

Local and Nonlocal Potentials for Low Energy Pion-Nucleon Scattering

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Abstract

Local and nonlocal potentials have been evaluated using the reaction matrix approach to the inverse scattering problem for low energy pion-nucleon scattering. The nonlocal potential gave the correct position for the P11 ground state nucleon mass, but the local potential did not. Incident pion energies up to 700 MeV were considered.

1. Introduction

Scattering theory is concerned with the determination of interparticle interactions, mostly with the aid of potentials. The traditional approach is to assume some parameterized form for the potential and to vary these parameters until suitable fits are obtained to cross sections or phase shifts. The inverse scattering problem is posed in the opposite manner. Given information about phase shifts, one tries to deduce directly the interparticle potential.

In a previous paper, Cook (1972) used reaction matrix theory to show that the reaction matrix parameters determined a class of nonlocal potentials. Clayton (1972) applied this theory to pion-nucleon scattering below 700 MeV and we quote his results in this paper. Every nonlocal potential has an equivalent energy-dependent form, and these are what Clayton evaluated. In a later paper, Cook (1973) showed how potentials can be described by a series of step functions whose heights are fitted to the experiments to yield a local form for the potential. In the present paper we show how these heights are fitted for the seven most important states of the pion-nucleon system.

Let us now consider the purpose behind calculating potentials from phase shifts. Firstly, it is a convenient way of summarizing the phase shift data. The phase shifts are determined by the asymptotic form of the radial wave function for large separations of the two bodies in the scattering system. Thus, secondly, a potential may be a useful way of parameterizing the wavefunction at short range. In momentum space, the phase shifts are determined by the on-shell properties of the two-body T matrix, but the off-shell behaviour is arbitrary unless particular assumptions are made about the form of the two-body interaction. Thus, one way of distinguishing between phase equivalent potentials is to look to data which depend upon off-shell properties, such as three-body interactions. The binding energies of two-body bound states should be given correctly by the R matrix prescription used, but this does not remove the ambiguity of the potential.

As regards the data used, we required the following restriction: there had to be a high density of experimental energy points below the nonelastic threshold of ~ 300 MeV. This was required because we used only a one-channel analysis and did not try to fit absorption coefficients. With this restriction, we were forced to choose the obsolete data of Roper *et al.* (1965) and to forego later compilations which did not provide enough points for us to perform a least squares fit to the reaction matrix. For this reason, the fitting to data at ~ 300 MeV should be viewed as just an academic exercise in proving our methods. Throughout this paper we use the Heaviside system of units.

2. Theory and Evaluation

A review of existing literature on the inverse scattering problem has been given by Cook (1972), while the standard theory relating to meson-nucleon scattering has been specified by Hamilton (1967). The nonspin- and spin-flip amplitudes for meson scattering are:

$$f(\theta) = q^{-1} \sum_{l=0}^{\infty} \{(l+1)A_{l+} + lA_{l-}\} P_l(\cos \theta), \quad (1)$$

$$g(\theta) = q^{-1} \sum_{l=0}^{\infty} \{A_{l+} - A_{l-}\} P_l^1(\cos \theta). \quad (2)$$

Here the $A_{l\pm}$ are partial wave amplitudes which, for each isospin state T , are given by

$$A_{l\pm}^T = -\frac{1}{2}i\{\eta_{l\pm}^T \exp(2i\alpha_{l\pm}^T) - 1\}, \quad (3)$$

where q is the centre-of-mass momentum of the pion, $\alpha_{l\pm}^T$ is the phase shift and $\eta_{l\pm}^T$ is the absorption coefficient.

We investigate the low energy region where, essentially, the amplitudes for the reactions

$$\pi^+ + p \rightarrow \pi^+ + p, \quad \pi^- + p \rightarrow \pi^- + p, \quad \pi^- + p \rightarrow \pi^0 + n \quad (4)$$

are respectively given by (dropping the subscripts l)

$$A^+ = A^{3/2}, \quad A^- = \frac{1}{3}(A^{3/2} + 2A^{1/2}), \quad A_{ce} = \frac{1}{3}\sqrt{2}(A^{3/2} - A^{1/2}),$$

where A_{ce} is the charge-exchange amplitude and the superscripts denote the total isotopic spin of the state. Our discussion is concerned mainly with the $A_{l\pm}^T$ amplitude.

From reaction matrix theory (Wigner and Eisenbud 1947) we may write the S matrix, in the absence of absorption, for each eigenstate of orbital angular momentum l and isospin T as

$$S = \exp(2i\alpha) = \Omega\{1 - R(L - B)\}^{-1}\{1 - R(L^* - B)\}\Omega, \quad (5)$$

where $L = s + iP$ and $\Omega = \exp(-2i\omega)$, while s is the level shift, P is the penetration factor, ω is the hard sphere phase shift and B is the boundary condition parameter. It is usual (Lane and Thomas 1958) to use the so-called square well boundary condition

$$B = -l. \quad (6)$$

Tables of the functions $s(q)$, $P(q)$ and $\omega(q)$ can be found in Lane and Thomas (1958). Wigner and Eisenbud (1947) showed that R could be written as a sum of poles of the form

$$R(q) = a^{-1} \sum_{\lambda} U_{\lambda}^2 / (q_{\lambda}^2 - q^2) + R_0, \quad (7)$$

where a is the hard sphere radius (taken to be the nucleon radius) and R_0 is the constant background, while the q_{λ}^2 are defined in the following paragraph.

Cook (1972) considered the Schrödinger equation for the problem

$$\left(\frac{d^2}{dr^2} + q^2 - \frac{l(l+1)}{r^2} - V(r) \right) \psi(q, r) = 0, \quad (8)$$

with a potential energy V , in the light of the inverse scattering problem. In essence this means that, given a , one wishes to determine a corresponding V from equation (8). There is an infinite set of orthonormal functions $U_{\lambda}(r)$ which satisfies

$$\left(\frac{d^2}{dr^2} + q_{\lambda}^2 - \frac{l(l+1)}{r^2} - V(r) \right) U_{\lambda}(r) = 0, \quad (9)$$

with the usual boundary condition

$$\left(\frac{dU_{\lambda}(r)}{dr} \right)_{r=a} = \frac{B}{a} U_{\lambda}(a). \quad (10)$$

Equations (9) and (10) determine the infinite set of eigenvalues q_{λ}^2 . Cook (1973) assumed an eigenvalue form for $V(r)$

$$V(r) = \sum_{\sigma=1}^N V_{\sigma} \theta(r_{\sigma} - r_{\sigma+1}) \theta(r_{\sigma-1} - r_{\sigma}), \quad (11)$$

where $\theta(x)$ is the Heaviside function defined by

$$\begin{aligned} \theta(x) &= 0, & x < 0, \\ &= 1, & x > 0, \end{aligned}$$

and he obtained the V_{σ} as unique eigenvalues of a matrix equation. This was done by using the expansions

$$\Psi(q, r) = \sum_{\lambda} A_{\lambda}(q^2) U_{\lambda}(r), \quad V \Psi(q, r) = \sum_{\lambda} A_{\lambda} \sum_{\mu} V_{\lambda\mu} W_{\mu}(r), \quad (12a, b)$$

$$U_{\lambda}(r) = \sum_{\mu} B_{\lambda\mu} W_{\mu}(r), \quad (12c)$$

where $W_{\mu}(r)$ is the eigenfunction of the corresponding free-particle equation

$$\left(\frac{d^2}{dr^2} + Q_{\mu}^2 - \frac{l(l+1)}{r^2} \right) W_{\mu} = 0, \quad (13)$$

in which

$$\left(\frac{dW_{\mu}(r)}{dr} \right)_{r=a} = \frac{B}{a} W_{\mu}(a).$$

From the equations (12) we find that the matrix equation

$$\mathbf{V}^T \mathbf{B} \mathbf{W} = \mathbf{W} \mathbf{V} \quad (14)$$

must be satisfied, where

$$V_{\lambda\mu} = (q_\lambda^2 - Q_\mu^2) B_{\lambda\mu}, \quad B_{\lambda\mu} = \int_0^a U_\lambda(r) W_\mu(r) dr, \quad W_{\mu\sigma} = \int_{r_\sigma}^{r_{\sigma+1}} W_\mu(r) dr. \quad (15)$$

Thus, if we specify the $B_{\lambda\mu}$, the eigenvalues V_σ are determined. The eigenvectors W_σ are all known. Unfortunately the $B_{\lambda\mu}$ are not known, so that any definition which satisfies can be applied from the equations (12), e.g.

$$U_\lambda(a) = \sum_\mu B_{\lambda\mu} W_\mu(a), \quad W_\mu(a) = \sum_\lambda B_{\lambda\mu} U_\lambda(a), \quad (16)$$

where $U_\lambda(a)$ and $W_\lambda(a)$ are known from equations (7) and (13) respectively. Thus there is a whole class of energy-dependent phase-equivalent potentials which reproduce the experimental phase shifts each defined by a different \mathbf{B} . In general, however, the matrices \mathbf{B} will not have associated eigenvectors \mathbf{W} as appears in equation (14). Only the \mathbf{B} which leads to an energy-independent V will have these eigenvectors. From this, it appears that there is only one such form (11) for $V(r)$ with unique eigenvalues V_σ . We have evolved a method for finding these.

Equation (11) for the potential necessarily means that the wavefunction in the region of interaction has the form applicable to a square well:

$$\Psi_l(q, r) = A_\rho j_l(Q_\rho r) + B_\rho n_l(Q_\rho r) \quad Q_\rho^2 = q^2 - V_\rho, \quad q^2 > V_\rho; \quad (17a)$$

$$= A_\rho i_l(Q_\rho r) + B_\rho i_{-l}(Q_\rho r) \quad Q_\rho^2 = V_\rho - q^2, \quad q^2 < V_\rho; \quad (17b)$$

where j_l , n_l , i_l and i_{-l} are modified Bessel functions given by Schelkunoff (1951). By choosing the external wavefunction to be

$$\Psi_l = j_l(qa) - \tan(\alpha_l) n_l(qa) \quad r = a, \quad (18)$$

and near the origin to be

$$\Psi_l = A_1 j_l(Q_1 r) \quad q^2 > V_1, \quad 0 < r < r_1; \quad (19a)$$

$$= A_1 i_l(Q_1 r) \quad q^2 < V_1, \quad 0 < r < r_1; \quad (19b)$$

we could match the wavefunctions and their derivatives for a set of $\{V_\rho\}$. We then varied the $\{V_\rho\}$ using the search program SPIRAL (Jones 1970) until the quantity

$$M^2 = \sum_{i=1}^N \left(\frac{\alpha_{\text{exp}} - \alpha_{\text{theor}}}{0.1 \alpha_{\text{theor}}} \right)^2 \quad (20)$$

was minimized, which implies a 10% error in the experimental values α_{exp} . These results were then checked by another program which used an 8000 step Fox–Goodwin (Fox 1962) central-difference approximation to integrate the wave equation (8) to beyond $r = a$, where the phase shifts were obtained by matching to the exterior solution. Satisfactory agreement was obtained in all cases. In this way we obtained a local potential.

The nonlocal potential was found from the expression (cf. Cook 1972)

$$V(r, r') = \sum_{\lambda, \mu} V_{\lambda\mu} U_{\lambda}(r') W_{\mu}(r). \quad (21)$$

Specifying $B_{\lambda\mu}$ as

$$B_{\lambda\mu} = \delta_{\lambda\mu} - (U_{\lambda} - W_{\lambda})(U_{\mu} - W_{\mu}) / \left(\sum_{\rho} U_{\rho}^2 - \sum_{\rho} U_{\rho} W_{\rho} \right), \quad (22)$$

we integrated the resulting wave equation (for details, see Clayton 1972), and the phase shifts were determined by the standard matching procedure. Clayton actually used the source term

$$\begin{aligned} \rho(q, r) &= \sum_{\lambda\mu} A_{\lambda}(q) V_{\lambda\mu} W_{\mu}(r) \\ &= \int_0^a V(r, r') \Psi(q, r') dr \end{aligned} \quad (23)$$

in the equation

$$\left(\frac{d^2}{dr^2} + q^2 - \frac{l(l+1)}{r^2} \right) \Psi(q, r) = \rho(q, r) \quad (24)$$

to evaluate his phase shifts.

Table 1. Reaction matrix parameters

q_{λ}^2	γ_{λ}^2	R_0	a	q_{λ}^2	γ_{λ}^2	R_0	a
(a) <i>S11</i>				(b) <i>S31</i>			
1.2053	1.1205	0.1176	1.3	1.1362	0.9009	0.9449	1.6
10.072	0.8003			10.125	0.6809		
26.076	1.6294	$[V_0 = 0.2547]$		21.899	0.7610	$[V_0 = 0.0]$	
(c) <i>P11</i>				(d) <i>P13</i>			
-1.0645	0.2272	0.06361	1.0	3.1303	0.6349	0.02305	1.8
6.1282	0.9062			12.746	0.6492		
22.790	3.6890	$[V_0 = 10.934]$		29.154	0.5678	$[V_0 = 0.0]$	
137.40	3.1777						
(e) <i>P31</i>				(f) <i>P33</i>			
3.6499	0.7339	0.02305	1.8	2.0756	0.3142	0.09822	1.5
13.026	0.5004			6.2188	0.7072		
27.0	0.8419	$[V_0 = 0.0]$		23.163	1.6452	$[V_0 = 2.3109]$	
(g) <i>D13</i>							
2.0756	0.3142	0.09822	1.5				
6.2188	0.7072						
23.163	1.6452	$[V_0 = 2.3109]$					

3. Potentials

The reaction matrix parameters calculated by Clayton (1972) are shown in Table 1. The experimental phase shifts used in the calculations are those of Roper *et al.* (1965). Apart from parameters already defined, Table 1 also contains the parameter $\gamma_{\lambda}^2 = U_{\lambda}^2(a)/a$, while V_0 is the effective local square well potential that determines the background R_0 in equation (7).

Table 2. Phase shifts

E_π (MeV)	α_{exp} ($^\circ$)	α_{en} ($^\circ$)	α_{loc} ($^\circ$)	E_π (MeV)	α_{exp} ($^\circ$)	α_{en} ($^\circ$)	α_{loc} ($^\circ$)
(a) $S11$				(b) $S31$			
31	5.48	5.48	5.79	31	-2.93	-2.97	-2.70
58	6.92	6.97	7.16	58	-5.14	-5.10	-4.76
98	8.29	8.29	8.21	98	-8.31	-8.32	-7.94
120	8.88	8.84	8.63	120	-10.00	-10.07	-9.69
144	9.46	9.39	9.06	144	-11.79	-11.91	-11.56
170	10.06	9.98	10.16	170	-13.64	-13.80	-13.49
220	11.24	11.27	10.86	220	-16.92	-17.06	-16.87
240	11.76	11.86	11.50	240	-18.10	-18.20	-18.07
270	12.62	12.85	12.59	270	-19.72	-19.73	-19.70
310	13.96	14.34	14.30	310	-21.56	-21.44	-21.53
333	14.86	15.31	15.4	330	-22.44	-22.24	-22.41
370	16.53	17.03	17.3	370	-23.56	-23.25	-23.52
410	18.69	19.17	19.7	410	-24.33	-23.97	-24.31
450	21.28	21.66	22.2	450	-24.62	-24.28	-24.62
490	24.35	24.58	25.1	490	-24.41	-24.14	-24.44
523	27.27	27.39	27.8	523	-23.83	-23.66	-23.89
572	32.34	32.33	32.4	572	—	—	—
581	—	—	—	581	-21.91	-21.89	-21.96
600	35.65	35.61	35.4	600	-21.02	-21.01	-21.05
650	42.37	42.19	41.3	650	-18.06	-17.95	-18.05
698	49.88	49.14	47.4	698	-14.33	-13.89	-14.44
(c) $P11$				(d) $P13$			
6	-0.15	-0.15	-0.05	6	-0.015	-0.014	-0.013
31	-1.22	-1.16	-0.53	31	-0.168	-0.167	-0.157
58	-2.045	-1.94	-1.08	58	-0.41	-0.42	-0.404
98	-2.06	-1.89	-1.38	98	-0.86	-0.87	-0.88
120	-1.458	-1.24	-1.11	120	-1.14	-1.15	-1.17
144	-0.344	-0.0196	-0.347	144	-1.45	-1.46	-1.51
170	1.375	1.86	1.12	170	-1.81	-1.81	-1.89
195	3.522	4.18	3.22	195	-2.15	-2.15	-2.25
220	6.166	6.99	6.00	220	-2.49	-2.49	-2.60
240	8.659	9.57	8.71	240	-2.76	-2.76	-2.88
270	13.09	14.01	13.51	270	-3.15	-3.16	-3.27
310	20.42	21.04	21.1	310	-3.66	-3.68	-3.75
333	25.39	25.72	26.0	333	-3.94	-3.97	-4.01
370	34.42	34.33	34.59	370	-4.37	-4.41	-4.39
410	45.12	45.16	44.84	410	-4.81	-4.86	-4.77
450	56.13	57.09	55.98	450	-5.24	-5.28	-5.15
490	67.12	69.01	67.49	490	-5.65	-5.68	-5.53
523	75.91	77.90	76.47	523	-5.98	-6.00	-5.87
550	82.55	84.13	82.99	550	-6.26	-6.27	-6.17
572	87.30	88.44	87.53	572	-6.49	-6.49	-6.43
581	89.03	90.00	89.18	581	-6.59	-6.58	-6.54
600	92.23	92.93	92.22	600	-6.79	-6.78	-6.80
616	94.44	95.02	94.38	616	-6.97	-6.96	-7.02
650	97.74	98.49	97.73	650	-7.37	-7.36	-7.55
689	99.60	101.07	99.83	689	-7.88	-7.88	-8.22
698	99.81	101.49	100.09	698	-8.00	-8.01	-8.40

Table 2 (Continued)

E_π (MeV)	α_{exp} (°)	α_{en} (°)	α_{loc} (°)	E_π (MeV)	α_{exp} (°)	α_{en} (°)	α_{loc} (°)
(e) P31				(f) P33			
6	-0.03	-0.03	-0.027	6	0.23	0.20	0.22
31	-0.34	-0.30	-0.32	31	2.82	2.63	2.81
58	-0.85	-0.78	-0.85	58	7.81	7.59	7.89
98	-1.82	-1.74	-1.86	98	20.48	20.45	20.61
120	-2.42	-2.36	-2.49	120	31.53	31.55	31.65
144	-3.11	-3.06	-3.21	144	48.15	48.09	48.46
170	-3.88	-3.84	-4.00	170	70.59	70.37	71.20
195	-4.61	-4.59	-4.74	195	91.70	91.57	91.63
200	—	—	—	200	108.12	108.24	106.03
220	-5.32	-5.31	-5.44	220	—	—	—
240	-5.85	-5.85	-5.97	240	117.68	117.95	113.3
270	-6.59	-6.59	-6.68	270	127.72	128.05	—
310	-7.39	-7.39	-7.43	310	136.27	136.48	—
333	-7.74	-7.73	-7.75	(g) D13			
370	-8.09	-8.09	-8.07	31	0.005	0.005	0.004
410	-8.15	-8.15	-8.11	98	0.122	0.111	0.103
450	-7.81	-7.84	-7.78	120	0.214	0.198	0.184
490	-7.02	-7.10	-7.03	170	0.659	0.540	0.517
523	-6.02	-6.13	-6.04	220	1.1866	1.161	1.148
550	-4.93	-5.08	-4.96	270	2.161	2.175	2.22
572	-3.87	-4.03	-3.89	333	4.14	4.27	4.55
581	-3.38	-3.54	-3.40	410	8.55	8.90	9.81
600	-2.26	-2.42	-2.27	490	18.19	18.63	20.4
616	-1.22	-1.36	-1.21	523	25.28	25.63	27.1
650	1.32	1.30	1.35	572	42.08	42.70	40.3
689	4.80	4.77	4.74	600	56.40	58.49	49.4
698	5.69	5.69	5.58				

Table 2 gives an intercomparison between the nonlocal α_{en} (energy-dependent), local α_{loc} and experimental α_{exp} (Roper *et al.* 1965) phase shifts. The nonlocal phase shifts were determined from equation (24), while the local phase shifts were determined by fitting to the local potential (equation 11). We do not discuss in detail the energy-dependent local potentials derived from equation (7), as these are to be the subject of a future work. The local potential is listed in Table 3.

The Clayton R matrix parameters for the S11 state are listed in Table 1*a*. For this state we note that the nonlocal phase shifts (Table 2*a*) fit to the experimental data better than do the local ones. For the S11 local potential (Table 3*a*), six eigenvalues were used at equal increments in the range $0 < r < 1.3$. The potential evidently behaves like a damped oscillation as r increases.

Reference to the S31 Clayton parameters in Table 1*b* indicates that there appear to be no bound levels for either of the S states. The agreement with experiment for both nonlocal and local phase shifts is quite good (Table 2*b*). For the local potential (Table 3*b*) there appears to be a large hard core, which is not evident in the other S state.

The P11 phase shifts (Table 1*c*) were rather harder to fit. For the nonlocal reaction matrix approach, four poles were required to give reasonable results. The bound

Table 3. Local potentials
Errors are absolute values unless given as explicit percentages

ρ	V_ρ	Error	ρ	V_ρ	Error	ρ	V_ρ	Error
(a) $S11$								
1	22.7	± 1.6	3	45.7	± 1.0	5	9.7	± 0.2
2	-33.4	± 0.9	4	-21.0	± 0.8	6	-2.18	± 0.13
(b) $S31$								
1	1322	± 145	3	21.5	± 0.3	5	1.79	± 0.07
2	-35.1	± 0.4	4	-6.8	± 0.1	6	-0.489	± 0.045
(c) $P11$ ($a = 1.5$)								
1	-49.8	$\pm 100\%$	6	-19.5	± 0.3	11	-6.4	± 0.1
2	-2.4	$\pm 100\%$	7	-1.7	± 0.3	12	-15.67	± 0.07
3	-8.4	± 2.1	8	-15.3	± 0.3	13	-3.53	± 0.06
4	-20.0	± 0.7	9	21.0	± 0.3	14	13.69	± 0.07
5	-28.5	± 0.4	10	12.2	± 0.2	15	-1.52	± 0.05
(d) $P13$								
1	-115.6	± 0.4	3	-5.65	± 0.15	5	-0.14	± 0.03
2	-15.1	± 0.9	4	0.447	± 0.057	6	-0.015	± 0.016
(e) $P31$								
1	0.07	$\pm 100\%$	5	-4.5	± 0.1	9	-2.6	± 0.2
2	-0.68	$\pm 100\%$	6	2.1	± 0.1	10	-3.5	± 0.1
3	-3.9	± 0.4	7	6.2	± 0.1	11	-4.1	± 0.1
4	-6.5	± 0.2	8	3.3	± 0.2	12	-1.21	± 0.07
(f) $P33$								
1	142	± 33	4	-5.4	± 0.4	7	3.0	± 0.5
2	-47.5	± 0.9	5	3.1	± 0.6	8	-3.6	± 0.3
3	-18.2	± 0.4	6	4.2	± 0.7			
(g) $D13$								
1	5.0	$\pm 100\%$	5	0.9	$\pm 100\%$	9	48	± 2
2	4.9	$\pm 100\%$	6	-56	± 2	10	-17	± 1
3	2.9	$\pm 100\%$	7	-62.8	± 1.5			
4	-100	± 11	8	-1.0	± 1.8			

level at $q_\lambda^2 = -1.0645$ is, in fact, the nucleon pole. Clayton (1972) showed that the corresponding pole in the S matrix occurred at about 928 MeV, which is slightly less than the correct value of 938 MeV, but since no error analysis was carried out, we do not know the significance of the disagreement. The fitted phase shifts are to be found in Table 2c and the local potential in Table 3c. The fit to the lower energies is seen to be rather poor for both potentials, but improves towards the higher energies.

Unlike the nonlocal potential, the local potential appears not to give a bound state corresponding to the nucleon pole. Two programs were written, one based upon the matching method discussed in Section 2, the other upon direct numerical integration. If a bound state were present, an exponentially decaying external solution should have been found near $q^2 = -1$. Neither program gave this result. In fact, no bound state was found at all, despite the double-well structure of the local potential. The fits were extremely insensitive to the core region where almost any eigenvalues within a 100% range gave a good fit.

The Clayton reaction matrix parameters for the P13 state are illustrated in Table 1*d*. The phase shifts for both potentials are given in Table 2*d*, and the fitted local potential appears in Table 3*d*.

The P31 state results resemble the P13 values, with significant differences. The Clayton R matrix parameters are listed in Table 1*e*. Note the longer range of the P13 and P31 potentials as obtained from the best fitting radius a . The local potential required 12 eigenvalues for a reasonable fit and the results are given in Table 3*e*.

The state P33 is the famous state which resonates strongly at an incident pion energy of about 195 MeV. Clayton's reaction matrix parameters are seen in Table 1*f*. One can see that the broad peak in the total cross section appears to arise from two low-energy poles and the effect of their interference, rather than from an isolated pole. This was how the reaction matrix behaved when we calculated it and is not an artificial fit. The phase shift fits are to be found in Table 2*f*. The high energy end could not be fitted satisfactorily for the local potential, and so the fit there was restricted to energies below 270 MeV. The local potential obtained is given in Table 3*f*.

Finally, the D state potential D13 has a resonant effect at an incident pion energy of about 600 MeV. The Clayton reaction matrix parameters are given in Table 1*g*. The source integration yielded the phase shifts shown in Table 2*g*. The local potential fit is not nearly as accurate as the nonlocal, possibly because too few eigenvalues were used. The fitted local potential is given in Table 3*g*, and it can be seen that four of the chosen ten eigenvalues are not unique.

4. Conclusions

Both local and nonlocal potentials have been found which generally give a satisfactory fit to the pion-nucleon phase shifts below 600 MeV. These potentials display similar features of alternate wells and barriers, sometimes with a small surface repulsion. Since Roper *et al.* (1965) did not give experimental errors for their phase shifts, we are unable to assess the statistical significance of the fits. Furthermore, because of the restriction of reproducing believable potentials below 300 MeV, we are unable to draw sound conclusions about the behaviour of wave functions at small distances of $r < 0.2$.

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