

Analyses of Inelastic Proton Scattering to High Spin States of Unnatural Parity

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Abstract

Distorted wave approximation analyses of 135 MeV inelastic proton scattering to high spin (4^- and 6^-) unnatural parity states in ^{24}Mg and ^{28}Si have been made to test the character of an effective two-nucleon interaction, assuming a transition spectroscopy of particle-hole excitation from a projected Hartree-Fock intrinsic ground state.

1. Introduction

Inelastic proton scattering data initiated by projectiles with energies in excess of 60 MeV have long been of interest since, given a good (microscopic) understanding of the spectroscopy of the target states involved, such data should reflect properties of the effective interaction between the projectile and any bound nucleon (Satchler 1967; Austin 1971).

At lower projectile energies there is now substantial evidence of important 'two-step' corrections (Geramb *et al.* 1975), especially those mediated by giant resonances and, in fact, the energy dependence of (p, p') data in this region has been used to delineate the gross properties of giant resonances. However, the use of reaction data requires that the important attributes of the effective two-nucleon interaction, by which direct (one-step) reaction processes occur, be well understood. To obtain such information it is therefore advantageous to analyse data that are 'free' of contributions from competing processes and, for most targets, this requires projectile proton energies in excess of 60 MeV.

The utility of (p, p') data to test any proposed effective two-nucleon interaction is further limited by any inadequacies that exist in the microscopic model of the transition spectroscopy. For example, core polarization corrections, as revealed by and directly correlated to, effective charges in spectroscopy (Love and Satchler 1967; Brown and Madsen 1975) are often required in reaction analyses. The need for these corrections has limited the use (Geramb and Amos 1970; Amos and Geramb 1971) of otherwise simple reactions, such as the excitations of the $9/2^+$ (0.908 MeV) state in ^{89}Y and of the $3/2^-$ (0.97 MeV) state in ^{207}Pb , for testing a two-nucleon interaction. Other natural parity transitions, for which one can trust the basic transition spectroscopy, are often dominated by these core polarization corrections, and with rare exception (Ford *et al.* 1971; Nesci and Amos 1977) cannot be readily improved (by using a bigger basis spectroscopy).

The problems discussed above are minimized when unnatural parity transitions are considered, since no core corrections are needed in most cases and the transition spectroscopy is usually quite simple. In particular, analyses of transitions to the 2^- state (8.88 MeV) in ^{16}O (Smith and Amos 1975; Lebrun *et al.* 1976) and to the 1^+ states (12.71 and 15.11 MeV) in ^{12}C (Amos *et al.* 1974), and analyses of charge exchange data (Amos and Geramb 1974; Rikus *et al.* 1977) have made credible an effective two-nucleon interaction between the projectile and a bound nucleon which can be reasonably represented by simple functional forms but possesses full operator character (central, tensor and two-body spin-orbit). This specification of the interaction has had further success in that, when used to analyse (p, p') data taken at energies in the giant resonance region (Geramb *et al.* 1975; Lebrun *et al.* 1976), the extracted gross properties of the resonances are consistent with those obtained from experiments. Nonetheless, new tests of our basic effective interaction are required; in particular of the high multipoles of this interaction. Such tests can be made now that 135 MeV data have been taken (Adams *et al.* 1977) from ^{24}Mg and ^{28}Si , in which 4^- and 6^- states are excited. The analyses of these new data are presented herein as is a reanalysis of the 61.2 MeV data (Scott *et al.* 1969) from the transition to the $9/2^+$ state in ^{89}Y , the results being given in Section 3. A brief description of the reaction theory and spectroscopy is first presented in Section 2.

2. Theory

For reactions initiated by protons with energies in excess of 15 MeV, a direct reaction theory that has been described in detail elsewhere (Geramb *et al.* 1975) is appropriate. Details of transition amplitudes associated with various reaction mechanisms in the distorted wave approximation (hereafter referred to as DWA) have been published (Amos *et al.* 1967; Amos and Geramb 1971) and therefore are not repeated here.

It is sufficient to note that for (p, p') analyses we must evaluate transition amplitudes that have the form

$$T_{if} = \sum_{j_1 j_2 m_1 m_2 I N} (2J_f + 1)^{-\frac{1}{2}} S(j_1 j_2; J_i J_f; I) \times (-)^{j_1 - m_1} \langle j_1 j_2 m_1 - m_2 | I - N \rangle \langle J_i I M_i N | J_f M_f \rangle \mathcal{M}_{if}^{(\alpha)}, \quad (1)$$

where the order of the reaction mechanism is denoted by α and, as defined previously (Amos and Geramb 1971), the spectroscopic amplitudes are

$$S(j_1 j_2; J_i J_f; I) = S_{j_1 j_2}^{(x)} = \langle J_f || [a_{j_2}^\dagger \times a_{j_1}]^I || J_i \rangle. \quad (2)$$

These amplitudes (for protons or neutrons in the target according as x is π or ν) carry all the multinucleon attributes of the reaction and are the same multinucleon numbers required to predict γ -ray transition rates, since we have

$$B(EI; J_i \rightarrow J_f) = \{(2I+1)(2J_i+1)\}^{-1} \left\{ \sum_{j_1 j_2} S_{j_1 j_2}^{(\pi)} (e + e_{\text{pol}}) \langle \phi_{j_2}^\pi || r^I Y_I || \phi_{j_1}^\pi \rangle + e_{\text{pol}} S_{j_1 j_2}^{(\nu)} \langle \phi_{j_2}^\nu || r^I Y_I || \phi_{j_1}^\nu \rangle \right\}^2. \quad (3)$$

In the studies reported here, only first-order valence matrix elements, denoted by $\mathcal{M}^{(1)}$ in equation (1), are involved and these are specified by

$$\mathcal{M}^{(1)} = \langle \chi_f^{(-)}(0) \phi_{j_2}(1) | t(0, 1) | \mathcal{A}_{01} \{ \chi_i^{(+)}(0) \phi_{j_1}(1) \} \rangle, \quad (4)$$

where $\mathcal{A}_{01}$ antisymmetrizes the initial two-particle state of continuum particle χ and bound state nucleon ϕ . We suppose that the effective two-nucleon interaction is

$$\begin{aligned} t(0, 1) = & -25 \exp(-0.2750 r^2) P^{01} \\ & -47 \exp(-0.3375 r^2) P^{10} \\ & + S_{01} [\{ -105.25 r^2 \exp(-1.0862 r^2) - 1.9481 r^2 \exp(-0.24174 r^2) \} P^{10} \\ & + \{ 17.918 r^2 \exp(-0.7612 r^2) - 2.3085 r^2 \exp(-0.52277 r^2) \\ & + 0.3831 r^2 \exp(-0.20041 r^2) \} P^{11}], \end{aligned} \quad (5)$$

where the tensor force operator is defined by

$$S_{01} = 3(\boldsymbol{\sigma}_0 \cdot \hat{\mathbf{r}})(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_0 \quad (6)$$

and the two-nucleon state projection operators are

$$P^{ST} = \{1 + 2\delta_{S_1} + (-)^{S+1} \boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_1\} \{1 + 2\delta_{T_1} + (-)^{T+1} \boldsymbol{\tau}_0 \cdot \boldsymbol{\tau}_1\}. \quad (7)$$

This tensor force was used in light nuclei calculations (Eikemeier and Hackenbroich 1971) and is referred to hereafter as the E-H force. We have included an $L \cdot S$ two nucleon force (E-H form) in some analyses, but as yet there is no strong evidence for its necessity in (p, p') analyses despite its expected role in analysing power predictions (Lebrun *et al.* 1976). In fact, our current analyses suggest that strong $L \cdot S$ forces should not be involved in the specification of the effective two-nucleon interaction.

In making calculations using the DWA, it is customary to generate the continuum wavefunctions from optical model potentials that are parameterized to best fit the pertinent elastic scattering data. These are not available to us and thus we have used the potential specified by Horowitz (1972) for the elastic scattering of 100 MeV protons from ^{24}Mg and ^{28}Si . The exact parameter values are not too important since, for these projectile energies, the wavefunctions are not too distorted (Amos 1966) and, in any event, are integrated in DWA calculations. To check these assumptions, calculations with a parameter set quoted for 165 MeV protons (Comparat 1975) were made. Little change in the (p, p') predictions result, at least as far as differential cross sections are concerned.

It thus remains to specify the spectroscopy, and transitions to high spin unnatural parity states not only are useful, in that no core polarization effects are required in the calculations, but they are also attractive, since the residual nuclear states may be well described by

$$|JT\rangle = N \sum_{j_1 j_2} \alpha_{j_1 j_2} O_{j_1 j_2}^+(JT) |00\rangle. \quad (8)$$

Here the sum extends over all possible 'pairs' (of particle-hole states) inherent in the state creation operator:

$$O_{j_1 j_2}^+(JT) = \sum (-)^{j_1 + \frac{1}{2} - m_1 - \tau_1} \langle j_1 j_1 m_1 - m_2 | JM \rangle \langle \frac{1}{2} \frac{1}{2} \tau_1 - \tau_2 | TM_T \rangle \\ \times a_{j_2 m_2 \tau_2}^+ a_{j_1 m_1 \tau_1}. \quad (9)$$

The high spin states in $N = Z$ nuclei are of particular interest since their isospin equivalence can be used while, for the 4^- and 6^- states in ^{24}Mg and ^{28}Si , we have the additional benefit that the particle creation in equation (9) must occur in extra core p-f shell orbits. Furthermore, angular momentum requirements reveal that the 6^- states can only have an extra core particle in the $f_{7/2}$ orbit. Of course, this prescription neglects the effects of mixtures of $(sd)^1-(p)^{-1}$ and $(pf)^3-(sd)^{-3}$ components. However, for ^{24}Mg and ^{28}Si , such admixtures are associated with large (unperturbed) energy values and should be small components in the 4^- and 6^- state description, albeit that such admixtures would reduce our predicted differential cross section strengths.

Thus, treating protons and neutrons equivalently, the normalization in our simple excitation model becomes

$$N = \left(\sum_{j_1 j_2} \alpha_{j_1 j_2}^2 \sum_{m_1 m_2} \langle j_1 j_2 m_1 - m_2 | JM \rangle^2 \sigma_{j_1 m_1 j_1 m_1} \right)^{-\frac{1}{2}}, \quad (10)$$

and involves the single-particle ground state density matrix which, for protons, is defined by

$$\sigma_{jmjm} = \langle 00 | a_{jm\frac{1}{2}}^+ a_{jm\frac{1}{2}} | 00 \rangle. \quad (11)$$

The appealing feature, so far as analyses of inelastic proton scattering to these high spin states are concerned, is that the spectroscopic amplitudes of equation (2) are simply

$$S_{jajb}^{(x)} = (2J+1)^{\frac{1}{2}} \langle JT(M_T) | \bar{O}_{jajb}^+(J, x) | 00(0) \rangle; \quad (12)$$

i.e. they are given by a matrix element involving operators $\bar{O}_{jajb}^+(J, x)$ that are the proton or neutron parts of the general p-h operator of equation (9) according as x is π or ν respectively, namely

$$\bar{O}_{j_1 j_2}^+(J, x) = (2)^{-\frac{1}{2}} \{ O_{j_1 j_2}^+(J, 0) + (\delta_{x\pi} - \delta_{x\nu}) O_{j_1 j_2}^+(J, 1) \}. \quad (13)$$

Upon using equation (8) for the final state, we have

$$S_{jajb}^{(x)} = N(2J+1)^{\frac{1}{2}} \sum_{j_1 j_2} \alpha_{j_1 j_2} \langle 00(0) | O_{j_1 j_2}(J, T) \bar{O}_{jajb}^+(J, x) | 00(0) \rangle \\ = \{ \frac{1}{2}(2J+1) \}^{\frac{1}{2}} \alpha_{jajb} N \{ \delta_{x\pi} + (-)^T \delta_{x\nu} \} \\ \times \sum_{m_a m_b} \{ \langle j_a j_b m_a - m_b | JM \rangle^2 \sigma_{j_a m_a j_a m_a} \}. \quad (14)$$

The density matrices σ were obtained from projected Hartree-Fock (PHF) calculations performed in the (s-d) shell and using the Chung-Wildenthal interaction (Chung 1976). The intrinsic minimum energy states that result are ellipsoidal for ^{24}Mg and oblate for ^{28}Si , and projecting out the O^+ components to describe the ground states yields density matrices that are independent of projection quantum numbers m_a , whence both the normalization N and the spectroscopic amplitudes

simplify to

$$N = \left(\sum_{j_1 j_2} \alpha_{j_1 j_2}^2 \sigma_{j_1} \right)^{-\frac{1}{2}} \quad (15a)$$

and

$$S_{j_a j_b}^{(x)} = \left\{ \frac{1}{2}(2J+1) \right\}^{\frac{1}{2}} \alpha_{j_a j_b} \sigma_{j_a} N \{ \delta_{x\pi} + (-)^T \delta_{x\nu} \}, \quad (15b)$$

where σ_k are the fractional occupancies of protons (neutrons) in the k shell.

We assume the particular cases of the 6^- states in ^{24}Mg and ^{28}Si to be described by an $f_{7/2}-(d_{5/2})^{-1}$ excitation from an (s-d) basis HF ground state, and for ^{24}Mg and ^{28}Si our HF calculations yield $\sigma_{5/2}$ values of 0.479 and 0.682 respectively. The 4^- ($T=1$) state in ^{28}Si is not as simple in that, while it should be dominated by an $f_{7/2}-(d_{3/2})^{-1}$ excitation (based upon a fractional occupancy $\sigma_{3/2}$ of 0.214), an $f_{7/2}-(d_{5/2})^{-1}$ component is to be expected. Whence, for the 4^- state, we have

$$N = (0.682 \alpha_{5/2 \ 7/2}^2 + 0.214 \alpha_{3/2 \ 7/2}^2)^{-\frac{1}{2}}$$

and

$$S_{j_7/2}^{(x)} = (1.45 \alpha_{5/2 \ 7/2} \delta_{j_5/2} + 0.454 \alpha_{3/2 \ 7/2} \delta_{j_3/2}) N (\delta_{x\pi} + (-)^T \delta_{x\nu}),$$

where

$$\alpha_{5/2 \ 7/2}^2 + \alpha_{3/2 \ 7/2}^2 = 1. \quad (16)$$

3. Results

We have analysed the inelastic scattering of 135 MeV protons leading to the 6^- states in ^{24}Mg and ^{28}Si , as well as to the 4^- state in ^{28}Si . This has been done using a fully antisymmetrized DWA (with target spectroscopy as described in Section 2) in which the single-particle states were those of a harmonic oscillator ($\hbar\omega = 13.5$ MeV). In previous analyses, Adams *et al.* (1977) did not specify the optical model parameters for the 135 MeV projectile, and so we used the values obtained by Horowitz (1972) from analyses of 100 MeV scattering and by Comparat (1975) from fits to the elastic scattering differential cross section from 165 MeV protons on ^{27}Al . The predictions for the inelastic scattering cross sections are not very sensitive to which parameter set was used, although the predictions of analyzing powers are.

For a projectile energy of 135 MeV, the reaction mechanism should be well described simply by the effective two-nucleon interaction that mediates the 'normal' first-order scattering processes. Previous analyses suggest that this can be well represented by the tensor force, given in equation (5), which has been deemed appropriate for light nuclei structure studies. In addition, a two-body spin-orbit force has been suggested by Eikemeier and Hackenbroich (1971) and there is some evidence, albeit scant, for its necessity in the prescription of the inelastic scattering effective interaction (Austin 1971; Lebrun *et al.* 1976). Thus we analysed the 135 MeV data both with and without a two-nucleon spin-orbit component in the effective interaction, with the E-H form being used.

Before analysing the ^{24}Mg and ^{28}Si data, however, we reanalysed the 61.2 MeV data of Scott *et al.* (1969) from ^{89}Y , since this 'isomeric' transition involves 'natural' parity states and high ($L=5$) multipoles of the effective interaction, and it can be described by a simple spectroscopy, namely a single proton outside of a closed ^{88}Sr core. Some core excitation must be involved to explain the empirical γ -ray decay rate, and earlier analyses by Amos and Geramb (1971) have shown that this explains the discrepancy observed at 40° scattering angle. Our main concern

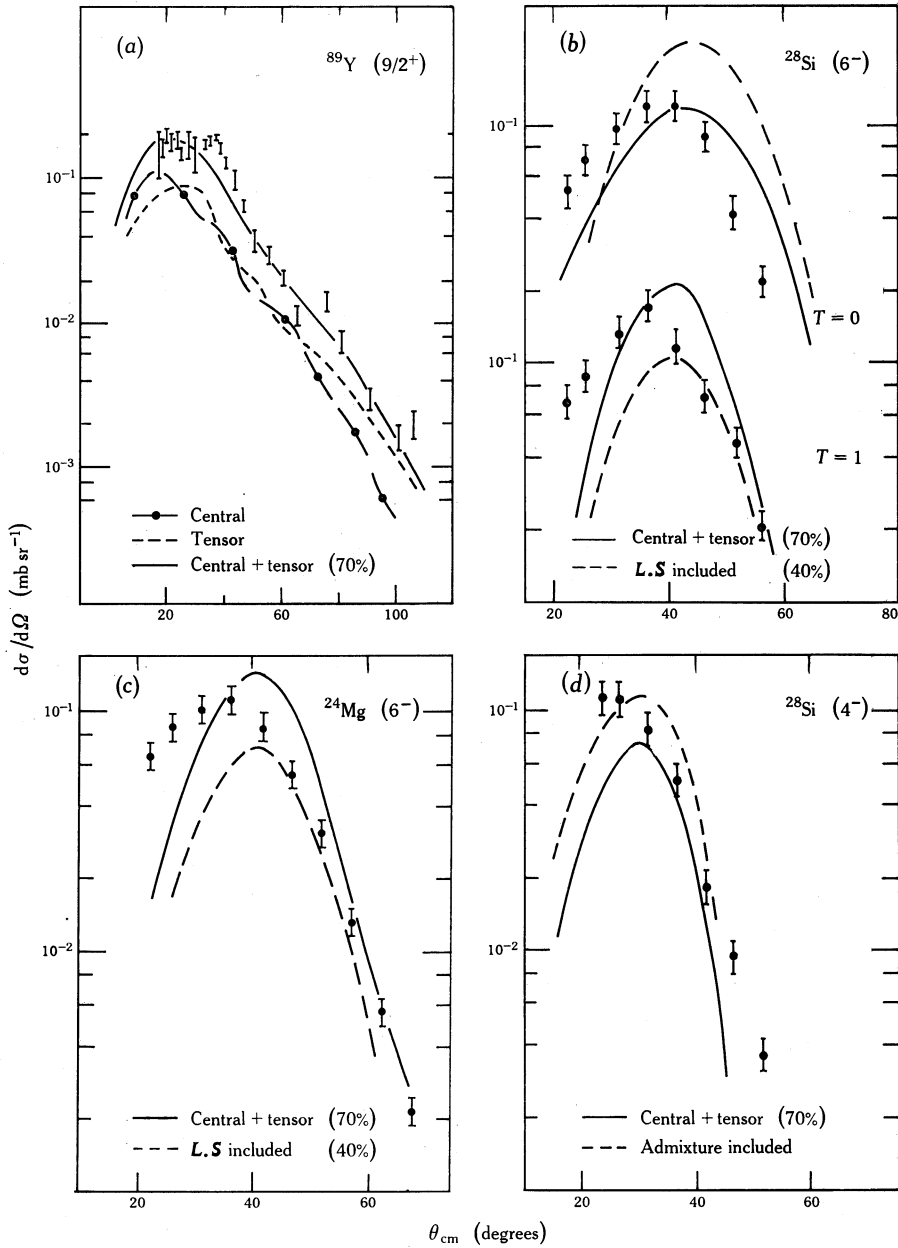


Fig. 1. Comparisons with experimental results of predicted angular distributions for inelastic proton scattering based on the effective interaction (equation 5):

(a) Scattering of 61.2 MeV protons from the $1/2^-$ to the $9/2^+$ (0.908 MeV) state in ^{89}Y . Experimental data are by Scott *et al.* (1969).

(b) Scattering of 135 MeV protons to the two 6^- states in ^{28}Si ($T = 0$, 11.6 MeV; $T = 1$, 14.4 MeV). Experimental data are by Adams *et al.* (1977).

(c) Scattering of 135 MeV protons to the 6^- ($T = 1$, 15.1 MeV) state in ^{24}Mg . Experimental data are by Adams *et al.* (1977).

(d) Scattering of 135 MeV protons to the 4^- ($T = 1$, 12.7 MeV) state in ^{28}Si . The curves allow for a pure $(d_{3/2})^{-1}-f_{7/2}$ excitation as well as a 25% $(d_{5/2})^{-1}-f_{7/2}$ admixture. Experimental data are by Adams *et al.* (1977).

is to show that our basic force is a reasonable one, and it is clearly seen to be so by the results given in Fig. 1a. Here, in contrast to earlier analyses, the tensor force contributions are significant, and they are comparable with those given by our central force. It could be expected then that some reduction in the tensor force strength might be necessary to accommodate an appropriate core polarization correction. However, the uncertainties relating to this prevent further analyses of these data from assisting us very much in our search for constraints upon the effective interaction prescription.

The results for the 6^- transition in ^{28}Si are shown in Fig. 1b, where they are compared with the data of Adams *et al.* (1977). Inclusion of a spin-orbit force (dashed curves) is certainly not favoured by this comparison (at least for the type used here), not only because of the need to reduce all strengths to 40% of the suggested values but also because the relative isospin state magnitudes are not predicted. As can be seen, the dashed curves respectively overestimate and underestimate the observed cross section for the $T = 0$ and 1 transitions. With no spin-orbit terms (solid curves) the transition data are seen to be quite well reproduced by a force that is 70% of that deemed appropriate in previous studies (see equation 5). However, no refinement of the present results should be made, since there are a number of as yet unresolved considerations. The first of these is: how good is the spectroscopy? The full force would yield the observed magnitudes if the $d_{5/2}$ fractional occupancy were changed from 0.682 to 0.333. A more deformed HF calculation would yield smaller values than 0.682, perhaps as little as 0.333. A second consideration, which influences the shape as well as the magnitude, is that, if the harmonic oscillator states were replaced by more realistic (e.g. Woods-Saxon) states then, particularly because the $f_{7/2}$ state would be weakly bound, the predicted cross sections should peak at smaller scattering angles. A third consideration is that the transitions are dominated by the tensor force components, with the exchange (knock-out) terms dominating the $T = 0$ excitation, while the direct terms dominate the $T = 1$ transition. Hence, changes not only in magnitude but also in the ranges of the even- and odd-state tensor force will influence results. Finally, we have assumed that isospin is a good quantum number. To test this we allowed up to 10% isospin mixing in each state, with the result that the magnitudes of the cross sections could vary by 10% (increasing or decreasing according to the admixture). This again is a direct result of the dominance of tensor force contributions, and we are unable to explain the data by a 'realistic' amount of isospin mixing in the residual nuclear states. Also, we cannot use the data to estimate any such isospin mixing. Such considerations, in the main, cannot be resolved by the current data, although the 6^- transition data from ^{24}Mg shown in Fig. 1c are quite consistent with our ^{28}Si results in that our central plus tensor force (solid curve) must be reduced to 70% of its initial value, while the 40% force (including $L \cdot S$) as shown by the dashed curve is too small. One can only assume that a 6^- $T = 0$ transition in ^{24}Mg would compare with an isovector excitation similar to that in ^{28}Si .

As a final search for consistency we analysed the 4^- $T = 1$ transition in ^{28}Si . Using the central plus tensor force at 70% of its assigned strength yields the solid curve shown in Fig. 1d, when a pure $(d_{3/2})^{-1}f_{7/2}$ excitation is assumed. The dashed curve gives the result one obtains when a 25% admixture of $(d_{5/2})^{-1}f_{7/2}$ excitation is allowed under the assumptions that $\alpha_{3/2\ 7/2}$ and $\alpha_{5/2\ 7/2}$ in equation (16) have values of 0.86 and 0.5 respectively.

4. Conclusions

The present analyses of transition to high spin unnatural parity states in ^{24}Mg and ^{28}Si (specifically to 6^- $T = 0$ and 1 states and to a 4^- $T = 1$ state) associated with the inelastic scattering of 135 MeV protons have shown that the general attributes of an effective two-nucleon interaction as used in other lower energy data analyses are quite reasonable. Specifically, the tensor force contributions are quite important, although the force used here seems to be too strong for high multipolarities. The role of an $L \cdot S$ force is inferred to be quite small, since use of a standard type gave incorrect relative magnitude predictions for the isovector and isoscalar 6^- transitions.

More data for this projectile energy and also from transitions to natural parity states whose spectroscopy can be described extremely well by a microscopic model (for example, the ^{20}Ne ground state bands) are needed. Such data should reveal the inadequacies of our effective interaction prescription and, it is to be hoped, should lead to a more pertinent specification.

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