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2^- States of ^{8}Be

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Abstract

The cross sections for several reactions populating ^{8}Be in the region of the ^{7}Be+n threshold show rapid changes, which have been attributed to a 2^- threshold state. An analysis of these data using *R* matrix theory shows that there is considerable isospin mixing in the threshold state, implying another 2^- state nearby. A two-level *R* matrix approximation provides a good fit to all the relevant data, with parameters consistent with shell model calculations and with properties of ^{8}Li. Proper account must be taken of the neutron threshold in order to explain the isospin mixing. The threshold state is expected to show different widths in different reactions.

1. Introduction

Rapid changes in the ^{7}Li(p, n)^{7}Be and ^{7}Li(p, p′)^{7}Li* cross sections and in the ^{7}Li+p elastic scattering 2^- phase shift near the ^{7}Be+n threshold have been attributed to a 2^- level situated very close to the threshold at 18·9 MeV in ^{8}Be, with width 50 ± 20 keV and isospin zero (Arnold *et al.* 1974). This appears to be a particular case of the phenomenon of Wigner cusps, which are expected at an s wave neutron threshold when a level close to threshold has a large reduced width for the neutron channel. An earlier fit to some of the data had given a much larger width ($\Gamma_p > 0{\cdot}5$ MeV) and considerable isospin mixing (Newson *et al.* 1957). Some other data not fitted by Arnold *et al.* also suggest a larger width. From ^{7}Li(p, γ)^{8}Be$_{16\cdot6,16\cdot9}$, Sweeney and Marion (1969) obtained a width of 150 ± 50 keV. In ^{10}B(d, α)^{8}Be, Callender and Browne (1970) obtained a weak peak at 18·9 MeV, with a width that they apparently regarded as being consistent with the then accepted width of $\sim$500 keV and that seems to be at least 100 keV, although a precise value is made difficult by an appreciable background and the presence of a contaminant group.

In the next section, we discuss the multichannel scattering length approximation that Arnold *et al.* (1974) used for the 2^- partial wave *S* matrix. In contrast with the conclusion of Arnold *et al.* that the threshold state 'is not a resonance in the usual sense of the term', we see in Section 3 that the data fitted by Arnold *et al.* may also be fitted in terms of normal resonance theory, by using the one-level approximation of *R* matrix theory. In Section 4 we consider the isospin mixing of the threshold state and the implied presence of a neighbouring 2^- state, using additional information from ^{8}Li measurements and from shell model calculations. This leads to a fit to the data using the two-level approximation of *R* matrix theory, given in Section 5. From the resultant values of the level parameters, cross sections are calculated in Section 6 for other reactions that involve the 18·9 MeV level but that are not included in the

fit. Some comments on the level parameter values concerned with isospin mixing and Coulomb matrix elements, and with the hard sphere phase shift, are made in Section 7.

2. Comments on Use of Scattering Length Approximation

With the form of the S matrix subjected to the constraints of open channel unitarity and symmetry above and below the neutron threshold, Arnold *et al.* (1974) wrote the diagonal elements of the 2^- S matrix in the scattering length approximation in terms of seven real constant parameters, so that all the energy dependence came through the explicit appearance of the neutron wave number k. The moduli of the off-diagonal elements were then given uniquely. Now, in a normal scattering length approximation, neglected terms are of order k^2; for example, the phase shift δ in an s wave neutral channel satisfies

$$k\cot\delta = -a^{-1} + \tfrac{1}{2}r_e k^2 + \ldots, \tag{1}$$

and the scattering length approximation retains only the first term. Similarly in the present case, the parameters $a = a_r + ia_i$, $b = b_r + ib_i$, η, δ^t_{pp} and $\delta^t_{p'p'}$ of Arnold *et al.* are to be regarded as the leading terms of expansions in powers of k^2. If the off-diagonal elements S_{ij} of the S matrix are written as $(1-iak)^{-1}\eta_{ij}(k)\exp(i\delta_{ij})$ then the phases δ_{ij} can in general be expanded as power series in k:

$$\delta_{ij} = \delta^{(0)}_{ij} + \delta^{(1)}_{ij}k + \delta^{(2)}_{ij}k^2 + \ldots,$$

where $\delta^{(1)}_{pn} = \delta^{(1)}_{p'n} = 0$, but $\delta^{(1)}_{pp'} \neq 0$. When these restrictions are imposed together with unitarity, we find that the seven parameters of Arnold *et al.* are not independent, but have to satisfy the relation

$$\eta^2(a_r - b_r)^2 = (1-\eta^2)(a_i^2 - \eta^2 b_i^2) \tag{2}$$

(unless $a_i = b_i = 0$, which gives $|S_{pn}| = |S_{p'n}| = 0$). The relation (2) is not satisfied by the best-fit numerical values of Arnold *et al.*, but it can be satisfied within their estimated uncertainties on these quantities (for example, $b_r/a_r = 1{\cdot}00$, $\eta^2 = 0{\cdot}94$, $b_i/a_i = 1{\cdot}03$). Thus the requirement of satisfying equation (2) probably does not much affect the quality of fit obtained by Arnold *et al.*

An inconsistency in the approach of Arnold *et al.* (1974) is that, although the scattering length approximation involved neglecting terms of $O(k^2)$ relative to the leading terms, other terms of $O(k^2)$ were retained in their formulae,† e.g. they took

$$(\sigma_{pn}/k) = (\sigma_{pn}/k)_{k=0}\{(1+a_i k)^2 + (a_r k)^2\}^{-1}. \tag{3}$$

The only justification for this appears to be the fact that the data can be fitted, but it does suggest that the fit may not be unique. The significance of terms of $O(k^2)$ may be seen by comparing the analysis of Newson *et al.* (1957) with the scattering length approximation. The cross section used by Newson *et al.* is of the form (3) with $a_r = 0$, as stated by Arnold *et al.* If, however, one makes the approximation of

† The open-channel S matrix of Arnold *et al.* (1974) is unitary below threshold only to $O(k)$ or $O(\alpha)$ (unless $b_i = 0$); therefore their inclusion in δ^{2-} of terms of higher order in α is unjustified.

Newson *et al.* in the elements of the scattering matrix and equates terms of $O(k)$, one gets $a_r = -a_n$ (the neutron channel radius). Some comments are given in Section 4 (below) on the arguments of Arnold *et al.* that the 2^- level has isospin zero, since they are not specifically related to the scattering length approximation.

3. One-level *R* Matrix Fit to Data

In the *R* matrix theory of nuclear reactions (Lane and Thomas 1958), the elements of the scattering matrix are expressed in terms of the eigenenergies E_λ for each level λ, the reduced width amplitudes $\gamma_{\lambda c}$ for each channel c, the channel radii a_c, the boundary condition parameters B_c, and the Coulomb functions P_c, S_c and $-\phi_c$, which are the penetration factor, shift factor and hard sphere phase shift respectively. In the one-level approximation for the 2^- S matrix for open channels c and c', we have

$$S_{cc'} = \exp(-i(\phi_c+\phi_{c'}))\left\{\delta_{cc'}+2iP_c^{\frac{1}{2}}P_{c'}^{\frac{1}{2}}\gamma_{1c}\gamma_{1c'}\Big/\left(E_1-E-\sum_{c''}(S_{c''}-B_{c''}+iP_{c''})\gamma_{1c''}^2\right)\right\}, \tag{4}$$

where the sum over c'' goes over all channels (open and closed).

As did Arnold *et al.* (1974), we assume that the 2^- contributions to the ^{7}Be(n, p), ^{7}Li(p, n), ^{7}Li(p, p′) and ^{7}Li+p elastic scattering data can be described in terms of a three-channel S matrix, where the three channels are all s wave and are labelled c = p, p′ or n. We measure E in the c.m. system from the neutron threshold, and we assume that over the energy range of interest (-100 keV $\lesssim E \lesssim$ 100 keV) we can take P_c, S_c and ϕ_c for c = p or p′ as constants (except for $P_{p'}$ as mentioned below). In fitting the data, the B_c may be chosen arbitrarily, but for convenience and in order that the values of the level parameters may be compared directly with shell model calculations, we choose $B_p = S_p$, $B_{p'} = S_{p'}$ and $B_n = 0$ (Barker 1972). Dependence on the neutron wave number $k = (2\mu_n E/\hbar^2)^{\frac{1}{2}}$ (for $E > 0$) comes through $P_n = ka_n$ and $\phi_n = ka_n$ (with $S_n = 0$), and dependence on $\alpha = (-2\mu_n E/\hbar^2)^{\frac{1}{2}}$ (for $E < 0$) through $S_n = -\alpha a_n$ (with $P_n = 0$). The a_c and γ_{1c} occur only in the quantities $\Gamma_p = 2P_p\gamma_{1p}^2$, $\Gamma_{p'} = 2P_{p'}\gamma_{1p'}^2$, $C_n = a_n\gamma_{1n}^2$ and ϕ_p, so that these together with E_1 are the five adjustable parameters that we use to fit the same data as Arnold *et al.* fitted with six adjustable parameters. These data are the values of δ^{2^-}, σ_{pn}/k and $\sigma_{pp'}$, together with the ^{7}Be destruction cross section for thermal neutrons $\sigma_{np}+\sigma_{np'} = (50\pm8)\times10^3$ b and the ratio $\sigma_{np'}/\sigma_{np} = 0{\cdot}02\pm0{\cdot}01$. Arnold *et al.* gave only a qualitative fit to $\sigma_{pp'}$ because of the large background from other partial waves. We represent this background by a linear function of energy, introducing two more parameters. We also allow for the rapidly varying d wave penetration factor $P_{p'}$ for the inelastic protons. In fitting the data, we have used the assigned errors for $\sigma_{pp'}$ from Presser and Bass (1972) and have assumed errors of $\pm2°$ in δ^{2^-} and $\pm0{\cdot}02$ in the ratios $(\sigma_{pn}/k)/(\sigma_{pn}/k)_{k=0}$. We have fixed $\sigma_{np}+\sigma_{np'} = 50\times10^3$ b and $\sigma_{np'}/\sigma_{np} = 0{\cdot}02$, giving $\Gamma_{p'}/\Gamma_p = 0{\cdot}02$. The best fits are obtained by minimizing

$$X = N^{-1}\sum_{i=1}^{N}|\{V_{calc}(i)-V_{exp}(i)\}/\varepsilon(i)|^2, \tag{5}$$

where V_{calc}, V_{exp} and ε are the calculated and experimental values and the error of the quantity i respectively.

The minimum values of X obtained by varying all parameters except Γ_p are shown in Fig. 1*a*, plotted as a function of Γ_p. The corresponding values of the other parameters (E_1, C_n and ϕ_p) are shown in Figs 1*b*–1*d*. It is seen that X_{min} stays essentially constant for $\Gamma_p \gtrsim 0{\cdot}4$ MeV. Also C_n/Γ_p stays approximately constant at about 12 fm. If we assume the conventional value of the channel radius $a_p = a_n = 1{\cdot}45 \times (7^{1/3}+1^{1/3}) = 4{\cdot}22$ fm then we find $\gamma_{1n}^2/\gamma_{1p}^2 \approx 4{\cdot}6$. We also have $E_1/\Gamma_p \approx 0{\cdot}25$.

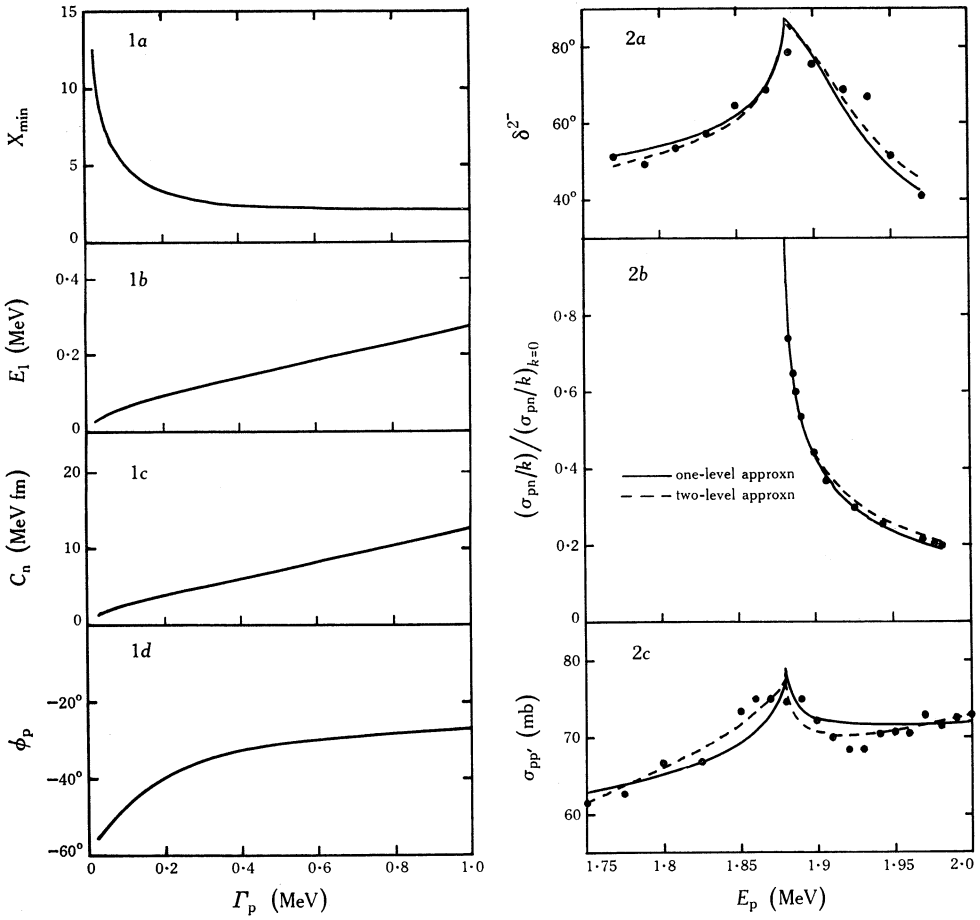

Fig. 1. Minimum values of X (equation 5) and corresponding values of the other parameters E_1, C_n and ϕ_p as functions of Γ_p, for the one level approximation.

Fig. 2. Best fits to the data as a function of the proton laboratory energy E_p (for both the one- and two-level approximations) of:

(*a*) real part of ^{7}Li(p, p)^{7}Li, 2^- phase shift δ^{2-};

(*b*) ^{7}Li(p, n)^{7}Be reduced cross section normalized to unity at the neutron threshold ($E_p = 1{\cdot}881$ MeV);

(*c*) ^{7}Li(p, p′)^{7}Li* cross section.

These results are essentially the same as those found by Newson *et al.* (1957) from fitting σ_{pn}/k and $\sigma_{pp'}$ values only. Since E_1 is small we refer to the 2^- state as the threshold state, or the 18·9 MeV state of ^{8}Be. One might put an upper limit on the

value of Γ_p by restricting the neutron reduced width to at most the single particle value (Lane 1960), that is,

$$\gamma_{1n}^2 \lesssim 0{\cdot}6(\hbar^2/\mu_n a_n^2) \approx 1{\cdot}6 \quad \text{MeV},$$

giving

$$\Gamma_p = 1{\cdot}63\,\gamma_{1p}^2 \lesssim 0{\cdot}6 \quad \text{MeV}.$$

Thus as reasonable values we take the set corresponding to $\Gamma_p = 0{\cdot}4$ MeV:

$$E_1 = 0{\cdot}139 \quad \text{MeV}, \qquad \Gamma_p = 0{\cdot}4 \quad \text{MeV}, \qquad \Gamma_{p'} = 0{\cdot}008 \quad \text{MeV}, \quad (6a\text{–}6c)$$

$$C_n = 5{\cdot}79 \quad \text{MeV fm}, \qquad \phi_p = -33{\cdot}0°, \qquad X = 2{\cdot}41. \quad (6d\text{–}6f)$$

The corresponding fits to the data are shown by the solid curves in Fig. 2. The main difference from the fits of Arnold *et al.* (1974) lies in the behaviour of $\sigma_{pp'}$ just below the threshold. Some of the discrepancy seen in Fig. 2*c* in the threshold region of $\sigma_{pp'}$ may be attributable to the target thickness, which was 25 keV (Presser and Bass 1972). Measurements with a 5 keV thick target (Newson *et al.* 1957) suggested that $\sigma_{pp'}$ stays constant within about 1% for at least the first 10 keV below threshold. Arnold *et al.* obtained $\sigma_{pp'}^{2^-} = \text{const.}$ for $E < 0$, but only by taking $a = b$ and $\eta^2 \neq 1$, which is in conflict with the requirement (2). Our parameter values (6a–6f) give in the scattering length approximation the following values for the parameters used by Arnold *et al.* (in their units, for which 1 fm $\equiv 0{\cdot}00647$ keV$^{-\frac{1}{2}}$):

$$a_r = 0{\cdot}058 \quad \text{keV}^{-\frac{1}{2}}, \qquad b_r = 0{\cdot}063 \quad \text{keV}^{-\frac{1}{2}}, \qquad \eta^2 = 0{\cdot}947, \quad (7a\text{–}7c)$$

$$a_i = 0{\cdot}126 \quad \text{keV}^{-\frac{1}{2}}, \qquad b_i = 0{\cdot}127 \quad \text{keV}^{-\frac{1}{2}}, \qquad \delta_{pp}^t = 88{\cdot}3°. \quad (7d\text{–}7f)$$

These values are not very different from those of Arnold *et al.* We can get $a_r = b_r$, which is required in order that the slope of $\sigma_{pp'}^{2^-}$ just below the threshold should be finite, only by taking $E_1 = 0$. This gives $a_r < 0$, however, contrary to what Arnold *et al.* required, and also we do not get zero slope. We also note that the value of ϕ_p calculated for $a_p = 4{\cdot}22$ fm is $+37°$, so that there is considerable discrepancy with the value given in equation (6e).

4. Isospin Mixing in 18·9 MeV State of ^{8}Be

For a state that is a mixture of pure $T = 0$ and 1 states, $\Psi = \alpha_0 \Psi_0 + \alpha_1 \Psi_1$, we can write

$$\gamma_{1n}^2/\gamma_{1p}^2 = \{(\alpha_0 x_0 - \alpha_1 x_1)/(\alpha_0 x_0 + \alpha_1 x_1)\}^2,$$

where x_T is the reduced width amplitude of the state Ψ_T apart from the isospin Clebsch–Gordan coefficient. In the particular case that Ψ_0 and Ψ_1 have the same structure apart from their T values we have $x_0 = x_1$, and a value of $\gamma_{1n}^2/\gamma_{1p}^2$ implies a value of the $T = 1$ intensity admixture $\alpha_1^2/(\alpha_0^2 + \alpha_1^2)$ (Barker and Mann 1957). The values (6b–6d) give $\gamma_{1n}^2/\gamma_{1p}^2 = 5{\cdot}6$, which implies about 14% admixture in the 2^-, 18·9 MeV state. Arnold *et al.* (1974) have used the ratio of the measured cross sections for ^{7}Be(n, γ)^{8}Be and ^{7}Li(p, γ)^{8}Be going to the 16·6 and 16·9 MeV levels to argue that the 2^- state is fairly pure isospin zero. In extracting reduced widths, they assumed a proton penetration factor corresponding to zero channel

radius, obtaining $\gamma_{1n}^2/\gamma_{1p}^2 \approx 3{\cdot}5/1{\cdot}5 \approx 2{\cdot}3$. If the penetration factor for $a_p = 4{\cdot}22$ fm is used, however, we obtain $\gamma_{1n}^2/\gamma_{1p}^2 \approx 5{\cdot}7$, in good agreement with the value from the equations (6), based on quite independent data.

Arnold *et al.* (1974) also argued that, unless the 2^- state is isopsin zero, one would expect to see a resonance at the position of the 2^- state in the ^{7}Li(p, γ)^{8}Be cross section to the 2^+ first excited state of ^{8}Be. This assumes, however, that any $T = 1$ admixture to the 2^- state would have an appreciable E1 matrix element coupling it to the 2^+ state. In the shell model calculations of Aswad *et al.* (1973), a 2^-, $T = 0$ state is predicted at 18·36 MeV, which is associated with the observed state at 18·9 MeV. The nearest 2^-, $T = 1$ state is at 19·38 MeV, and this is the one most likely to be mixed into the $T = 0$ state (Aswad *et al.*). The calculated E1 partial radiative width of the 19·38 MeV level to the 2^+ first excited state is, however, only 0·057 eV (Aswad *et al.*), which is very small, implying that even a large admixture of this state is consistent with nonobservation of a resonance in the ^{7}Li(p, γ)^{8}Be cross section.

Thus the experimental data suggest considerable isospin mixing. The magnitude of the Coulomb matrix element coupling the pure $T = 0$ and 1 states is not expected to be more than about 0·5 MeV (Barker 1961), so that 14% admixture implies a separation of the pure states (and of the mixed states) of not more than 1·5 MeV. It has been assumed that the 18·9 MeV state is mainly $T = 0$, since a $T = 1$ state of ^{8}Be near 18·9 MeV would have an analogue in ^{8}Li near 2·1 MeV excitation, and no such state is seen (Ajzenberg–Selove and Lauritsen 1974). For the same reason, the 2^-, $T = 1$ state in ^{8}Be is expected to be above the 2^-, $T = 0$ state. Then the mixing suggests a 2^-, mainly $T = 1$ state of ^{8}Be below about 20·4 MeV, and therefore a 2^- level in ^{8}Li below about 3·6 MeV. No such level has been identified (Ajzenberg–Selove and Lauritsen). We now consider, however, some evidence for its existence, which comes from the measured values of the ^{7}Li+n scattering lengths, taken in conjunction with the shell model calculations of Aswad *et al.* (1973).

The values of the scattering lengths a_s, where the channel spin s has the values 1 and 2, may be obtained from the measured values of the ^{7}Li+n elastic scattering thermal cross section $\sigma = 1{\cdot}07 \pm 0{\cdot}04$ b and of the coherent scattering length (thermal, bound) $\bar{b} = -2{\cdot}1 \pm 0{\cdot}1$ fm (Ajzenberg–Selove and Lauritsen 1974). We have

$$\sigma = 4\pi(\tfrac{5}{8}a_2^2 + \tfrac{3}{8}a_1^2) \quad \text{and} \quad \bar{b} = \tfrac{8}{7}(\tfrac{5}{8}a_2 + \tfrac{3}{8}a_1), \tag{8}$$

giving

$$a_2 = -3{\cdot}59 \pm 0{\cdot}06 \text{ fm} \quad \text{and} \quad a_1 = 1{\cdot}09 \pm 0{\cdot}20 \text{ fm}. \tag{9}$$

Measured values for $a_2 - a_1$ of $-4{\cdot}5 \pm 0{\cdot}2$ fm and $-4{\cdot}7 \pm 0{\cdot}2$ fm (Roubeau *et al.* 1974) are in good agreement with the values (9), and exclude an alternative set of a_s values that also fit the σ and $\bar{b}$ values.

Now we use R matrix theory (Lane and Thomas 1958) to express the scattering lengths a_s in terms of the ^{8}Li level parameters E_λ^s and $\gamma_{\lambda c}^s$ (since $J = s$):

$$a_s = a_n\left(1 - \sum_{\lambda\mu} \gamma_{\lambda n}^s \gamma_{\mu n}^s A_{\lambda\mu}^s(0)\right), \tag{10}$$

where

$$[\{\mathbf{A}^s(E)\}^{-1}]_{\lambda\mu} = (E_\lambda^s - E)\delta_{\lambda\mu} - \sum_c (S_c - B_c + iP_c)\gamma_{\lambda c}^s \gamma_{\mu c}^s, \tag{11}$$

with energies measured from the ^{7}Li$_0$+n threshold (in this section only). In the

summation over c in equation (11), we include for $s = 2$ only the s wave ground state channel (since the excited state channel is d wave), but for $s = 1$ we include both the ground and excited state s wave channels. For all these neutron channels, we assume the same channel radius a_n and take $B_c = 0$. Aswad *et al.* (1973) predicted one low-lying 2^-, $T = 1$ level and two low-lying 1^-, $T = 1$ levels, each with an appreciable spectroscopic amplitude for the $^7\mathrm{Li}_0 + \mathrm{n}$ channel.

We assume that the predicted energy differences of the states and their spectroscopic amplitudes are more reliable than their absolute energies, and take from Aswad *et al.* (1973) the results

$$E_1^1 - E_1^2 = 0{\cdot}78 \quad \mathrm{MeV}, \qquad E_2^1 - E_1^2 = 2{\cdot}74 \quad \mathrm{MeV}, \tag{12a,b}$$

$$(\mathscr{S}_{1\mathrm{n}}^2)^{\frac{1}{2}} = 0{\cdot}8404, \qquad (\mathscr{S}_{1\mathrm{n}}^1)^{\frac{1}{2}} = -0{\cdot}6434, \qquad (\mathscr{S}_{2\mathrm{n}}^1)^{\frac{1}{2}} = -0{\cdot}6176, \tag{12c–12e}$$

$$(\mathscr{S}_{1\mathrm{n}'}^1)^{\frac{1}{2}} = -0{\cdot}5767, \qquad (\mathscr{S}_{2\mathrm{n}'}^1)^{\frac{1}{2}} = 0{\cdot}5617. \tag{12f,g}$$

The reduced width amplitudes are given by

$$\gamma_{\lambda\mathrm{n}}^s = (\mathscr{S}_{\lambda\mathrm{n}}^s)^{\frac{1}{2}}(0{\cdot}6\hbar^2/\mu_\mathrm{n} a_\mathrm{n}^2)^{\frac{1}{2}}. \tag{13}$$

Then the values (9) are fitted with

$$a_\mathrm{n} = 3{\cdot}86 \pm 0{\cdot}15 \quad \mathrm{fm}, \qquad E_1^2 = 0{\cdot}71 \pm 0{\cdot}05 \quad \mathrm{MeV}. \tag{14}$$

This value of a_n is not far from the conventional value of 4·22 fm. The value of E_1^2 implies a 2^- level of ^{8}Li at an excitation energy of 2·74 MeV, and 1^- levels at 3·52 and 5·48 MeV. The calculated widths of these levels are 1·8, 2·5 and 3·8 MeV respectively. These levels may be related to the 2^- level assumed by Lane *et al.* (1964) at 5·0 MeV to explain their ^{7}Li(n, n) angular distribution and polarization data, and to the 1^- level assumed by Freeman *et al.* (1955) at 3·2 MeV to fit their ^{7}Li(n, n′) data. The 2^- level in ^{8}Be corresponding to the values (14) would be at an excitation energy of 19·53 MeV, in good agreement with the value of 19·38 MeV predicted by Aswad *et al.*

Therefore considerable evidence exists for a 2^-, mainly $T = 1$ level of ^{8}Be not far above the threshold region of interest here, so that the use of a one-level approximation for the 2^- S matrix is suspect. In the next section we use a two-level approximation to fit the data.

5. Two-level *R* Matrix Fit to Data

The two-level approximation of R matrix theory for the 2^- S matrix is

$$S_{cc'} = \exp\bigl(-\mathrm{i}(\phi_c + \phi_{c'})\bigr)\Bigl(\delta_{cc'} + 2\mathrm{i}P_c^{\frac{1}{2}} P_{c'}^{\frac{1}{2}} \sum_{\lambda\mu} \gamma_{\lambda c}\gamma_{\mu c'} A_{\lambda\mu}\Bigr), \tag{15}$$

where

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\,\delta_{\lambda\mu} - \sum_{c''} (S_{c''} - B_{c''} + \mathrm{i}P_{c''})\,\gamma_{\lambda c''}\gamma_{\mu c''}. \tag{16}$$

Here λ and μ take on the values 1 and 2 for the two levels (we assume $E_1 < E_2$), while c takes on the values p, p′ and n. Inclusion of the second level introduces four new parameters E_2, $\gamma_{2\mathrm{p}}$, $\gamma_{2\mathrm{p}'}$ and $\gamma_{2\mathrm{n}}$. Not all of these would be well determined by fitting the present data. In view of the similarity of the spectroscopic amplitudes given by Aswad *et al.* (1973) for their states at 18·36 and 19·38 MeV, it seems

reasonable to assume that the two levels 1 and 2 are produced by the mixing of pure $T = 0$ and 1, 2^- states that have the same structure apart from their T values. This restricts the values of the reduced width amplitudes of the mixed levels, giving

$$\gamma_{2p} = \gamma_{1n}, \qquad \gamma_{2p'} = \gamma_{1p'}\gamma_{1n}/\gamma_{1p}, \qquad \gamma_{2n} = -\gamma_{1p}, \tag{17}$$

and the only adjustable new parameter is E_2, making the total number of parameters used in fitting the data the same as used by Arnold *et al.* (1974).

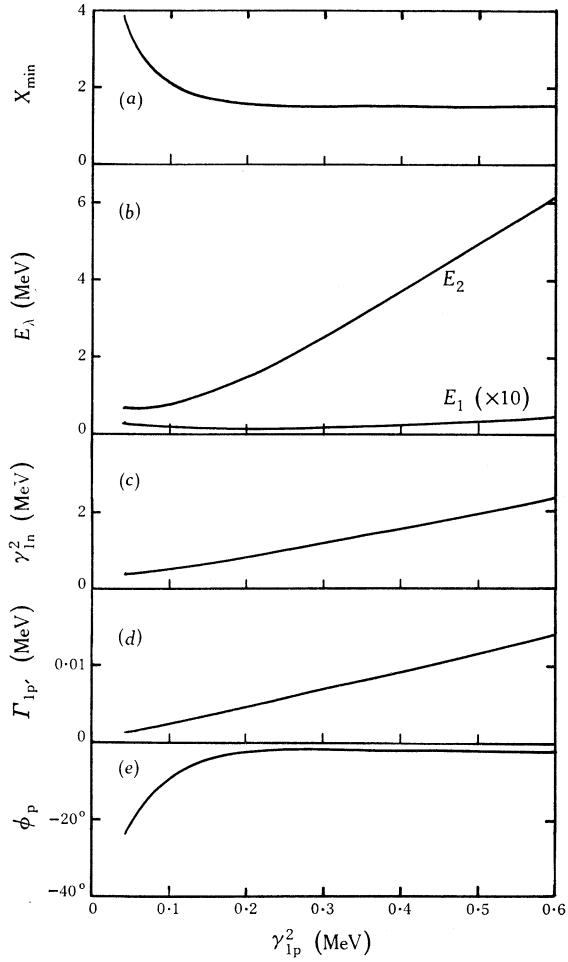

Fig. 3. Minimum values of X and corresponding values of the other parameters E_λ, γ^2_{1n}, $\Gamma_{1p'}$ and ϕ_p as functions of γ^2_{1p}, for the two-level approximation.

For comparison with the one-level fit of Section 3 above, the minimum values of X obtained by varying all parameters except γ^2_{1p} are shown in Fig. 3*a*, plotted as a function of γ^2_{1p} (for $a_p = a_n = 4{\cdot}22$ fm† and $P_p = 0{\cdot}814$). The corresponding values of the other parameters (E_λ, γ^2_{1n}, $\Gamma_{1p'}$ and ϕ_p) are also shown in Figs 3*b*–3*e*. Here the

† Other choices of channel radii give essentially the same fits and predictions, provided $a_p\gamma^2_{1p}$ and $a_n\gamma^2_{1n}$ are kept constant, due to the fact that P_p at the threshold is closely proportional to a_p.

values of $\sigma_{np}+\sigma_{np'}$ and of $\sigma_{np'}/\sigma_{np}$ are included in the fits, and the best fits are obtained with calculated values for them of about 48×10^3 b and 0·014 respectively. Again X_{min} stays essentially constant for $\gamma_{1p}^2 \gtrsim 0{\cdot}2$ MeV. We take as reasonable values the set for $\gamma_{1p}^2 = 0{\cdot}25$ MeV, which corresponds approximately to the set of values (6):

$$E_1 = 0{\cdot}019 \text{ MeV}, \quad E_2 = 1{\cdot}94 \text{ MeV}, \quad \gamma_{1p}^2 = 0{\cdot}25 \text{ MeV}, \quad \text{(18a–18c)}$$

$$\Gamma_{1p'} = 0{\cdot}006 \text{ MeV}, \quad \gamma_{1n}^2 = 1{\cdot}006 \text{ MeV}, \quad \phi_p = -1{\cdot}9°, \quad X = 1{\cdot}51. \quad \text{(18d–18g)}$$

The corresponding fits to the data are shown by the dashed curves in Fig. 2. Although the two-level fits may not look significantly better than the one-level fits, it should be remembered that the two-level fits involve only one additional adjustable parameter and that the inclusion of two levels is necessary for a consistent description of the ^{8}Be and ^{8}Li 2^- levels. The parameters (18) give in the scattering length approximation:

$$a_r = 0{\cdot}069 \text{ keV}^{-\frac{1}{2}}, \quad b_r = 0{\cdot}069 \text{ keV}^{-\frac{1}{2}}, \quad \eta^2 = 0{\cdot}945, \quad \text{(19a–19c)}$$

$$a_i = 0{\cdot}120 \text{ keV}^{-\frac{1}{2}}, \quad b_i = 0{\cdot}123 \text{ keV}^{-\frac{1}{2}}, \quad \delta_{pp}^t = 86{\cdot}7°. \quad \text{(19d–19f)}$$

In the two-level approximation, the requirement that the slope of $\sigma_{pp'}$ just below the threshold should be finite gives $E_1 E_2 = 0$, and thus we might look for other fits with $E_2 \approx 0$. This condition gives $a_r < 0$, however, and the best fits with $E_2 \approx 0$ are much poorer than those with $E_1 \approx 0$. Also $E_2 \approx 0$ would imply a 2^-, $T = 1$ level below the neutron threshold, contrary to the evidence from ^{8}Li.

We note that the value of ϕ_p in equation (18f) is closer than that in (6e) to the calculated value of 37°, but there is still considerable discrepancy. Also from equations (18c) and (18e), we have $\gamma_{1n}^2/\gamma_{1p}^2 \approx 4$, implying about 10% isospin mixing.

6. Cross Sections for Reactions not included in Fit

We now consider the contributions of the 2^- levels to the cross sections for other reactions in which the 18·9 MeV level has been identified, i.e. as a compound state in ^{7}Li(p, γ)^{8}Be$_{16\cdot6,16\cdot9}$ and as a product state in ^{10}B(d, α)^{8}Be, and also for the reaction ^{7}Li(^{3}He, d)^{8}Be, from which some evidence might be expected. The ^{7}Li(p, γ)^{8}Be cross section is of the form (Lane and Thomas 1958)

$$\sigma_{p\gamma} \propto \left| \sum_{\lambda\mu} \gamma_{\lambda p} \Gamma_{\mu\gamma}^{\frac{1}{2}} A_{\lambda\mu} \right|^2, \quad (20)$$

with $A_{\lambda\mu}$ given by equation (16). We assume that the 16·6 and 16·9 MeV 2^+ levels of ^{8}Be are formed by isospin mixing of pure $T = 0$ and 1 states (Barker 1966), and for simplicity we assume that the pure states differ only in their isospin and that there is an equal amount of each in the 16·6 and 16·9 MeV levels. Then for transitions to the 16·6 MeV level, we have $\Gamma_{1\gamma}^{\frac{1}{2}}/\Gamma_{2\gamma}^{\frac{1}{2}} = \gamma_{1p}/\gamma_{1n}$, while for transitions to the 16·9 MeV level, we have $\Gamma_{1\gamma}^{\frac{1}{2}}/\Gamma_{2\gamma}^{\frac{1}{2}} = -\gamma_{1n}/\gamma_{1p}$.

For reactions of the type ^{10}B(d, α)^{8}Be and ^{7}Li(^{3}He, d)^{8}Be, the cross section as a function of the ^{8}Be energy E may be taken to have a similar form (Barker 1967),

$$\sigma_f \propto \sum_{c^+} P_{c^+} \left| \sum_{\lambda\mu} G_{\lambda f}^{\frac{1}{2}} \gamma_{\mu c^+} A_{\lambda\mu} \right|^2, \quad (21)$$

where the initial channel partial width amplitude $(2P_p)^{\frac{1}{2}}\gamma_{\lambda p} \equiv \Gamma^{\frac{1}{2}}_{\lambda p}$ in the relation (20) is replaced by a feeding amplitude $G^{\frac{1}{2}}_{\lambda f}$, with f labelling the reaction, and $\Gamma^{\frac{1}{2}}_{\mu\gamma}$ for the final channel is replaced by $\Gamma^{\frac{1}{2}}_{\mu c+} = (2P_{c+})^{\frac{1}{2}}\gamma_{\mu c+}$, and the sum goes over all open decay channels c^+. With the simple model for the levels that we have chosen

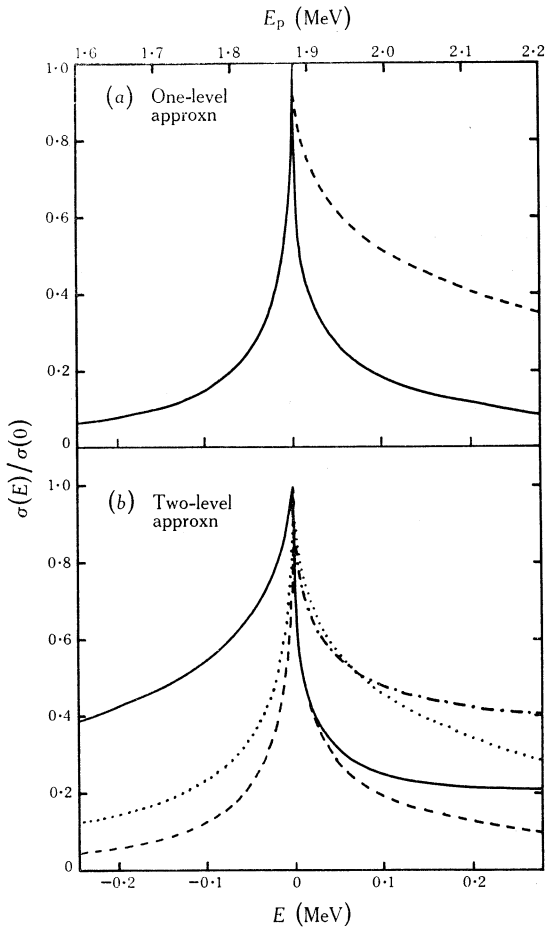

Fig. 4. Contributions of 2^- levels to the normalized cross sections $\sigma(E)/\sigma(0)$ for the following reactions, as functions of the neutron channel energy E (or equivalent proton laboratory energy E_p).

(*a*) One level approximation:
^{7}Li(p, γ)^{8}Be$_{16\cdot6}$ or ^{7}Li(p, γ)^{8}Be$_{16\cdot9}$ (solid curve);
^{10}B(d, α)^{8}Be or ^{7}Li(^{3}He, d)^{8}Be (solid curve for $E < 0$, dashed curve for $E > 0$).

(*b*) Two-level approximation:
^{7}Li(p, γ)^{8}Be$_{16\cdot6}$ (solid curve);
^{7}Li(p, γ)^{8}Be$_{16\cdot9}$ (dashed curve);
^{10}B(d, α)^{8}Be (dotted curve);
^{7}Li(^{3}He, d)^{8}Be (solid curve for $E < 0$, dot–dash curve for $E > 0$).

in the two-level approximation, the $G^{\frac{1}{2}}_{\lambda f}$ for the ^{10}B(d, α)^{8}Be reaction, assuming isospin conservation, are proportional to the $T = 0$ amplitudes in the wave functions, giving $G^{\frac{1}{2}}_{1f}/G^{\frac{1}{2}}_{2f} = (\gamma_{1p}-\gamma_{1n})/(\gamma_{1p}+\gamma_{1n})$. For the ^{7}Li(^{3}He, d)^{8}Be reaction, assumed to proceed by stripping, they are proportional to the $\gamma_{\lambda p}$, giving $G^{\frac{1}{2}}_{1f}/G^{\frac{1}{2}}_{2f} = \gamma_{1p}/\gamma_{1n}$.

The parameter values (18) do not give the relative sign of γ_{1p} and γ_{1n} but, because of the form of the relations (20) and (21), this is significant only for the ^{10}B(d, α)^{8}Be reaction. We assume that the 2^-, $T = 1$ state lies above the 2^-, $T = 0$ state, for the reasons given in Section 4. We then have $\gamma_{1p}/\gamma_{1n} < 0$, giving $|G^{\frac{1}{2}}_{1f}/G^{\frac{1}{2}}_{2f}| > 1$ as expected for ^{10}B(d, α)^{8}Be.

The calculated contributions $\sigma(E)$ of the 2^- levels to the cross sections for these reactions are shown in Fig. 4*b*, for the parameter values (18) and the normalization $\sigma(0) = 1$ (for comparison, Fig. 4*a* shows the corresponding contributions in the one-level approximation with the parameter values (6)). It is seen that different shapes, and in particular different values of the full width at half-maximum, are expected for different reactions. For ^{7}Li(p, γ)^{8}Be, the cross section to the 16·6 MeV level is expected to be wider than that to the 16·9 MeV level, and this agrees with the observation of Sweeney and Marion (1969). Comparison is difficult because the resonant contribution for the 16·6 MeV level is superimposed on an appreciable background, which is attributed to direct capture, and interference between these two contributions is possible. The correct relative normalization of the calculated 16·6 and 16·9 MeV contributions is obtained by multiplying the 16·9 MeV contribution by

$$\{\gamma_{1p}\gamma_{1n}(E_2-E_1)/(\gamma_{1p}^2 E_2+\gamma_{1n}^2 E_1)\}^2 \approx \gamma_{1n}^2/\gamma_{1p}^2 \approx 4,$$

and by the cube of the ratio of the γ-ray energies. With allowance for an experimental energy resolution of $\sim$20 keV, the expected ratio of the 16·9 and 16·6 MeV peak heights would be about 2. The observed ratio given by Sweeney and Marion is 0·6, but this neglects any interference between the resonant and nonresonant contributions. For ^{10}B(d, α)^{8}Be the calculated 2^- contribution smeared by an energy resolution of about 60 keV would have a FWHM of about 200 keV, which is probably consistent with the observed peak (Callender and Browne 1970).

7. Coulomb Matrix Elements and Hard Sphere Phase Shifts

Two comments may be made about the parameter values (18) that give the best fits. The first concerns the isospin mixing. The Coulomb matrix element that is required to produce the values (18) is

$$H_{01}^{\text{exp}} = \tfrac{1}{2}\{(\gamma_{1n}^2-\gamma_{1p}^2)/(\gamma_{1n}^2+\gamma_{1p}^2)\}(E_2-E_1) = 0{\cdot}58 \quad \text{MeV}, \tag{22}$$

with the convention that the pure $T = 0$ and 1 states have the same sign for the s wave ^{7}Li$_0$ + p spectroscopic amplitude. A conventional calculation of the Coulomb matrix element, using the shell model wave functions of Aswad *et al.* (1973) for the 18·36 and 19·38 MeV states, gives $H_{01}^{\text{calc}} \approx -0{\cdot}11$ MeV for harmonic oscillator single-particle wave functions, and $-0{\cdot}46$ MeV for wave functions in a finite depth potential (Barker 1961). The sign of H_{01}^{calc} is opposite to that of H_{01}^{exp}. In other apparently similar pairs of levels in ^{14}N and ^{16}O, the calculated Coulomb matrix elements are similar to those obtained here, but there is agreement with observation because it is the upper member of the pair that has the neutron reduced width larger than the proton reduced width (Barker 1961).

It seems that the discrepancy in the present case is due to neglect of the effect of the neutron threshold; the levels here are close to or above the neutron threshold

while the ^{14}N and ^{16}O levels are below the neutron thresholds. An alternative calculation of the Coulomb matrix element uses the potential model proposed by Dalton and Robson (1966), which allows for the effects of the nearby neutron and proton thresholds. The assumptions of this model appear to be well justified for the 2^- levels of ^{8}Be. We then have

$$H_{01}^{\text{calc}} = \tfrac{1}{2}\int_0^{a_n} u^2(r)\{V_c(r)-\Delta_c\}\,\mathrm{d}r + \tfrac{1}{2}(\hbar^2/2\mu_n a_n)u^2(a_n)\{S_n(E_1)-S_p(E_1)\}, \quad (23)$$

where $u(r)$ is a neutron s wave radial wave function, normalized by

$$\int_0^{a_n} u^2(r)\,\mathrm{d}r = 1,$$

$V_c(r)$ is the one-body Coulomb interaction and Δ_c is the Coulomb displacement energy (1·644 MeV). Qualitatively, the first term of equation (23) gives a small contribution and the second term is positive because $S_n(E_1)$ is zero for an s wave neutron channel with $E_1 \geqslant 0$. This differs from the situation in all other cases (s wave channel with $E_1 < 0$, or channels with $l \neq 0$), for which $S_n - S_p < 0$. For quantitative results we use $u(r)$ calculated for a Woods–Saxon potential with standard parameter values $r_0 = 1{\cdot}25$ fm and $a = 0{\cdot}65$ fm, and with depth chosen to produce a 2s state of zero binding energy, and we take $V_c(r)$ for a uniformly charged sphere of radius $1{\cdot}25A^{1/3}$ fm. Then equation (23) gives $H_{01}^{\text{calc}} = 0{\cdot}19$ MeV for the conventional value of the channel radius $a_n = 4{\cdot}22$ fm, while agreement with the value (22) is obtained for $a_n = 3{\cdot}4$ fm.

The other comment is that the fitted value of the phase ϕ_p is different from the calculated value by about 40°. This difference may not be surprising, since it has been shown (Cugnon 1975) that the use of the hard sphere phase shift in equation (15) is arbitrary and that it may be replaced, at least formally, by any other phase shift. The difference from the hard sphere value is interpreted, however, as being due to distant levels (Gupta 1976), and we would expect only a small effect since we have included explicitly one level that is outside the energy range being fitted, and we would not expect other 2^- levels with large s wave proton reduced widths except at very much higher energies (Aswad *et al.* 1973). An alternative explanation may be possible in terms of the experimental values of the ^{7}Li(p, p)^{7}Li, 2^- phase shift which were obtained (Brown *et al.* 1973) from analysis of scattering data for both polarized and unpolarized protons, and which determine the value of ϕ_p in equation (18f). Other sets of phase shifts fitting these data exist, and Brown *et al.* mention one such set, which was unfavoured because its 1^- phase shift varied less smoothly with energy in the region $E_p \approx 1$ MeV. Such behaviour might be expected from the shell model calculations of Aswad *et al.*, since they predict a 1^-, $T = 0$ level just 0·9 MeV below the lowest 2^-, $T = 0$ level. Thus the alternative sets of phase shifts should be investigated further.

8. Conclusions

We have shown that the two-level R matrix approximation is able to fit the experimental data from the ^{7}Be(n, p), ^{7}Li(p, n) and ^{7}Li(p, p′) reactions and ^{7}Li+p elastic scattering, and that it is consistent with observations of the 18·9 MeV level

from the ^{7}Li(p, γ) and ^{10}B(d, α) reactions. It is also consistent with the shell model calculations of Aswad *et al.* (1973) and with observed properties of ^{8}Li. There is appreciable (10%) isospin mixing in the 18·9 MeV 2^- level of ^{8}Be, and its FWHM as observed in different reactions is expected to vary from about 20 to 200 keV, for good energy resolution. It may be noted that such values of the FWHM are consistent with a partial proton width $\Gamma_p = 2P_p\gamma_{1p}^2$ of about 0·4 MeV, so that there is no inherent contradiction in the width values quoted by Newson *et al.* (1957) and by Arnold *et al.* (1974).

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References

Ajzenberg–Selove, F., and Lauritsen, T. (1974). *Nucl. Phys.* A **227**, 1.
Arnold, L. G., Seyler, R. G., Brown, L., Bonner, T. I., and Steiner, E. (1974). *Phys. Rev. Lett.* **32**, 895.
Aswad, A., Kissener, H. R., Jäger, H. U., and Eramzhian, R. A. (1973). *Nucl. Phys.* A **208**, 61.
Barker, F. C. (1961). *Phys. Rev.* **122**, 572.
Barker, F. C. (1966). *Nucl. Phys.* **83**, 418.
Barker, F. C. (1967). *Aust. J. Phys.* **20**, 341.
Barker, F. C. (1972). *Aust. J. Phys.* **25**, 341.
Barker, F. C., and Mann, A. K. (1957). *Phil. Mag.* **2**, 5.
Brown, L., Steiner, E., Arnold, L. G., and Seyler, R. G. (1973). *Nucl. Phys.* A **206**, 353.
Callender, W. D., and Browne, C. P. (1970). *Phys. Rev.* C **2**, 1.
Cugnon, J. (1975). *Phys. Rev.* C **11**, 291.
Dalton, B. J., and Robson, D. (1966). *Phys. Lett.* **20**, 405.
Freeman, J. M., Lane, A. M., and Rose, B. (1955). *Phil. Mag.* **46**, 17.
Gupta, S. K. (1976). *Phys. Rev.* C **13**, 1326.
Lane, A. M. (1960). *Rev. Mod. Phys.* **32**, 519.
Lane, R. O., Elwyn, A. J., and Langsdorf, A. (1964). *Phys. Rev.* B **136**, 1710.
Lane, A. M., and Thomas, R. G. (1958). *Rev. Mod. Phys.* **30**, 257.
Newson, H. W., Williamson, R. M., Jones, K. W., Gibbons, J. H., and Marshak, H. (1957). *Phys. Rev.* **108**, 1294.
Presser, G., and Bass, R. (1972). *Nucl. Phys.* A **182**, 321.
Roubeau, P., *et al.* (1974). *Phys. Rev. Lett.* **33**, 102.
Sweeney, W. E., and Marion, J. B. (1969). *Phys. Rev.* **182**, 1007.

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