

Vibrational Excitations in the Even-mass Cadmium Isotopes

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Abstract

The backward-angle scattering of ^4He ions from the even- A stable isotopes of cadmium has been investigated at bombarding energies above the Coulomb barrier. At 17.5 MeV bombarding energy the gross features of the spectra are strikingly regular from one isotope to another, and this may be interpreted in terms of vibrational excitations. Other evidence is presented to support the concept of a uniform vibrational character for all the even- A stable cadmium isotopes. The spectra show no significant evidence for previously unreported levels in the two-quadrupole-phonon excitation region. However, a new level is found at 2304 ± 4 keV in ^{106}Cd .

1. Introduction

The even-mass stable isotopes of cadmium have been widely regarded as good examples of vibrational nuclei (see e.g. Alder *et al.* 1956). Some of these isotopes have been examined more fully than others, but in every case considerable effort has been devoted to identifying those excited states which could be attributed to two-quadrupole-phonon vibrations (the 0^+ , 2^+ , 4^+ triplet) and the single-octupole-phonon 3^- state. These efforts have yielded candidates for the two-quadrupole-phonon triplet in each nucleus except in the case of ^{106}Cd where no candidate for the 0^+ member has yet been observed. Furthermore, in each of the nuclei a candidate for the 3^- state has been proposed, although in some cases the spin and parity assignment is tentative and the evidence for vibrational character has not been overwhelming.

One of the main problems with the vibrational model interpretation has been the observation of relatively large quadrupole moments Q_{2+} for the first excited 2^+ states. Attempts have been made to explain these large quadrupole moments within the general framework of the vibrational model by introducing anharmonicities arising, for example, from mixing of the one- and two-quadrupole-phonon states (Tamura and Udagawa 1966). These attempts have met with partial success. The problem has been complicated by confusion among the experimental results, which have been obtained by various techniques based upon the reorientation effect in Coulomb excitation. Recently Esat *et al.* (1976a, 1976b) attempted to clarify the experimental situation by measuring Q_{2+} for all the even-mass stable cadmium isotopes using a common technique; they found no significant variation of Q_{2+} with mass number.

Esat *et al.* (1976a, 1976b) used beams of ^4He and ^{16}O ions to bombard cadmium targets at energies below the Coulomb barrier. Scattered particles were detected with an annular counter at backward angles, and the ratio of inelastic to elastic peak

areas was used to extract Q_{2+} . In their analysis, they allowed for second-order interference terms arising from excitation of the 2^+ state through higher 2^+ states. In each isotope, the existence of a 2^+ state at approximately twice the excitation energy of the first 2^+ state was known (in the language of the vibrational model, the ‘two-phonon 2^+ state’ 2^{++}). For both ^{112}Cd and ^{114}Cd , an additional 2^+ state (2^{++}) was known to exist in the two-phonon region, and in both cases contributions from this state were included in the analysis. At that time, no 2^{++} states had been identified in the two-phonon region for the other isotopes, and Esat *et al.* therefore made no allowance for such a state in their analysis for these isotopes. Examination of the level schemes as then known showed that more levels had been reported in the two-phonon region for ^{112}Cd and ^{114}Cd than for the other isotopes. There was a particular paucity of levels in ^{106}Cd and ^{108}Cd .

The present work was undertaken, in the first instance, to search for previously unreported levels in the two-phonon region of the cadmium isotopes; in particular, for levels which were strongly populated in inelastic α particle scattering and might therefore contribute significantly to the interference terms involved in Coulomb excitation of the first-excited 2^+ state via higher states. In fact, the relevant matrix elements for the 2^{++} states in ^{112}Cd and ^{114}Cd are such that the effects of these states on the determination of Q_{2+} are almost negligible. Nevertheless, further investigation of the situation in the other isotopes was deemed desirable. In the course of the work, there emerged other results of interest for the vibrational model interpretation of the cadmium isotopes, and these also are reported here, and discussed in Section 4.

Table 1. Isotopic compositions of cadmium targets

Isotopes in target	Composition (atomic per cent) of target:					
	^{106}Cd	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd
^{106}Cd	82.33	0.57	<0.01	0.04	<0.02	<0.02
^{108}Cd	0.82	73.68	<0.01	0.03	<0.02	<0.02
^{110}Cd	2.93	5.63	97.2	0.20	0.10	0.18
^{111}Cd	2.64	4.10	1.04	0.59	0.12	0.20
^{112}Cd	3.68	6.58	0.9	96.97	0.32	0.46
^{113}Cd	1.72	2.67	0.27	1.26	0.47	0.56
^{114}Cd	3.85	5.33	0.49	0.80	98.80	4.30
^{116}Cd	2.01	1.44	0.09	0.11	0.18	94.30

2. Experimental Procedure

The equipment used was basically as described by Esat *et al.* (1976*b*). Beams of $^4\text{He}^{2+}$ ions from the ANU EN tandem accelerator were used to bombard targets of CdCl_2 evaporated onto thin self-supporting carbon foils. The partial thickness of cadmium was approximately $8\text{ }\mu\text{g cm}^{-2}$ for all isotopes except ^{114}Cd , for which it was approximately $25\text{ }\mu\text{g cm}^{-2}$. The isotopic compositions of the targets (taken from the assay provided by the supplier of the target material, Oak Ridge Separated Isotope Division) are listed in Table 1. Scattered particles were detected with a $200\text{ }\mu\text{m}$ thick annular surface barrier detector which subtended a solid angle of about 40 msr at a mean scattering angle of 171.6° (lab.). The overall energy resolution of the system was typically 30 keV (FWHM). Spectra were obtained at several bombarding energies above the effective Coulomb barrier, known to be at approximately 10.5 MeV (Esat *et al.* 1976*b*). Most data were taken at 17.0 and 17.5 MeV. These

energies were chosen to minimize pile-up from α particles scattered from ^{12}C , and they correspond to minima in the cross sections at backward angles for $^{12}\text{C}(\alpha, \alpha_0)^{12}\text{C}$ (Carter *et al.* 1964) and $^{12}\text{C}(\alpha, \alpha_1)^{12}\text{C}$ (Mitchell *et al.* 1964). Absolute differential cross sections were determined by comparing elastic yields with those from Rutherford scattering with the same target-detector arrangements at 10 MeV bombarding energy. These results for elastic scattering at 171.6° from the most abundant isotope in each target are:

$$(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\text{lab}} = \begin{array}{cccccc} & ^{106}\text{Cd} & ^{108}\text{Cd} & ^{110}\text{Cd} & ^{112}\text{Cd} & ^{114}\text{Cd} & ^{116}\text{Cd} \\ = & 3.30 & 3.02 & 2.87 & 2.66 & 2.38 & 2.22 \end{array} \text{ mb sr}^{-1}$$

In each case the error is about 5% due to possible variations in target thickness.

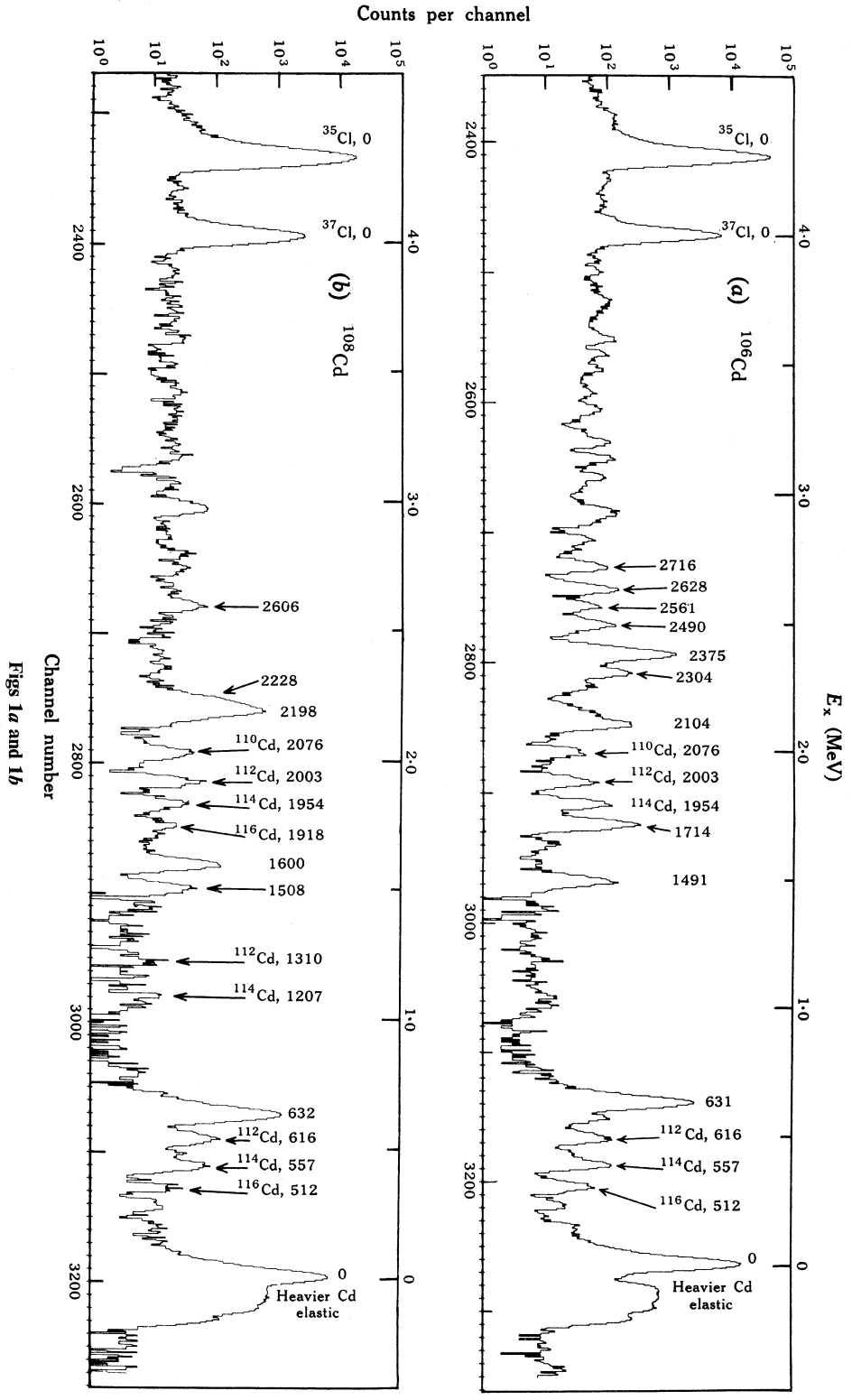
3. Results

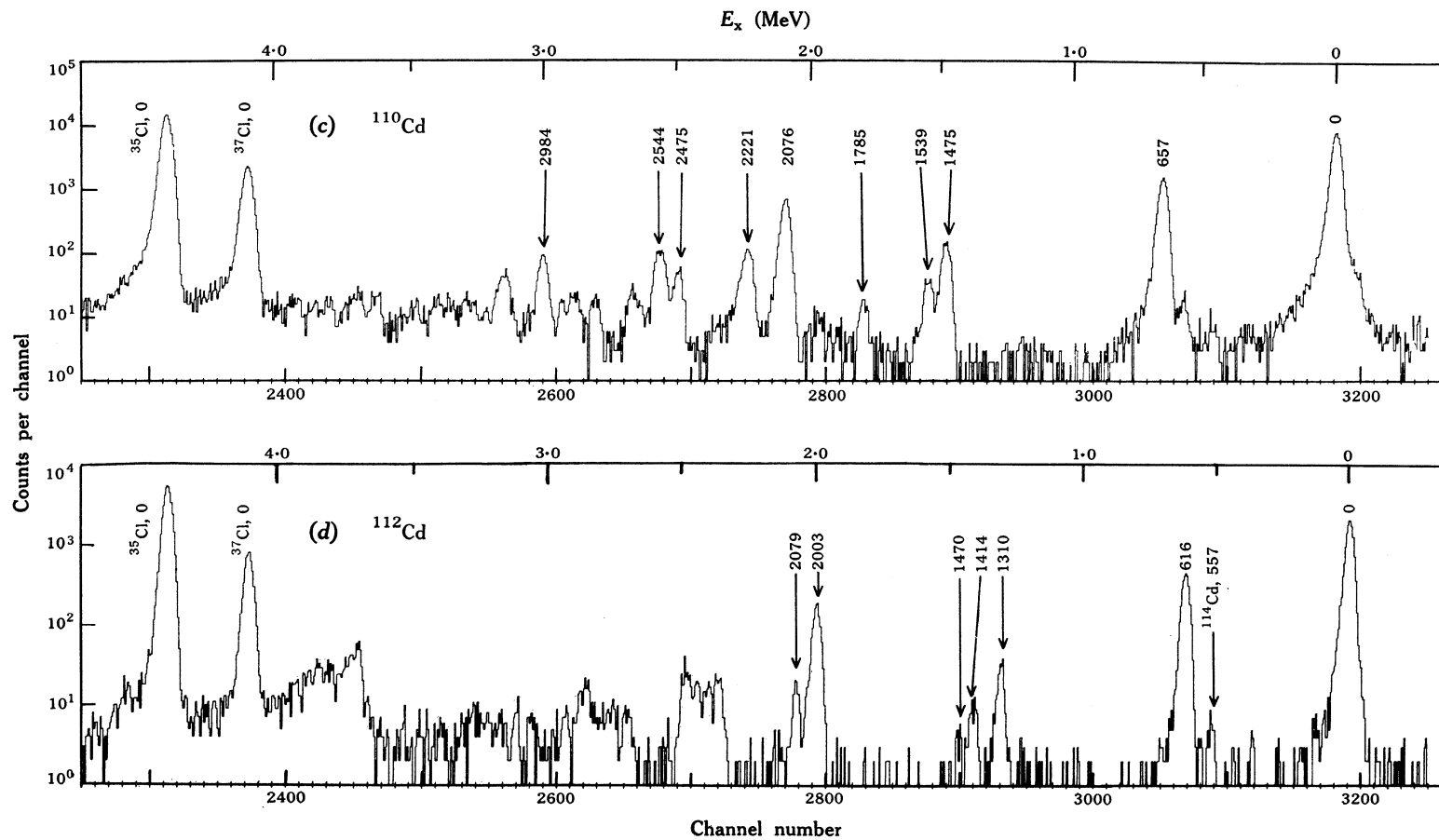
Logarithmic plots of spectra obtained at 17.5 MeV bombarding energy are shown in Fig. 1. Spectra were also obtained for ^{106}Cd , ^{112}Cd and ^{116}Cd at 17.0 MeV bombarding energy. These were not essentially different from those at 17.5 MeV and are not shown, although they were used in obtaining excitation energies. Energy calibration of the spectra was obtained by using as references the groups corresponding to elastic scattering from the most abundant cadmium isotope and from ^{35}Cl and ^{37}Cl in the target. From inspection of the plots in Fig. 1 it is immediately apparent in every case that, apart from elastic scattering, two peaks stand out prominently. One of these groups is clearly due to inelastic scattering to the first excited (2^+) state of the most abundant isotope. As is argued in the following section, the other prominent group is attributed to inelastic scattering to the putative 3^- single-octupole-phonon state at about 2 MeV excitation energy. For purposes of identification in subsequent discussion this group is referred to as the 3^- group.

All significant peaks between the main Cd elastic scattering group and the 3^- group have been identified and labelled in Fig. 1. The ^{106}Cd and ^{108}Cd targets were of relatively low isotopic purity (Table 1) and consequently their spectra show significant contributions from other isotopes. For each target many peaks are observed at excitation energies above that corresponding to the 3^- group. However, because of rapidly increasing level densities there is a considerable probability that peaks in this region may contain contributions from more than one level. Thus with the exception of several prominent peaks (see Fig. 1) no attempt has been made to deduce level energies in this region.

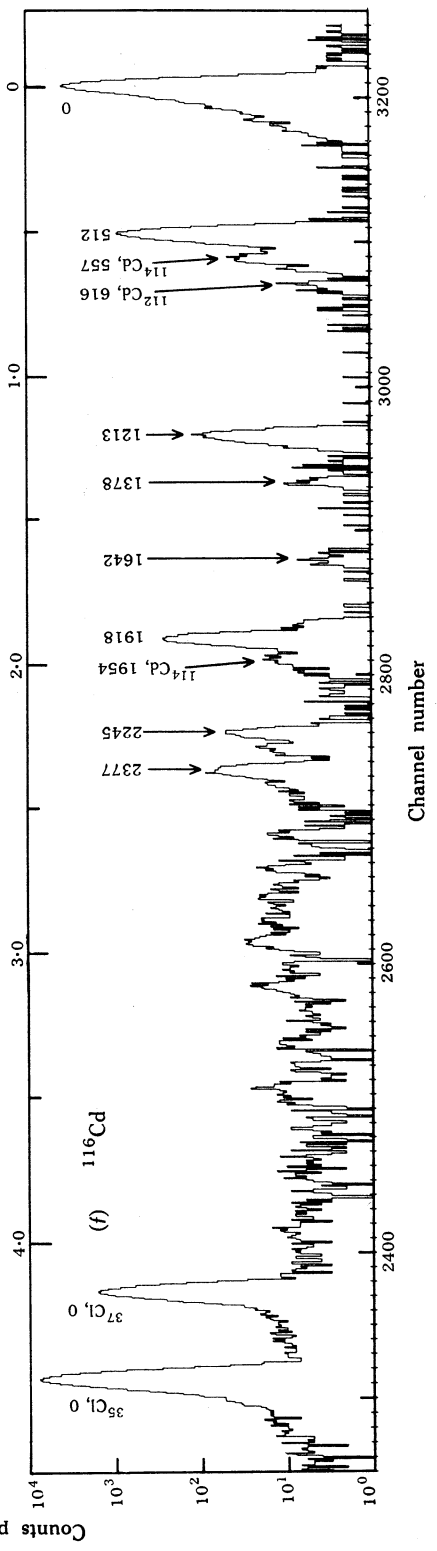
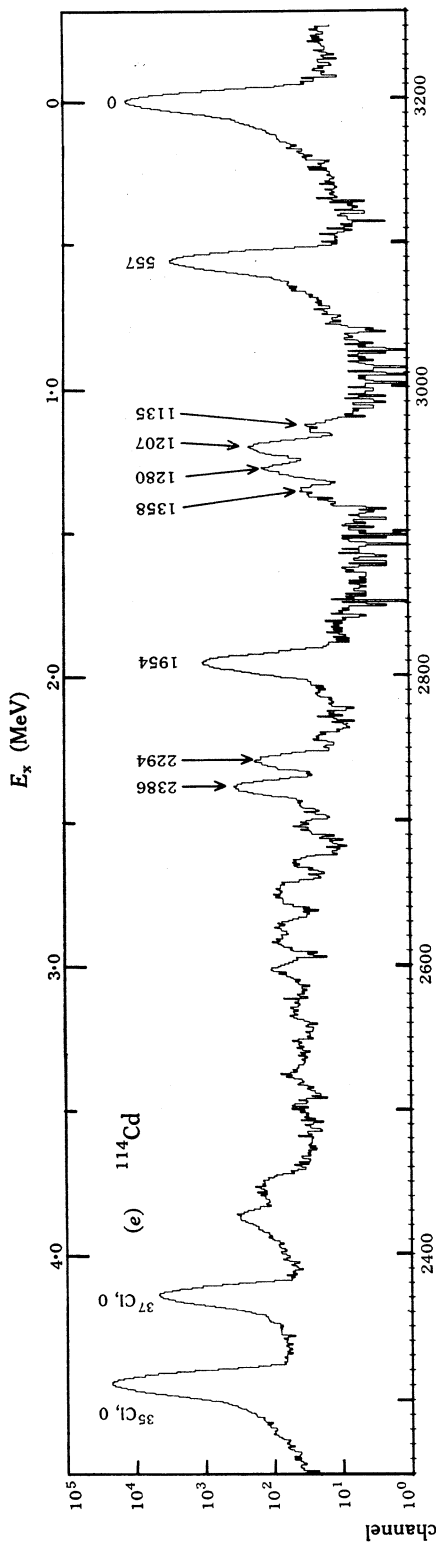
Excitation energies obtained in the present work for levels up to that corresponding to the 3^- group are given in Table 2, where they are compared with level information from all previously reported work other than that of Gill *et al.* (1974a, 1974b), which is discussed in the following section. With one exception the existence of all levels listed in Table 2 appears to have been established prior to the present work.

Figs 1a-1f (pp. 136-8). Spectra obtained at 17.5 MeV bombarding energy and a laboratory scattering angle of 171.6° for inelastic α particle scattering by the even- A stable cadmium isotopes: (a) ^{106}Cd , (b) ^{108}Cd , (c) ^{110}Cd , (d) ^{112}Cd , (e) ^{114}Cd , (f) ^{116}Cd . Peaks corresponding to scattering from the dominant isotope of the target are labelled with the excitation energy (keV) corresponding to that isotope, while this scale of excitation energies (MeV) is reproduced along the upper horizontal axis. Peaks from contaminants are identified.





Figs 1c and 1d



Figs 1e and 1f

Table 2. Low-lying levels of even-*A* stable cadmium isotopes

The spin-parity assignments J^π and excitation energies E_x from previous work have been taken from the following compilations: Bertrand and Raman (1971), Auble *et al.* (1972), Bertrand (1972), Raman and Kim (1972), Bertrand (1974), Carlson *et al.* (1975) and Kim (1975). The stated errors in excitation energies obtained from the present work are from uncertainties in peak centroids. Larger errors reflect weak or partially obscured peaks

Nucleus	J^π (previous work)	E_x (keV) (previous work)	E_x (keV) (present work)	Nucleus	J^π (previous work)	E_x (keV) (previous work)	E_x (keV) (present work)	Nucleus	J^π (previous work)	E_x (keV) (previous work)	E_x (keV) (present work)
^{106}Cd	0^+	0	0	^{110}Cd	0^+	0	0	^{114}Cd	0^+	0	0
	2^+	632.7 ± 0.3	631 ± 3		2^+	657.72 ± 0.02	657 ± 3		2^+	558.29 ± 0.03	557 ± 3
	(4^+)	1494.2 ± 0.7	1491 ± 3		(0^+)	1473.2 ± 0.3	1475 ± 3		0^+	1134.18 ± 0.03	1135 ± 5
	2^+	1716.2 ± 0.5	1714 ± 3 (1792 $\pm$ 5)		2^+	1475.71 ± 0.04			2^+	1209.28 ± 0.03	1207 ± 3
		2100	2104 ± 3		4^+	1542.38 ± 0.03	1539 ± 3		4^+	1283.30 ± 0.03	1280 ± 3
		(2305)	2304 ± 4		$0^+ - 2^+$	1732 ± 1			0^+	1305.18 ± 0.06	
	(3^-)	2366	2375 ± 3		$1, 2$	1783.3 ± 1.3	1785 ± 5		2^+	1363.94 ± 0.03	1358 ± 3
						1809.2 ± 0.8				1841.46 ± 0.06	
					(5^-)	2004.24 ± 0.03			0^+	1863.64 ± 0.06	
					3^-	2078.8 ± 0.4	2076 ± 3		3^-	1957 ± 5	1954 ± 3
^{108}Cd	0^+	0	0	^{112}Cd	0^+	0	0	^{116}Cd	0^+	0	0
	2^+	632.98 ± 0.05	632 ± 3		2^+	617.4 ± 0.3	616 ± 3		2^+	513.9 ± 0.1	512 ± 3
	(4^+)	1509.0 ± 0.5	1508 ± 4		0^+	1223.0 ± 0.7			2^+	1213.6 ± 0.2	1213 ± 3
	2^+	1607.0 ± 1.0	1600 ± 3		2^+	1311.6 ± 0.5	1310 ± 3		4^+	1220.2 ± 0.2	
	$0^+ - 2^+$	1721 ± 9			4^+	1414.2 ± 0.6	1414 ± 4		$(0, 2^+)$	1283.4 ± 0.3	
	$(0^+ - 2^+)$	1911 ± 10			(0^+)	1431.7 ± 0.6			0^+	1381.3 ± 0.3	1378 ± 5
	$(0^+ - 2^+)$	2157 ± 11			2^+	1468.2 ± 0.6	1470 ± 5			1641 ± 15	1642 ± 5
	(3^-)	2199 ± 11	2198 ± 3			(1812 $\pm$ 8)			(3^-)	1920 ± 15	1918 ± 3
					0^+	1869.7 ± 0.6					
					3^-	1971 ± 2					
						2003.8 ± 0.7	2003 ± 3				

The level at 2304 ± 4 keV in ^{106}Cd reported here confirms the tentative proposal of a level at 2305 keV in the work of Starke *et al.* (1969). Subtraction of appropriately normalized contributions to the spectrum from other even- A isotopes of cadmium showed that the corresponding group did not arise from these isotopes. Its prominence in the ^{106}Cd spectrum and absence from the ^{108}Cd spectrum showed that it could not be due to either of the odd- A cadmium isotopes in the target; both of these were more abundant in the ^{108}Cd target than in the ^{106}Cd target (see Table 1). Furthermore, comparison of the spectra obtained from the ^{106}Cd target at 17.0 and 17.5 MeV bombarding energy showed that the group was not due to elastic scattering from a target contaminant other than cadmium. It may be noted that since the mean detection angle was close to 180° , and the projectile and targets both have zero spin and even parity, all levels strongly populated in the present work have natural parity, that is, $\pi = (-)^J$ (Litherland 1961).

4. Discussion of Results

For each of the isotopes studied, no evidence is obtained for significant population of new levels in the two-phonon region. There is inconclusive evidence for excitation of a hitherto unreported level in ^{106}Cd at 1792 ± 5 keV. The corresponding group in the spectrum would coincide with a group from inelastic scattering to the 1954 keV level of ^{114}Cd , but the observed intensity cannot be fully accounted for by this. However, even if this level in ^{106}Cd exists, it is very weakly excited. Hence it may be said for each isotope studied that, if any 2^+ states exist in the two-phonon region in addition to those previously reported, they are either unresolved or very weakly populated. There is therefore no cause to modify the Q_{2+} values obtained by Esat *et al.* (1976b).

The most striking aspect of the present data is the remarkable consistency of the major features of the spectra for all isotopes. This is illustrated in Fig. 2 which shows a composite linear plot of all the spectra obtained at 17.5 MeV. The spectra have been normalized such that the elastic scattering peaks all have the same height. In each spectrum the outstanding groups are those corresponding to scattering to the ground state, the first 2^+ state and, with the exception of ^{112}Cd , to a state which has been definitely or tentatively assigned 3^- in previous work. The relative strengths of these three groups are remarkably constant from one isotope to another. It is well known that the inelastic scattering process preferentially populates collective states, with cross sections closely related to electromagnetic multipole transition probabilities (see e.g. Hodgson 1971; Morrison *et al.* 1975). Thus the most reasonable interpretation of the data shown in Fig. 2 is that in each case the major peaks are due to elastic scattering, the one-quadrupole-phonon transition to the first 2^+ state, and the one-octupole-phonon transition to the 3^- state. The transitions to the two-quadrupole-phonon states are substantially weaker, but again their strengths relative to elastic scattering vary little from one isotope to another.

The uniformity of the principal features of the spectra is more quantitatively illustrated in Fig. 3. In Fig. 3a, the ratio $\Sigma J^\pi / \Sigma 0^+$ (where ΣJ^π is the number of counts in the peak corresponding to a level of spin J and parity π , and $\Sigma 0^+$ is the number of counts in the elastic peak) is plotted against mass number A for the presumed vibrational states of the various isotopes. Also shown, in Fig. 3b, is the variation with A of the double ratio $(\Sigma J^\pi / \Sigma 0^+) / B(E2; 0^+ \rightarrow 2^+)$ for the first excited

states, the $B(E2)$ values being taken from Esat *et al.* (1976b). The absolute cross sections for elastic scattering at 17.5 MeV (quoted in Section 2) also show a smooth variation with mass number.

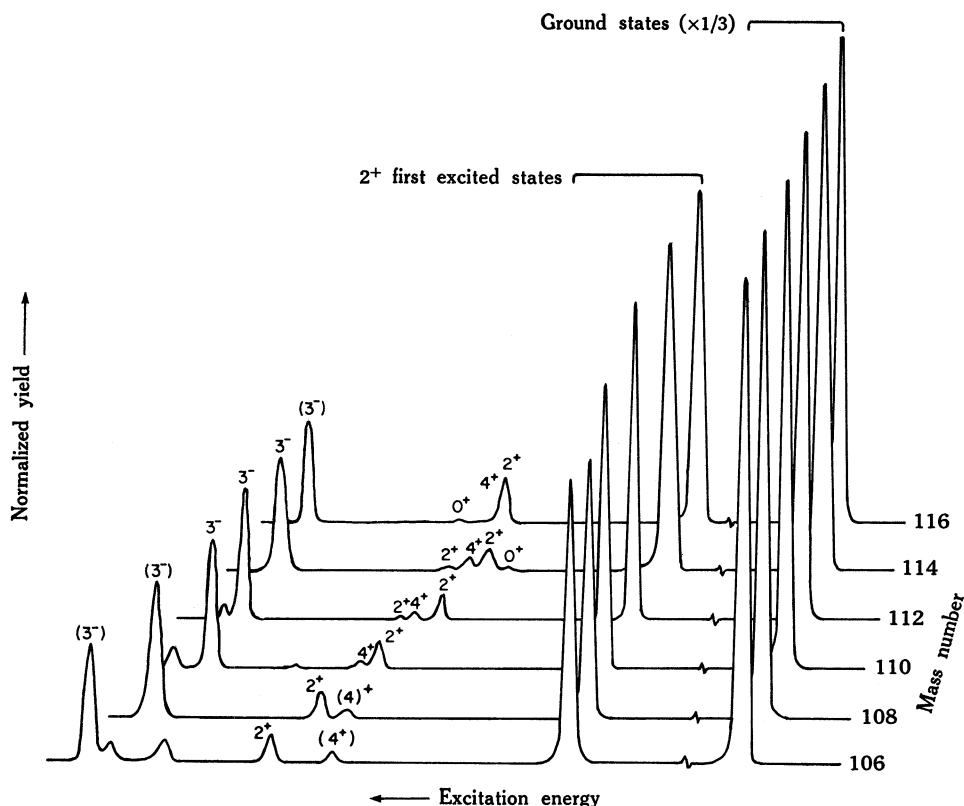


Fig. 2. Composite linear plot of spectra obtained at 17.5 MeV bombarding energy and a laboratory scattering angle of 171.6° for the inelastic scattering of α particles by the even- A stable isotopes of cadmium, here designated by their mass numbers. Contributions from even- A isotopes other than the dominant isotope of the target have been subtracted from the ^{106}Cd and ^{108}Cd data. The intensity scales of the spectra have been normalized so that all elastic peaks have the same height. The spin and parity assignments have been taken from the previous compilations listed in Table 2.

The 3^- assignments from previous work shown in Table 2, are based upon a large amount of model-dependent evidence. The assignments for ^{110}Cd , ^{112}Cd and ^{114}Cd appear to be more firmly established than those for ^{106}Cd , ^{108}Cd and ^{116}Cd . Nevertheless all the assignments have been widely assumed to be correct. In ^{112}Cd we find that the prominent ' 3^- group' in the spectrum corresponds not to the previously reported 3^- level at 1971 keV but to another level at 2004 keV. It thus appears that the previous 3^- assignment (Hansen and Nathan 1963; McGowan *et al.* 1965) was made to the wrong level.

Gill *et al.* (1974a, 1974b) have questioned the existence of the 3^- octupole states in the even cadmium isotopes. In high resolution studies of $^{114}\text{Cd}(n, n'\gamma)^{114}\text{Cd}$, they found no evidence for a 3^- level at about 1960 keV, but rather reported the existence of a single $J = 1$ level at 1959 keV. In similar work on ^{116}Cd , they found no 3^- state near 1920 keV, but rather a triplet of levels at 1917, 1923 and 1930 keV. The 1917 keV

level was assigned $J = 0$, but apparently no value of J would fit their data for the other two levels. Thus Gill *et al.* suggested a qualitative difference between the level structures of ^{114}Cd and ^{116}Cd in the region where the 3^- states had been assumed to exist. This suggestion appears to be incompatible with the present data, which demonstrate a strong correspondence between the level structures in the appropriate regions, not only of ^{114}Cd and ^{116}Cd , but of all the even- A cadmium isotopes.

Recently Gillespie *et al.* (1976) have argued that their inelastic electron scattering data support the 3^- assignment in ^{114}Cd , and Seyfarth (1975) reported a definite 3^- assignment for this level. If the 3^- assignment is accepted in any one of the even- A cadmium isotopes then the striking similarities of the present spectra would confirm that 3^- states of a collective nature exist at 2375 ± 3 keV in ^{106}Cd , at 2198 ± 3 keV

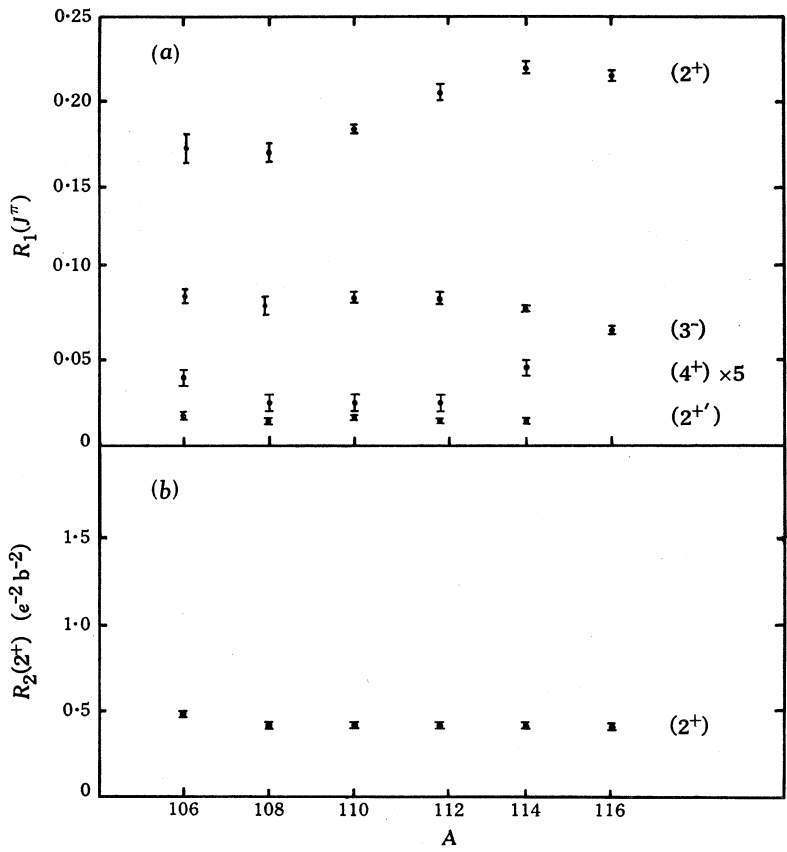


Fig. 3. Variation with mass number A (for the even- A stable cadmium isotopes) of:
(a) the dimensionless ratio $R_1(J^\pi) = \sum J^\pi / \sum 0^+$, the intensity of a group corresponding to an excited level normalized to the intensity of the elastic scattering group;
(b) the double ratio $R_2(2^+) = (\sum 2^+ / \sum 0^+) / B(E2; 0^+ \rightarrow 2^+)$ for the first excited state (see text).

The groups in (a) are labelled as follows:

- 2^+ , first excited 2^+ state; $2^{+'}$, second excited 2^+ state;
- 4^+ , first excited 4^+ state; 3^- , first excited 3^- state.

For ^{116}Cd the $2^{+'}$ and 4^+ groups are not sufficiently resolved to permit separation of their individual intensities.

in ^{108}Cd , at 2076 ± 3 keV in ^{110}Cd , at 2003 ± 3 keV in ^{112}Cd , at 1954 ± 3 keV in ^{114}Cd and at 1918 ± 3 keV in ^{116}Cd . It can be seen that the 3^- excitation energies exhibit a smooth decrease with mass number.

Koike *et al.* (1969) using 55 MeV protons, and Gillespie *et al.* (1976) using 112 MeV electrons have both presented evidence for single-phonon-hexadecapole excitation of a 4^+ state at about 2.39 MeV in ^{114}Cd . In the present ^{114}Cd data, a level at 2386 ± 3 keV is the next most strongly populated after the single-phonon excitations of the first 2^+ and 3^- states. Furthermore, in ^{110}Cd a state at 2220 keV, known to have $J^\pi = 4^+$ (Bertrand and Raman 1971), is also strongly populated. It may be that both these cases are single-phonon-hexadecapole excitations. The similarity (Fig. 1) between the excitation of the 2377 keV state in ^{116}Cd , of unknown J^π , and the 2386 keV in ^{114}Cd , suggests that the former excitation may also be hexadecapole. But even if these speculations are correct, it is certainly true that the hexadecapole excitation is not as systematic across the cadmium isotopes as are the quadrupole and octupole excitations.

Table 3. Experimental reduced E2 matrix elements in cadmium isotopes

Data are from the following references: McGowan *et al.* (1965), Milner *et al.* (1969), Grabowski and Robinson (1973) and Esat *et al.* (1976b)

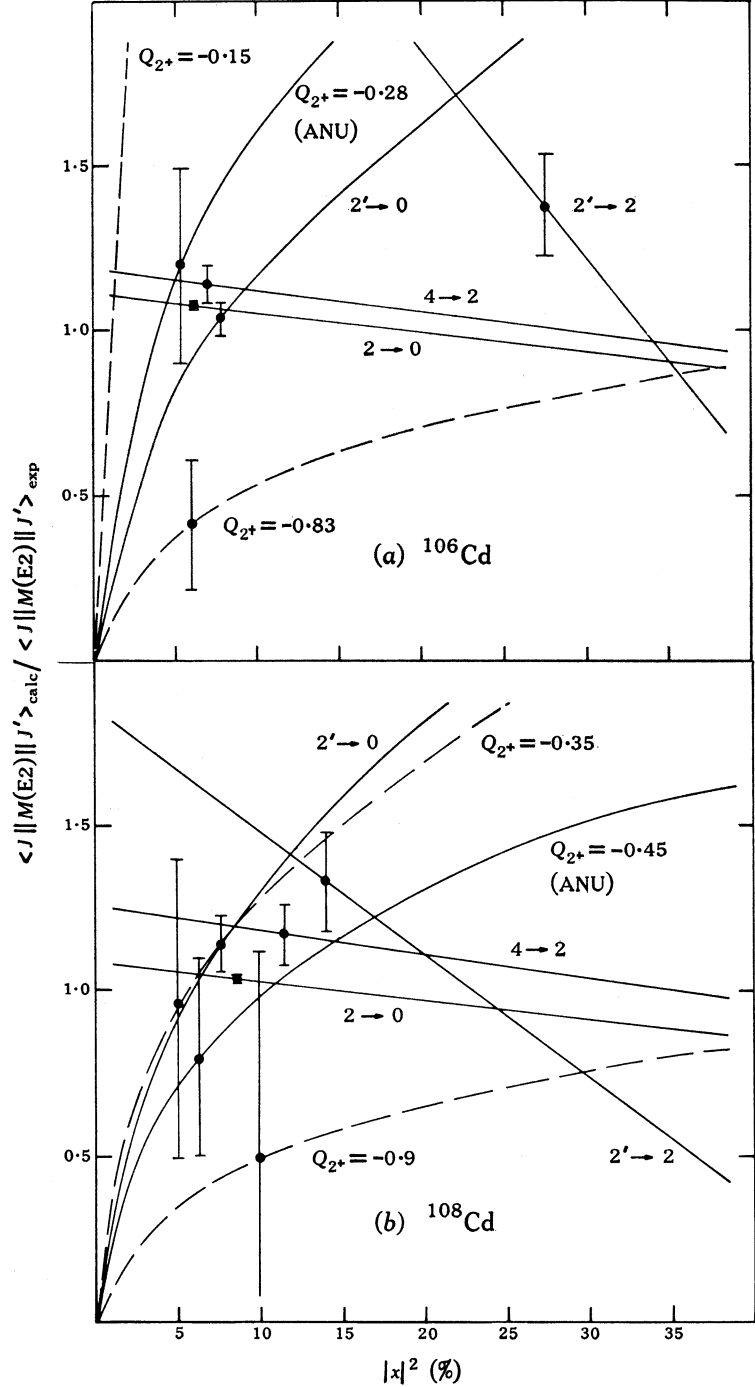
J_i	J_f	Reduced E2 matrix element ($e\text{fm}^2$) for the cadmium isotope:					
		^{106}Cd	^{108}Cd	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd
2	0	61.97 ± 0.3	63.80 ± 0.3	65.35 ± 0.3	69.57 ± 0.3	72.66 ± 0.3	72.94 ± 0.3
2'	0	19 ± 1	17 ± 1	15 ± 1	10 ± 1	9.8 ± 0.8	14 ± 1
2''	0	—	—	—	7 ± 1	8.4 ± 0.7	—
2'	2	32 ± 5	53 ± 8	71 ± 10	54 ± 5	64 ± 7	60 ± 8
2''	2	—	—	—	4 ± 3	2 ± 1	—
4	2	111 ± 7	106 ± 6	113 ± 7	131 ± 8	138 ± 8	133 ± 14
0'	2	—	—	—	—	31 ± 3	32 ± 3
0''	2	—	—	—	—	31 ± 5	—
2	2	37 ± 11	59 ± 11	48 ± 11	52 ± 11	48 ± 11	55 ± 11

Tamura and Udagawa (1966) attempted to explain the large observed value for Q_{2+} in ^{114}Cd and other properties of ^{114}Cd which deviate from the predictions of the purely harmonic vibrational model, by considering the wave functions of the first and second 2^+ states to be orthogonal linear combinations of the one- and two-phonon harmonic vibrational states. The wave function of the first excited 2^+ state was assumed to take the form

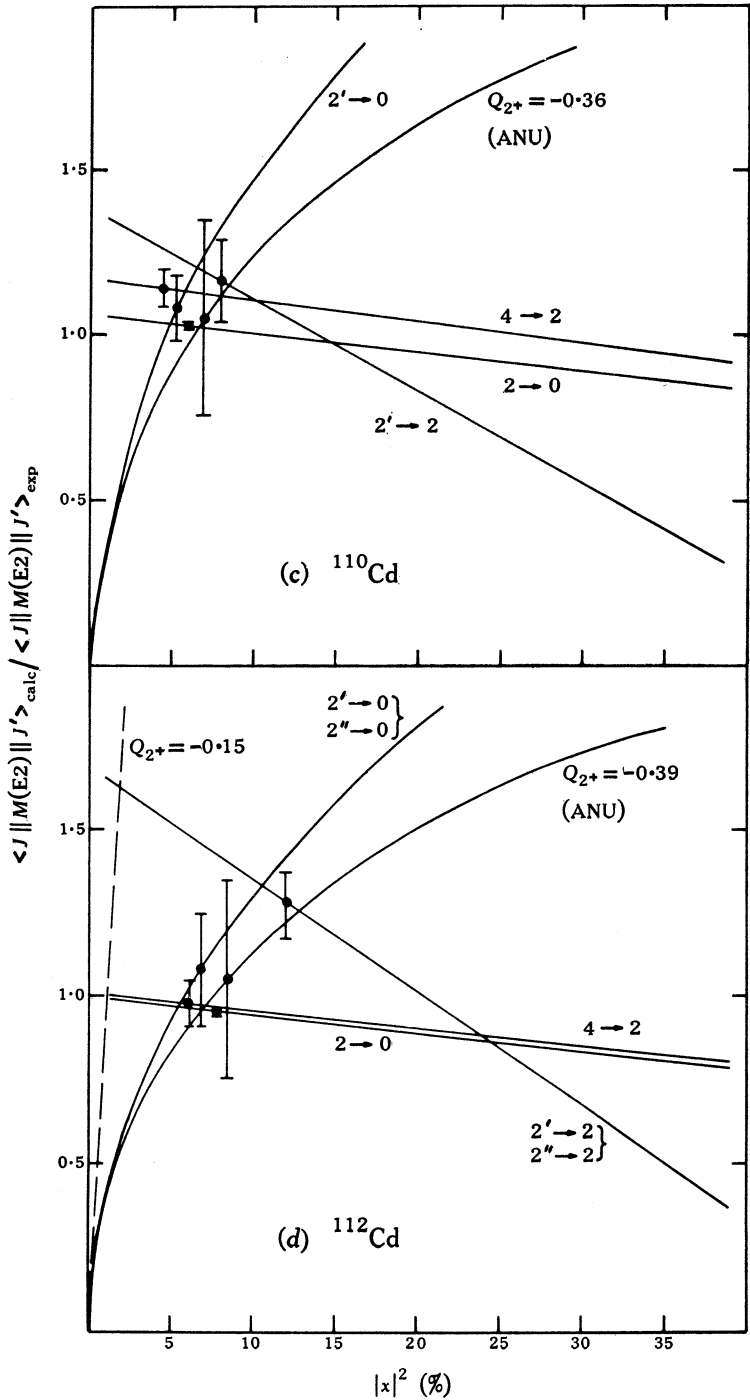
$$|2^+\rangle = (1-x^2)^{\frac{1}{2}}|1,2\rangle + x|2,2\rangle,$$

which is orthogonal to the wave function for the second 2^+ state $|2^{+'}\rangle$. Here $|N,J\rangle$ denotes the wave function of the harmonic N -phonon state with spin J .

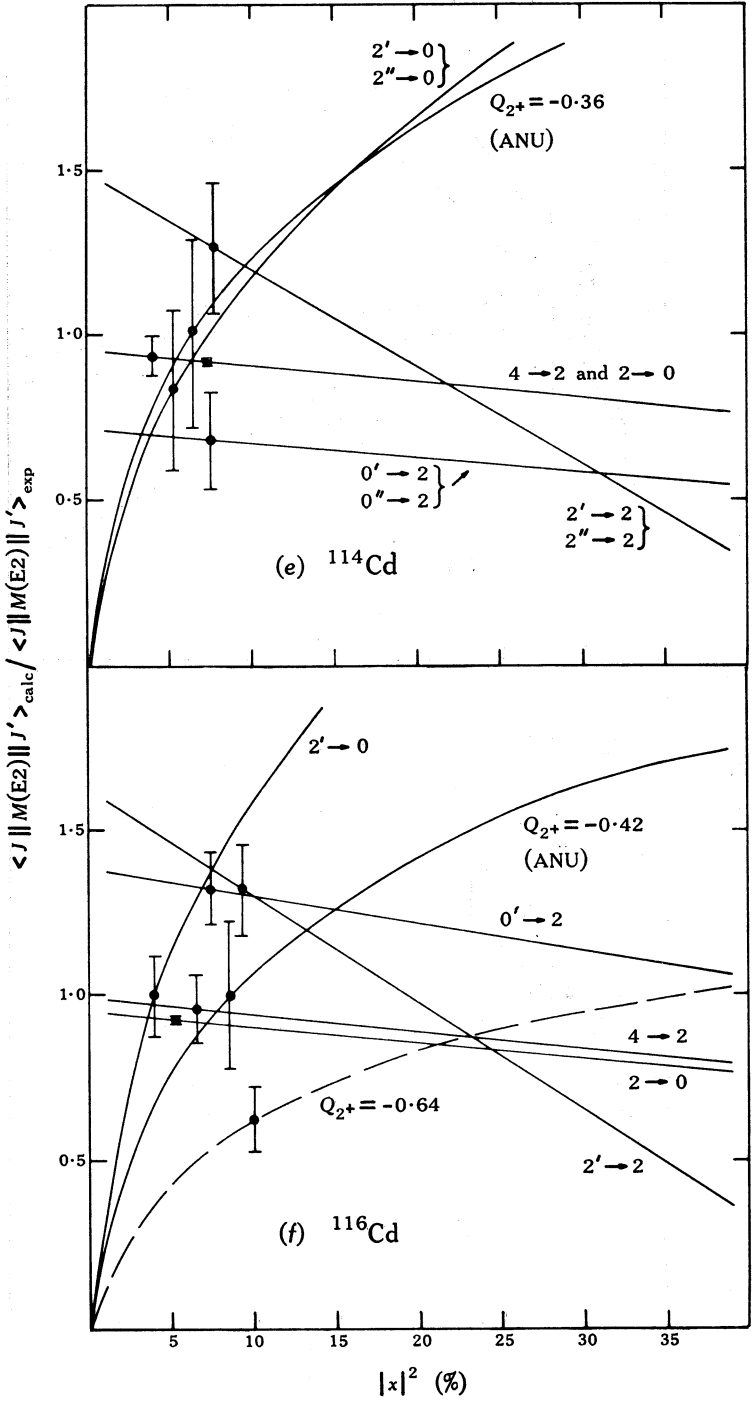
Figs 4a-4f (pp. 144-6). Ratios of theoretical and experimental reduced E2 matrix elements in the even- A stable cadmium isotopes. The data were taken from McGowan *et al.* (1965), Milner *et al.* (1969), Grabowski and Robinson (1973) and Esat *et al.* (1976b). Error bars are indicated at arbitrary values of $|x|^2$; each line thus indicates the centre of a band with a vertical span determined by these experimental errors. The dashed lines refer to Q_{2+} data from work other than the ANU results of Esat *et al.*, as specified in the text.



Figs 4a and 4b



Figs 4c and 4d



Figs 4e and 4f

Häusser *et al.* (1971) applied the above phenomenological model to ^{112}Cd and found that they could reproduce the experimental E2 matrix elements, including Q_{2+} , with the relatively small mixing parameter $|x|^2 \approx 6\%$. We have performed a similar analysis of all available E2 matrix element data (Table 3) for transitions among the zero-, one-, and two-phonon states of the even cadmium isotopes. The E2 transition operator is assumed to be of the form

$$E2 = \alpha(b^\dagger + b),$$

where $b^\dagger$ is a phonon creation operator, b is a phonon annihilation operator and α is a constant chosen to match experimental E2 matrix elements.

Fig. 4 shows the ratios of calculated to experimental E2 matrix elements plotted as functions of $|x|^2$. The value $\alpha = 31 \text{ efm}^2$ was used for all the isotopes. For ^{112}Cd and ^{114}Cd , where a third 2^+ state ($2^{+''}$) is known in the two-phonon region, we have followed Häusser *et al.* (1971) in adding the $2^{+'}$ and $2^{+''}$ matrix elements before comparison with the calculation. The Q_{2+} values are those of Esat *et al.* (1976b). For each isotope it is possible to reproduce all the data with one value of $|x|^2$ (including the crossover $2^{+'} \rightarrow 0^+$ transition). The only exception to this is the $2^{+'} \rightarrow 2^+$ transition in ^{106}Cd , where we have used the value of Grabowski and Robinson (1973); the earlier value of Milner *et al.* (1969) is much more consistent with the other data. The values of $|x|^2$ required to achieve consistency among the data range from about 7% for ^{106}Cd (ignoring the $2^{+'} \rightarrow 2^+$ case) to about 11% for ^{108}Cd . In fact, the data for all isotopes would be reasonably consistent with $|x|^2 \approx 8\%$.

It is significant that some older Q_{2+} results, which are in disagreement with those of Esat *et al.* (1976b), are inconsistent with the other matrix element data. Some examples are shown in Fig. 4: for ^{106}Cd the values $Q_{2+} = -0.15$ (Hall *et al.* 1974) and $Q_{2+} = -0.83$ (Kleinfeld *et al.* 1970); for ^{108}Cd , the value $Q_{2+} = -0.9$ (Steadman *et al.* 1970); for ^{112}Cd , the value $Q_{2+} = -0.15$ (Steadman *et al.*); and for ^{116}Cd , the value $Q_{2+} = -0.64$ (Hall 1975).

In summary, three pieces of information suggest that the vibrational character of the even-mass cadmium isotopes is remarkably uniform from one isotope to another:

- (1) the regularity of the one-quadrupole-phonon and one-octupole-phonon excitations in inelastic α particle scattering (Fig. 2);
- (2) the consistency of the mixing of one- and two-phonon 2^+ states required to fit the E2 matrix element data (Fig. 4);
- (3) the constancy of the quadrupole moments Q_{2+} among the isotopes (Esat *et al.* 1976b).

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