural renormalization point? Assume that the quarks stop at the charmed ones. Then one could renormalize the mass of the charmed quark such that $m_c(M) = M$, with $M \approx 2$ GeV. This would then give a natural mass scale for the hadron problem. We also obtain then a fixed coupling constant, and indications are that this is $g^2/4\pi \approx \frac{1}{4}$.

There are two other mass scales in the theory besides $M$, namely $M\exp\{-(2bg^2)^{-1}\}$ and $M\exp\{+(2bg^2)^{-1}\}$, with $M \approx 2$ GeV and $g^2/4\pi \approx \frac{1}{4}$, which are 100 MeV and 40 GeV respectively. The small mass corresponds to the distance at which confinement ought to set in; the large mass is where the radiative corrections to Bjorken scaling are appreciable. In this theory Bjorken scaling works, apart from logarithmic corrections. It is the only type of theory in which you get it to work, but that is rather trivial because we have a small coupling constant. But this is a theory which also gives strong binding.

So the picture that emerges if confinement really comes out of the theory is one in which the strong interaction is strong only because of the confinement. Otherwise, the coupling between the quarks and the glue is quite small. Inside the confinement region the quarks are relativistic but practically free.

## Unified Weak, Electromagnetic and Associated Interactions

Now consider the broken Yang–Mills theory for which you can make a more convincing case for application to electromagnetic, weak and neutral current interactions. You can use this theory for three levels of unification:

The first level of unification is fairly well established and the general idea could hardly be wrong. This is the unification of electromagnetic, weak and associated interactions including only intermediate bosons in the range 50–100 GeV. You get then interactions of the strength of the Fermi constant, besides electromagnetism. The smallest such scheme is the Weinberg scheme with $\gamma$, $X^{\pm}$ and $Z^0$ for the neutral current. The corresponding group is $SU_2 \times U_1$. The scheme can be enlarged by adding more $Z^0$.

The second stage of unification involves superheavies ($\geqslant$ 500 GeV). At this stage we are still using only flavour and handedness. We are not using anything coupled to colour or anything that changes a quark into a lepton. Here we want to explain the Weinberg angle $\theta_W$ and the lepton mass ratio $m_e/m_\mu$. The group of this higher unification might be $SU_4 \times SU_4$: an $SU_4$ acting on flavour of right-handed particles and an $SU_4$ acting on corresponding left-handed particles.

The more speculative third stage of unification would bring in the strong interaction. Here one would have intermediate bosons that couple quarks to leptons and refer to colour. In the standard quark picture, colour is held in, so a lot of the bosons in stage three would be confined like quarks and gluons.

At every level of unification there is a gigantic violation of symmetry. Even at the first stage the photon has zero mass while the other three particles have mass around 50 GeV. Not only is local gauge invariance broken but global symmetry is also violated, because the photon is the isotopic partner of something of 50 GeV mass. However, their violation has to be introduced in a soft way, so that at very high frequencies the Yang–Mills character of the theory reasserts itself. This happens with the Higgs $\phi$-breaking mechanism.

The divergences in weak interaction theory are associated with non-conservation of weak charges. The only way to get rid of the divergences is to have a conserved weak charge at high frequencies. We achieve this by unifying with the electromagnetic interaction. Now the weak charges $W$ and $W^+$ and the electric charge $Q$ do not commute, so we need a theory at high frequencies of non-commuting simultaneously conserved charges—and that is the Yang–Mills theory.

The Yang–Mills Lagrangian is

$$\begin{aligned}\mathscr{L} = & -\bar{\psi}_a \gamma_\alpha(\partial_\alpha \delta_{ab} - igT^i_{ab} B_{i\alpha})\psi_b \\ & -[(\partial_\alpha \delta_{AB} - igR^i_{AB} B_{i\alpha})\phi_B]^\dagger [(\partial_\alpha \delta_{AB} - igR^j_{AB} B_{j\alpha})\phi_B] \\ & -\tfrac{1}{4} G_{i\mu\nu} G_{i\mu\nu} + \mu^2 \phi_B \phi_B \\ & + \text{group invariant couplings of } \bar{\psi}\psi\phi \text{ and } \phi^4 \text{ type.}\end{aligned}$$

Notice that the $\phi$s have wrong sign for their mass term. A group invariant $\psi$ mass could be put in, but at a stage-two unification that would be prohibited.

Call

$$V(\phi) = \lambda |\phi|^4 - \mu^2 |\phi|^2$$

which is a sort of potential. Suppose the minimum occurs for $\phi \neq 0$, and thus at the bottom of some multidimensional trough. Now if the vacuum expected value $\langle 0 | \phi_B | 0 \rangle$ is nonzero, corresponding to some point at the bottom of the trough, then the group symmetry will be violated. There are many equivalent equilibrium positions, but in choosing one arbitrarily the symmetry is broken. Hence define new fields

$$\phi'_B = \phi_B - \langle 0 | \phi_B | 0 \rangle \qquad \text{for which} \qquad \langle 0 | \phi'_B | 0 \rangle = 0 .$$

When this substitution is made in the Lagrangian we find that the $\psi$s acquire a mass term, which is group violating. The bosons also get a symmetry-violating effective mass. This is the Higgs soft mass-breaking mechanism. In introducing this, one of the $\phi$s disappears as an effective field, but in doing so the vector mesons, originally massless and with two states of polarization, acquire mass and the extra degree of freedom appropriate for massive vector particles.

Let us consider the minimal unification of $\gamma$, $X^\pm$ and $Z^0$ ($SU_2 \times U_1$). This requires both charm and a neutral current interaction. We have the weak charges $W$, $W^+$ and $W_3$, which commute like $J_+$, $J_-$ and $J_3$ of $SU_2$, and $S$ which commutes with all components of $W$. The weak charge acts only on left-handed states and carries $\nu_{eL} \to e_L$, $\nu_{\mu L} \to \mu_L$, $u_L \to d'_L$ and $c'_L \to s'_L$, where $d'$ and $s'$ are as defined in equation (4) above and

$$W_3 \sim (\tfrac{1}{2} \nu^\dagger_{eL} \nu_{eL} - e^\dagger_L e_L + \nu^\dagger_{\mu L} \nu_{\mu L} - \mu^\dagger_L \mu_L + u^\dagger_L u_L - d'^\dagger_L d'_L + c^\dagger_L c_L - s'^\dagger_L s'_L) .$$

Now if we did not have charm then $d'^\dagger_L d'_L$ would give $(d^\dagger s + s^\dagger d)\cos\theta \sin\theta$ which is strangeness changing. Experimentally there is no strangeness-changing neutral current. For example $K \to \pi + \nu$ is inhibited. When the charm term is added we also add in $-s'^\dagger_L s'_L$ and, since

$$d'^\dagger_L d'_L + s'^\dagger_L s'_L = d^\dagger_L d_L + s^\dagger_L s_L ,$$

there is no strangeness-changing term.

On the minimal unification scheme we have only one neutral intermediate boson and so only one extra charge $S$. We construct the electric charge out of $W_3$ and $S$, with $S = Q_3 - W_3$:

$$\begin{aligned}S = & -\tfrac{1}{2}(\nu^\dagger_{eL} \nu_{eL} + e^\dagger_L e_L + \nu^\dagger_{\mu L} \nu_{\mu L} + \mu^\dagger_L \mu_L) + \tfrac{1}{6}(u^\dagger_L u_L + d^\dagger_L d_L + c^\dagger_L c_L + s^\dagger_L s_L) \\ & - e^\dagger_R e_R - \mu^\dagger_R \mu_R + \tfrac{2}{3}(u^\dagger_R u_R + c^\dagger_R c_R) - \tfrac{1}{3}(d^\dagger_R d_R + s^\dagger_R s_R) ,\end{aligned}$$

with quarks summed over colour. The simplest representation is to put $\phi$ in as $j = 1/2$, and so there are only two components $\phi_-$ and $\phi_0$. It is a complex field so we also have $\bar{\phi}_0$ and $\phi_+$ giving a total of four $\phi$ fields.

We introduce coupling constants $g$ and $g'$ for $W$ and $S$, since each factor group has its own coupling constant. The two bosons, the one coupled to $S$ and the one coupled to $W_3$, must be orthogonal linear combinations of the two physical particles $\gamma$ and $Z^0$. The bosons coupled to $S$ and $W_3$ are respectively

$$A_\mu \cos\theta_W - Z_\mu \sin\theta_W \qquad \text{and} \qquad A_\mu \sin\theta_W + Z_\mu \cos\theta_W .$$

The coupling of the neutrino to the photon is

$$\tfrac{1}{2}g(A_\mu \sin\theta_W + Z_\mu \cos\theta_W) - \tfrac{1}{2}g'(A_\mu \cos\theta_W - Z_\mu \sin\theta_W)$$

and the charged part must be zero, so

$$\tan\theta_W = g'/g ,$$

and this is the effective definition of the Weinberg angle $\theta_W$. Then we find the electric charge $e = 2g\sin\theta_W$ and the Fermi constant $\sqrt{2}\,G = g^2/M_X^2$. Since $e$ and $G$ are known we conclude

$$M_X = \frac{37}{|\sin\theta_W|}\ \text{GeV} \qquad \text{and} \qquad M_{Z^0} = \frac{M_X}{|\cos\theta_W|}\ \text{GeV}.$$

Experiments have determined $\theta_W$ near 35°.

Detailed accounts of the theories outlined in this review are given by Yang and Mills (1954), Utiyama (1956), Gell-Mann and Glashow (1961), Gell-Mann (1964), Weinberg (1967), Salam (1968), Abers and Lee (1973) and Berstein (1974).

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