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Isospin Mixing in Light Nuclei

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Abstract

Isospin mixing matrix elements are calculated for several pairs of mixed $T = 0$ and 1 states in ^{8}Be, ^{12}C and ^{16}O, using an extension of the method of Dalton and Robson (1966). This includes contributions due to differences between neutron and proton wavefunctions produced in the asymptotic region by the Coulomb force, as well as the internal Coulomb contribution. These cases provide no evidence for a charge-dependent nuclear interaction other than the Coulomb interaction.

1. Introduction

For some time there has been interest in the size of the charge-dependent Hamiltonian matrix elements that cause isospin mixing in states of certain light nuclei, in particular ^{8}Be, ^{12}C and ^{16}O. It seems that the Coulomb matrix elements calculated for shell model states are too small to account for the observed mixing. For example, the Coulomb matrix element coupling 2^+, $T = 0$ and 1 states of ^{8}Be, producing the states observed at 16·6 and 16·9 MeV, is calculated to be -67 keV (Barker 1966), whereas the observed properties of the states require a value of about -150 keV (Oothoudt and Garvey 1977). The situation is similar for pairs of 1^+ and 3^+ states of ^{8}Be (Barker 1966). For the 1^+ states of ^{12}C at 12·7 and 15·1 MeV, the calculated magnitude is given as 50 keV (Reisman *et al.* 1970) and observed magnitudes are (179 ± 75) keV (Lind *et al.* 1977) and (110 ± 30) keV (Adelberger *et al.* 1977). Such discrepancies have been cited as evidence for charge-dependent nuclear interactions in addition to the Coulomb interaction.

The calculated values quoted above were obtained from shell model states of the lowest configurations, with harmonic oscillator single-particle wavefunctions. Various attempts have been made to see if the discrepancies can be attributed to the inadequacies of these single-particle wavefunctions. Dalton and Robson (1966) used neutron and proton wavefunctions with the correct asymptotic form and fitted the observed splitting of the 2^+ levels of ^{8}Be, but they used the cluster model of Marion (1965), which includes only the channels involving ^{7}Li and ^{7}Be ground states, whereas shell model calculations give appreciable parentage in higher $A = 7$ states. Barker (1966) treated the contributions due to the different asymptotic forms of the corresponding neutron and proton wavefunctions in the ^{8}Be cases as perturbations, giving rise to level shifts of the Thomas–Ehrman type, and found them to have a small net effect when all nucleon channels were included. It was assumed, however, that the single-particle dimensionless reduced width θ_0^2 was the same for all channels, which is not

in agreement with results from realistic models. In a quite different approach, Anderson and Goldhammer (1971) and Anderson *et al.* (1972) used correlated wavefunctions derived from realistic potentials to obtain larger matrix elements in the ^{8}Be cases, but this work was criticized by McCarthy and Walker (1974) and by Bertsch and Shlomo (1974). After further calculations, Goldhammer (1975) obtained matrix elements for ^{8}Be agreeing closely with experimental values.

For the ^{12}C case, Braithwaite *et al.* (1972) and Wagner (1977) gave various expressions for the mixing matrix element in terms of energy differences of levels of $A = 11$ and 12 nuclei, by assuming that the 1^+ states belong to the configuration $(1p_{1/2})(1p_{3/2})^{-1}$ relative to the closed shell $(1p_{3/2})^8$ configuration of the ^{12}C ground state. Values of the matrix element obtained from these expressions were -240 keV (Braithwaite *et al.*) and 0, -131 and -150 keV (Wagner), the different values resulting from different approximations. Charge-dependent interactions other than the Coulomb interaction may, however, be contributing to these values. Also the assumed shell model descriptions of the states are not very realistic. The calculations of Cohen and Kurath (1965) give less than 40% of ^{12}C ground state belonging to the $(1p_{3/2})^8$ configuration and less than 70% of the lowest 1^+, $T = 0$ and 1 states belonging to the $(1p_{3/2})^7(1p_{1/2})$ configuration; also these 1^+ states contain only about 50% of the states formed by a $(1p_{1/2})(1p_{3/2})^{-1}$ excitation from the actual ground state.

There is better justification for the $(1d_{5/2})(1p_{1/2})^{-1}$ configuration assumed by Wagner (1977) to explain the isospin mixing of the 2^- states of ^{16}O at 12·53 and 12·97 MeV. His expressions in terms of energy differences of $A = 15$ and 16 levels give a matrix element of about 140 keV, which is comparable with his experimental lower limit. In cases such as this, involving particle–hole states of non-normal parity, one can also increase greatly the shell model value of the mixing matrix element by using single-particle wavefunctions for a finite depth potential rather than for an oscillator potential (Barker 1961); this is not the case for states of normal parity such as the ^{8}Be and ^{12}C cases mentioned above (Barker 1966). The present paper discusses the isospin mixing in these and other cases in which it is reasonable to assume a two-state mixing model, using an extension of the Dalton and Robson (1966) approach that includes contributions from many channels.

2. Formulae for Isospin Mixing

In the conventional treatment of the isospin mixing of two shell model states (Barker 1966, 1975) the eigenstates Ψ_λ (with $\lambda = a$ or b) of the total Hamiltonian H, where

$$H\Psi_\lambda = E_\lambda \Psi_\lambda, \tag{1}$$

are written

$$\Psi_\lambda = \sum_T A_{\lambda T} \Psi_T \tag{2}$$

in terms of the orthonormal basis states Ψ_T of good isospin T ($T = 0$ or 1, say). These are eigenstates of the charge-independent part H^0 of the Hamiltonian, where

$$H^0\Psi_T = E_T^0 \Psi_T, \tag{3}$$

with

$$\langle \Psi_{T'} | \Psi_T \rangle = \delta_{T'T}. \tag{4}$$

Then we have

$$\sum_T \{(E_T^0 - E_\lambda)\delta_{T'T} + \langle \Psi_{T'} | H^c | \Psi_T \rangle\} A_{\lambda T} = 0, \tag{5}$$

where the charge-dependent interaction H^c is given by

$$H = H^0 + H^c. \tag{6}$$

The integrations are over all space, and H^c is assumed to be hermitian, resulting in orthogonal states Ψ_λ, so that equation (2) can be written in the usual form (Barker 1966)

$$\Psi_a = \alpha\Psi_0 + \beta\Psi_1, \qquad \Psi_b = \beta\Psi_0 - \alpha\Psi_1, \tag{7}$$

where $\alpha^2 + \beta^2 = 1$ and we take $E_a < E_b$. The mixing matrix element is

$$H_{01}^c = \langle \Psi_0 | H^c | \Psi_1 \rangle = \langle \Psi_1 | H^c | \Psi_0 \rangle. \tag{8}$$

In the above treatment the Ψ_T, and therefore the Ψ_λ, are expressed in terms of single-particle wavefunctions, such as harmonic oscillator functions, which make no allowance for the presence of nearby thresholds of nucleon channels and for the consequent difference in asymptotic forms of neutron and proton wavefunctions. In the cases we consider, the levels are close to thresholds of nucleon channels for which they have appreciable spectroscopic factors.

Dalton and Robson (1966) showed how thresholds of the above type may be taken into account. They used a particular model for the 2^+ and 1^+ levels of ^{8}Be, which included contributions from only a single nucleon channel. We generalize this to take account of many contributing channels. The states Ψ_λ and Ψ_T are defined by equations (1) and (3) over a restricted region of space (the internal region of R-matrix theory (Lane and Thomas 1958)), and they are made to join smoothly onto the correct asymptotic wavefunctions at the surface of the internal region by imposing the boundary conditions (Bloch 1957; Lane and Robson 1966)

$$\mathscr{L}(S_\lambda)\Psi_\lambda = 0 \tag{9}$$

and

$$\mathscr{L}(\bar{S}_T)\Psi_T = 0, \tag{10}$$

where the Bloch operator is defined by

$$\mathscr{L}(S) = \sum_c |c) \frac{\hbar^2}{2m_c} \delta(r_c - a_c) \left(\frac{\partial}{\partial r_c} - \frac{S(c)-1}{a_c} \right) (c|. \tag{11}$$

Here $|c)$ is a channel wavefunction, which is a function of intrinsic and angular variables but not r_c, while $S_\lambda(c)$ is the logarithmic derivative of the appropriate external radial wavefunction in the channel c evaluated at the channel radius $r_c = a_c$ and at the channel energy $E_{\lambda c} = E_\lambda - E_{\mathrm{th},c}$, where $E_{\mathrm{th},c}$ is the threshold energy of the channel c. $\bar{S}_T(c)$ is similar except that for mirror neutron and proton channels it is evaluated at the average channel energy $\frac{1}{2}(E_{T\mathrm{n}} + E_{T\mathrm{p}})$ and for the average charges of the neutron and proton channels. The essential point is that $\bar{S}_T(\mathrm{n}) = \bar{S}_T(\mathrm{p})$.

Because $H+\mathscr{L}(S_\lambda)$ is realizable over the internal region (Lane and Robson 1966), one can write

$$\begin{aligned} 0 &= \{H+\mathscr{L}(S_\lambda)-E_\lambda\}\Psi_\lambda \\ &= \sum_T A_{\lambda T}\{H+\mathscr{L}(S_\lambda)-E_\lambda\}\Psi_T \\ &= \sum_T A_{\lambda T}\{E_T^0+H^c+\mathscr{L}(S_\lambda)-\mathscr{L}(\bar{S}_T)-E_\lambda\}\Psi_T, \end{aligned}$$

giving

$$\sum_T \{(E_T^0-E_\lambda)\delta_{T'T}+\langle\Psi_{T'}|H^c+\mathscr{L}(S_\lambda)-\mathscr{L}(\bar{S}_T)|\Psi_T\rangle\}A_{\lambda T}=0. \tag{12}$$

Integration over the internal region only is now implied for matrix elements included within angular brackets, including those in equations (4), (8) and (12). Equation (12) is similar to (5), but the mixing matrix element now depends on λ, so that the states Ψ_a and Ψ_b are not orthogonal over the internal region and cannot be written in the form (7). Instead we may write

$$\Psi_a=\alpha\Psi_0+\beta\Psi_1, \qquad \alpha^2+\beta^2=1, \tag{13a}$$

$$\Psi_b=\beta'\Psi_0-\alpha'\Psi_1, \qquad \alpha'^2+\beta'^2=1, \tag{13b}$$

with $\alpha'\neq\alpha$, $\beta'\neq\beta$.

The mixing matrix elements have contributions from the internal region and from the surface, and may be written

$$V_{01}^\lambda=H_{01}^c+L_{01}^\lambda. \tag{14}$$

The surface contribution is

$$\begin{aligned} L_{01}^\lambda &= \langle\Psi_0|\mathscr{L}(S_\lambda)-\mathscr{L}(\bar{S}_1)|\Psi_1\rangle \\ &= \sum_c \langle\Psi_0|c\rangle(\hbar^2/2m_c a_c)\delta(r_c-a_c)\{\bar{S}_1(c)-S_\lambda(c)\}\langle c|\Psi_1\rangle. \end{aligned} \tag{15}$$

We include contributions from nucleon channels only, and write $(c)\equiv(\tilde{c},m_t)$, where $m_t=+\frac{1}{2}$ for neutron channels and $-\frac{1}{2}$ for proton channels, while $\tilde{c}$ includes all other channel labels. Then we have

$$\langle c|\Psi_T\rangle = u_{T\tilde{c}}(r_c)\,\mathscr{S}^{\frac{1}{2}}_{T\tilde{c}}(\tilde{T}\tfrac{1}{2}-m_t m_t|T\,0), \tag{16}$$

where $u_{T\tilde{c}}(r)/r$ is the normalized radial wavefunction

$$\int_0^{a_{\tilde{c}}} u^2_{T\tilde{c}}(r)\,\mathrm{d}r=1, \tag{17}$$

and $\mathscr{S}^{\frac{1}{2}}_{T\tilde{c}}$ is the spectroscopic amplitude, excluding the isospin Clebsch–Gordan coefficient, which is written explicitly in equation (16). $\tilde{T}$ is the isospin of the residual nucleus in the channel $\tilde{c}$, which must be $\frac{1}{2}$ in the present case. Equation (15) then becomes

$$L_{01}^\lambda=\sum_{\tilde{c}}(\hbar^2/4m_{\tilde{c}}a_{\tilde{c}})u_{0\tilde{c}}(a_{\tilde{c}})\,u_{1\tilde{c}}(a_{\tilde{c}})\,\mathscr{S}^{\frac{1}{2}}_{0\tilde{c}}\,\mathscr{S}^{\frac{1}{2}}_{1\tilde{c}}\{S_\lambda(\tilde{c},\tfrac{1}{2})-S_\lambda(\tilde{c},-\tfrac{1}{2})\}. \tag{18}$$

In order to obtain numerical values of L_{01}^{λ} for given channel radii $a_{\tilde{c}}$, we take the $u_{T\tilde{c}}(r)$ as wavefunctions at the channel energy $E_{T\tilde{c}} \equiv \frac{1}{2}(E_{Tn}+E_{Tp})$ in a Woods–Saxon potential with the depth adjusted to make the logarithmic derivative equal to $\bar{S}_T(\tilde{c})$ at $r = a_{\tilde{c}}$. The $a_{\tilde{c}}$ must be chosen so that there is no polarizing interaction for $r > a_{\tilde{c}}$; then the external radial wavefunctions are Coulomb functions and the $S_\lambda(\tilde{c}, m_t)$ are the shift factors of R-matrix theory (Lane and Thomas 1958).† The $\mathscr{S}_{T\tilde{c}}^{\frac{1}{2}}$ are obtained from shell model calculations.

We calculate H_{01}^{c}, the internal contribution to the mixing matrix element, from equation (8) by assuming that H^c is the two-body Coulomb interaction coupling pairs of protons, that Ψ_T is a shell model state with harmonic oscillator single-particle wavefunctions, and that the integration is extended over all space. These approximations should be accurate provided that the channel radii $a_{\tilde{c}}$ can be chosen so that on the one hand they are sufficiently small that the harmonic oscillator wavefunctions adequately represent the true wavefunctions for $r < a_{\tilde{c}}$, and on the other hand they are sufficiently large that the oscillator wavefunctions are small in the additional region of integration. Agreement between calculated and experimental values of V_{01}^{λ} can therefore be expected only for a small range of $a_{\tilde{c}}$ values. (Note that the J^π dependence of most quantities introduced in this section has not been made explicit.)

3. Application to Special Cases

(*a*) 2^+ *Levels of* ^{8}*Be at 16·63 and 16·91 MeV*

The shell model calculations of Barker (1966), with single-particle wavefunctions in a harmonic oscillator potential, gave $H_{01}^{c} = -67$ keV. The oscillator length parameter $b = 1{\cdot}69$ fm was obtained by fitting the observed energy difference of the ground states of ^{8}Li and ^{8}B as a Coulomb energy difference, neglecting any contributions from level shifts of the Thomas–Ehrman type. Since such level shifts are closely related to the surface contributions to the mixing matrix element that we are considering, this is not a consistent way of obtaining b for the present purpose. In subsection (*d*) below, we consider these and other level shifts for levels of ^{8}Li, ^{8}Be and ^{8}B and obtain the best overall fit for $b \approx 1{\cdot}60$ fm. The usual alternative method of determining b in light nuclei from elastic electron scattering is not available for $A = 8$ nuclei. For neighbouring nuclei, electron scattering gives values of b from about $1{\cdot}7$ to $1{\cdot}90$ fm for ^{7}Li (Ajzenberg–Selove and Lauritsen 1974) and $1{\cdot}61$ to $1{\cdot}76$ fm for ^{9}Be (Lapikas *et al.* 1975). Energy differences of mirror levels, neglecting level shifts, give $b \approx 1{\cdot}64$ fm for ^{7}Li–^{7}Be ground states, $1{\cdot}78$ fm for ^{9}Be–^{9}B ground states and $1{\cdot}61$ fm for the lowest $T = 3/2$ levels of ^{9}Be and ^{9}B. The level shift contributions, which would tend to reduce the value of b, are expected to be least in the last case. As a reasonable compromise value, we assume $b = 1{\cdot}65$ fm for $A = 8$ nuclei, giving $H_{01}^{c} = -68$ keV, and note that a 10% change in b would change H_{01}^{c} by only 7 keV, since $H_{01}^{c} \propto b^{-1}$.

We include contributions to L_{01}^{λ} given by equation (18) from channels $\tilde{c}$ corresponding to the levels of ^{7}Li and ^{7}Be with assigned spin and parity (Ajzenberg–Selove and

† This is correct if there is at most one open channel at the energy of the level λ, as happens for several of the cases discussed here. If there is more than one open channel then the $S_\lambda(\tilde{c}, m_t)$ for an open channel becomes complex. We approximate by neglecting the part proportional to the penetration factor in the channel $(\tilde{c}, m_t)$, which is small for the levels considered here; then $S_\lambda(\tilde{c}, m_t)$ becomes real and is again equal to the shift factor.

Lauritsen 1974), and thus we write $\tilde{c} \equiv \tilde{J}s$, where $\tilde{J}$ is the spin of the $\tilde{T} = \frac{1}{2}$, $A = 7$ level (with asterisks denoting higher states of the same $\tilde{J}$) and s is the channel spin. All nucleons are p wave. As the excitation energy of the $A = 7$ level increases, the corresponding contribution to L_{01}^{λ} tends to decrease, because both $u_{Tc}(a_{\tilde{c}})$ and $S_{\lambda}(\tilde{c}, \frac{1}{2}) - S_{\lambda}(\tilde{c}, -\frac{1}{2})$ decrease, and $\mathscr{S}_{\tilde{T}\tilde{c}}^{\frac{1}{2}}$ tends to decrease. Thus contributions from higher unidentified levels should be negligible.

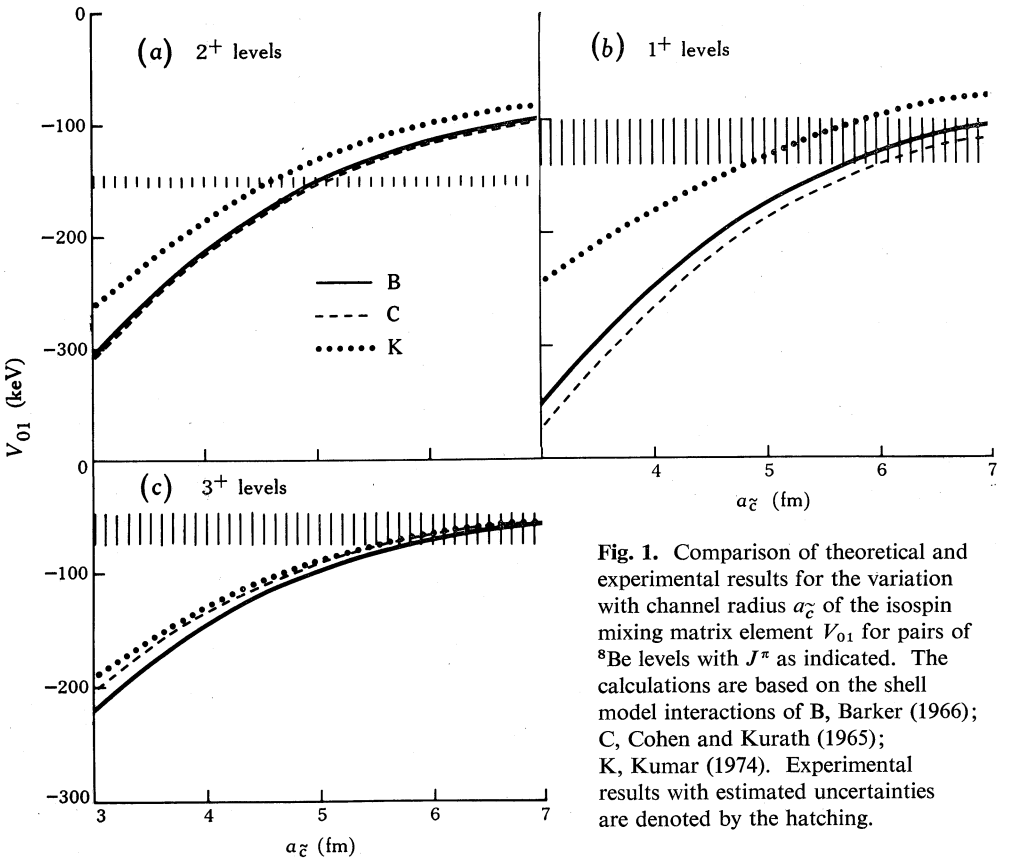

Fig. 1. Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the isospin mixing matrix element V_{01} for pairs of ^{8}Be levels with J^{π} as indicated. The calculations are based on the shell model interactions of B, Barker (1966); C, Cohen and Kurath (1965); K, Kumar (1974). Experimental results with estimated uncertainties are denoted by the hatching.

Because the separation of the 16·63 and 16·91 MeV levels of ^{8}Be is small, we assume them to be degenerate to the extent that $u_{0\tilde{c}} = u_{1\tilde{c}}$ and

$$S_a(\tilde{c}, \tfrac{1}{2}) - S_a(\tilde{c}, -\tfrac{1}{2}) = S_b(\tilde{c}, \tfrac{1}{2}) - S_b(\tilde{c}, -\tfrac{1}{2}).$$

Then we have $L_{01}^a = L_{01}^b = L_{01}$ say, and Ψ_a and Ψ_b are orthogonal and are described by equation (7). Table 1 shows the contributions to

$$L_{01} = \sum_{\tilde{c}} L_{01}(\tilde{c})$$

coming from the different channels $\tilde{c}$, for different values of the channel radii $a_{\tilde{c}}$ (assumed the same for all channels) and for the shell model interaction of Barker

(1966). We note the sum rule

$$\sum_{\tilde{c}} \mathscr{S}^{\frac{1}{2}}_{T'\tilde{c}} \mathscr{S}^{\frac{1}{2}}_{T\tilde{c}} = 4\delta_{T'T}, \tag{19}$$

where the sum is over all p-wave nucleon channels. The wavefunctions $u_{T\tilde{c}}(r)$ are taken as 1p wavefunctions in a Woods–Saxon potential with the standard parameter values $r_0 = 1{\cdot}25$ fm and $a = 0{\cdot}65$ fm, and a uniform charge distribution with $r_{0c} = 1{\cdot}25$ fm. The resultant value of V_{01} is shown by the solid curve in Fig. 1*a* as a function of channel radius.

Table 1. Contributions to L_{01} for 2^+ levels of ^{8}Be

The calculations use the shell model wavefunctions of Barker (1966), and are for various values of the channel radius $a_{\tilde{c}}$

$a_{\tilde{c}}$ (fm)	$\tilde{J}$	$\sum_s \mathscr{S}^{\frac{1}{2}}_{0\tilde{J}s} \mathscr{S}^{\frac{1}{2}}_{1\tilde{J}s}$	$\frac{\hbar^2}{4m_{\tilde{c}} a_{\tilde{c}}} u^2_{\tilde{c}}(a_{\tilde{c}})$ (keV)	$S(\tilde{c}, \frac{1}{2}) - S(\tilde{c}, -\frac{1}{2})$	$L_{01}(\tilde{c})$ (keV)
4·0	3/2	0·946	702	−0·203	−135
4·0	1/2	0·273	661	−0·170	−31
4·0	7/2	0·008	419	−0·099	0
4·0	5/2	−0·062	350	−0·100	2
4·0	5/2*	−0·788	328	−0·0576	15
4·0	7/2*	−0·292	277	−0·0424	3
4·0	3/2*	−0·034	263	−0·0400	0
				Total:	−146
5·0	3/2		243	−0·322	−74
5·0	1/2		219	−0·272	−16
5·0	7/2		103	−0·159	0
5·0	5/2		75·4	−0·156	1
5·0	5/2*		68·4	−0·101	5
5·0	7/2*		52·2	−0·0792	1
5·0	3/2*		47·4	−0·0755	0
				Total:	−83
6·0	3/2		96·4	−0·450	−41
6·0	1/2		83·4	−0·380	−9
6·0	7/2		27·4	−0·222	0
6·0	5/2		17·8	−0·215	0
6·0	5/2*		15·2	−0·146	2
6·0	7/2*		9·9	−0·117	0
6·0	3/2*		8·9	−0·0945	0
				Total:	−48

To indicate the dependence on the shell model interaction, Fig. 1*a* also shows values of V_{01} for two other interactions, the (6–16)2BME interaction of Cohen and Kurath (1965) and the interaction of Kumar (1974). It should be noted that these three interactions do not all give equally good fits to the energies and other properties of the relevant $A = 7$ and 8 levels (see Kumar 1974).

Observed properties of these 2^+ levels of ^{8}Be require values of V_{01} close to −150 keV (Barker 1966; Oothoudt and Garvey 1977). This is shown in Fig. 1*a*, with the hatching indicating the estimated uncertainty. It is seen that such values

are given by the calculations for $a_{\tilde{c}} \approx 5$ fm, which is a reasonable value (the conventional value of the channel radius for the channels involved is $1{\cdot}45(7^{1/3}+1^{1/3}) = 4{\cdot}22$ fm).

In the calculations for these 2^+ levels made by Dalton and Robson (1966), the model of Marion (1965) was used. Essentially this corresponds to the calculation of L_{01}^{λ} from equation (18) with only the lowest channel ($\tilde{J} = 3/2$) included, and this with

$$\sum_s \mathscr{S}^{1/2}_{0\,3/2\,s}\mathscr{S}^{1/2}_{1\,3/2\,s} = 1\,.$$

The contribution H_{01}^{c} was neglected. This resulted in a smaller value of $a_{\tilde{c}} = 3{\cdot}6$ fm being required to fit the experimental splitting.

The results of Barker (1966) can be obtained by making the approximations that the contributions from the different boundary conditions in equation (12) can be treated as perturbations, so that they produce an independent energy shift for each of the levels a and b, and that the value of $u_{T\tilde{c}}(a_{\tilde{c}})$ is the same for each channel $\tilde{c}$. The effect of the latter approximation is to increase the relative contributions of the higher channels, thus increasing the cancellation and reducing the net surface contribution to the mixing matrix element.

If the values of Goldhammer (1975) were used for H_{01}^{c}, much larger values of $a_{\tilde{c}}$ would be required in order to make the surface contributions small. The same remark applies to the two cases described in the following subsections (*b*) and (*c*).

(*b*) 1^+ *Levels of* ^{8}*Be at 17·64 and 18·15 MeV*

Considerations similar to those of subsection (*a*) give for the 1^+ levels of ^{8}Be the values of V_{01} shown in Fig. 1*b*. The appreciable differences in the calculated values for the three interactions appear to be due mainly to the different amounts of the [31] ^{33}P and [31] ^{31}P components in the 1^+, $T = 1$ state, and these are closely connected with the channel spin ratio for the 17·64 MeV level (Barker 1966). To the extent that the Kumar interaction best fits the observed channel spin ratio (Kumar 1974), it may be appropriate to prefer the Kumar values of V_{01}, for which agreement with the experimental value of about -120 keV (Barker 1966; Oothoudt and Garvey 1977) is obtained for $a_{\tilde{c}} \approx 5$–6 fm.

Dalton and Robson (1966) also considered this case, using Marion's (1965) model in which only the first excited state channel ($\tilde{J} = 1/2$) is included with

$$\sum_s \mathscr{S}^{\frac{1}{2}}_{0\,\frac{1}{2}s}\mathscr{S}^{\frac{1}{2}}_{1\,\frac{1}{2}s} = 1$$

(in comparison, shell model values of this quantity are 0·68 (Barker 1966), 0·69 (Cohen and Kurath 1965) and 0·59 (Kumar 1974)). With a channel radius of 3·6 fm, as obtained from the 2^+ levels, Dalton and Robson obtained $|V_{01}| = 185$ keV, which is much bigger than the observed value.

(*c*) 3^+ *Levels of* ^{8}*Be at 19·06 and 19·22 MeV*

Values of V_{01} calculated similarly to the above for the 3^+ levels are shown in Fig. 1*c*. Here there is very little difference between the different interactions. The experimental value of V_{01} has been given as -63 keV (Barker 1966) but it has an appreciable uncertainty (Oothoudt and Garvey 1977). A channel radius value $a_{\tilde{c}} \gtrsim 6$ fm is suggested.

(d) Relative Energies of 2^+, 1^+ and 3^+ Levels of ^{8}Li, ^{8}Be and ^{8}B

The 2^+, 1^+ and 3^+, $T = 1$ states of ^{8}Be that we have considered are the analogues of the lowest three states of ^{8}Li and ^{8}B, and a model essentially the same as that described in Section 2 may be used to calculate the differences in energies of these states. Of particular interest is the observed difference of ~ 200 keV in the excitation energies of the first excited states of ^{8}Li and ^{8}B (Ajzenberg-Selove and Lauritsen 1974). Since only $T = 1$ levels are involved, there is a one-level approximation for each J^π value, and the contributions to the relative energies come from Coulomb energy differences and from Thomas–Ehrman shifts.

With notation similar to that of equations (14) and (18), except that the level label λ is omitted and the dependence on J^π and M_T is shown explicitly, where $M_T = +1$, 0 and -1 for ^{8}Li, ^{8}Be and ^{8}B respectively, the observed level energies may be written

$$E(J^\pi, M_T) = E_1^0(J^\pi) + H_{11}^c(J^\pi, M_T) + L_{11}(J^\pi, M_T), \tag{20a}$$

$$L_{11}(J^\pi, M_T) = -\sum_{\tilde{c}} (\hbar^2/2m_{\tilde{c}}\, a_{\tilde{c}})\, u_{1\tilde{c}}^2(J^\pi, a_{\tilde{c}})\, \mathscr{S}_{1\tilde{c}}(J^\pi)\, \{S(J^\pi, M_T, \tilde{c}) - \bar{S}_1(J^\pi, \tilde{c})\}. \tag{20b}$$

Contributions come from the $\tilde{T} = 1/2$ and $\tilde{T} = 3/2$ channels. For ^{8}Be, both ^{7}Li+p and ^{7}Be+n channels contribute and $S(J^\pi, 0, \tilde{c})$ is the average of the shift factors for the proton and neutron channels. For ^{8}Li, the $\tilde{T} = 1/2$ channels are purely ^{7}Li+n, while the $\tilde{T} = 3/2$ channels include both ^{7}He+p and ^{7}Li+n, and $S(J^\pi, +1, \tilde{c})$ is a weighted average of the shift factors for the proton and neutron channels. Similar remarks apply for ^{8}B. Since the dependence of L_{11} on M_T comes only through $S(J^\pi, M_T, \tilde{c})$, the value of $\bar{S}_1(J^\pi, \tilde{c})$ is irrelevant in the energy differences

$$\Delta(J^\pi, M_T, M_T') = E(J^\pi, M_T) - E(J^\pi, M_T'). \tag{21}$$

Since the level energies are better known in ^{8}Li than in ^{8}Be or ^{8}B, we consider only values of $\Delta(J^\pi, M_T, +1)$ for $M_T = 0$ and -1, and write these as $\Delta(J^\pi, M_T)$. Also for $J^\pi = 1^+$ and 3^+, we use differences of excitation energies rather than of absolute energies:

$$\Delta_x(J^\pi, M_T) = \Delta(J^\pi, M_T) - \Delta(2^+, M_T). \tag{22}$$

Thus we compare calculated and expermental values of $\Delta(2^+, M_T)$, $\Delta_x(1^+, M_T)$ and $\Delta_x(3^+, M_T)$ for both $M_T = 0$ and -1. The same value of $a_{\tilde{c}}$ is assumed for all J^π, although in principle this is not necessary.

We again include contributions from channels $\tilde{c}$ corresponding to identified $A = 7$ levels. Unidentified levels are simulated by a fictitious level (labelled J in Table 2) with spectroscopic factors that exhaust the sum rule (19) and situated 14 MeV above the lowest $\tilde{T} = 1/2$ level (approximately the weighted mean energy of such levels from shell model calculations). To indicate the relative importance of contributions from different channels, Table 2 shows these for the particular case of the Barker (1966) interaction, with $a_{\tilde{c}} = 5{\cdot}0$ fm and $b = 1{\cdot}65$ fm. We have written

$$L_{11}(J^\pi, M_T) - L_{11}(J^\pi, +1) = \sum_{\tilde{c}} L(J^\pi, M_T, \tilde{c}),$$

$$H_{11}^c(J^\pi, M_T) - H_{11}^c(J^\pi, +1) = \Delta H^c(J^\pi, M_T).$$

Calculated values of $\Delta(2^+, M_T)$, $\Delta_x(1^+, M_T)$ and $\Delta_x(3^+, M_T)$ are shown in Fig. 2 as

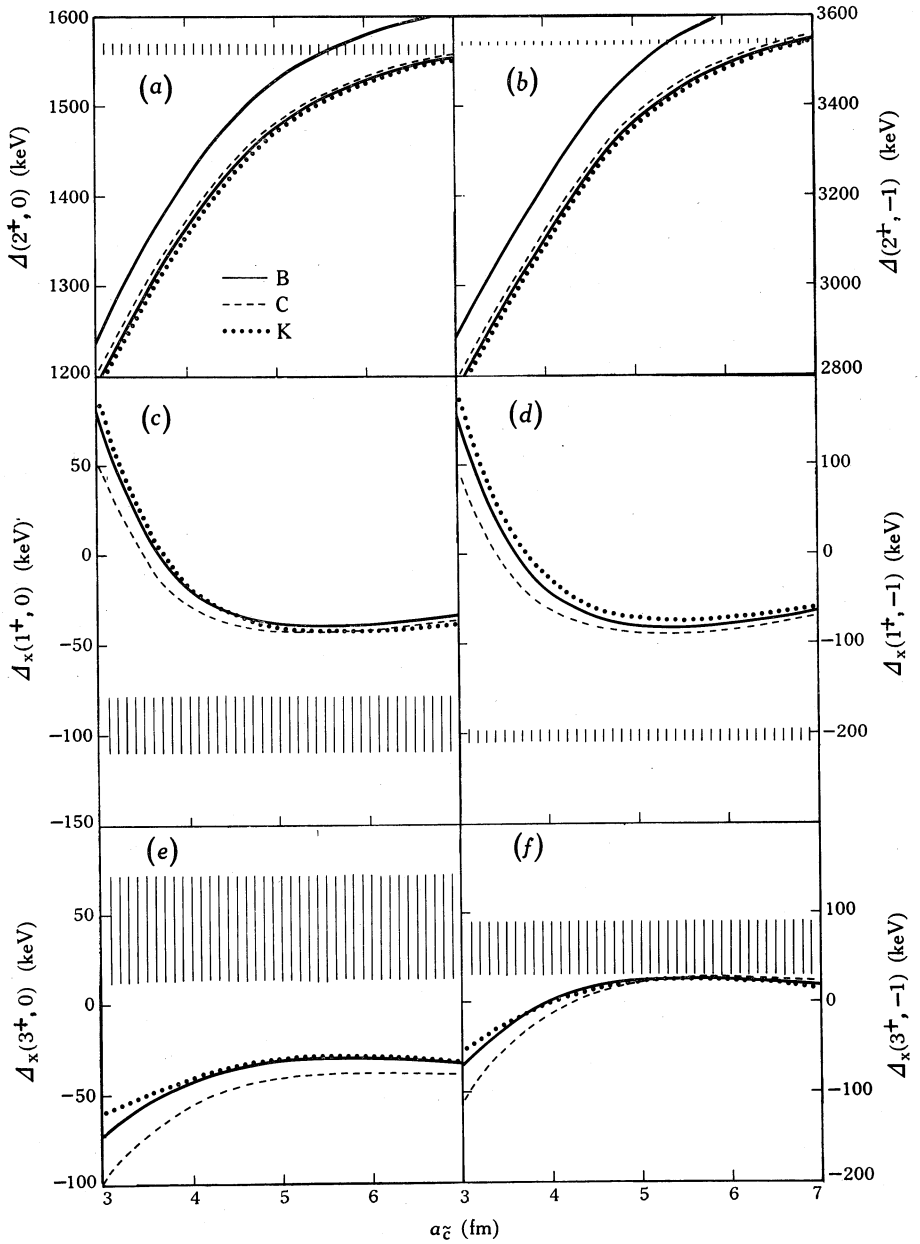

Fig. 2. Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the energy difference $\Delta(J^\pi, M_T)$ and the excitation energy difference $\Delta_x(J^\pi, M_T)$ for the indicated values of J^π and M_T. Left-hand boxes show ^{8}Be$-^8$Li differences and right-hand boxes show ^{8}B$-^8$Li differences. The calculations are based on shell model interactions of B, Barker (1966); C, Cohen and Kurath (1965); K, Kumar (1974); and use $b = 1{\cdot}65$ fm, except that the uppermost solid curves in (*a*) and (*b*) are for $b = 1{\cdot}60$ fm. Experimental results with estimated uncertainties are denoted by the hatching.

functions of the channel radius for the same three interactions as before. All the curves are for $b = 1{\cdot}65$ fm, except that the uppermost solid curves for $\Delta(2^+, M_T)$ in Figs 2*a* and 2*b* are for $b = 1{\cdot}60$ fm. Values of $\Delta_x(J^\pi, M_T)$ for $b = 1{\cdot}60$ fm are virtually indistinguishable from those shown.

Table 2. Contributions to energy differences of ^{8}Li, ^{8}Be and ^{8}B levels

The calculations use the shell model wavefunctions of Barker (1966) and are for $a_{\tilde{c}} = 5{\cdot}0$ fm and $b = 1{\cdot}65$ fm. The $M_T = 0$ and -1 values refer to ^{8}Be–^{8}Li and ^{8}B–^{8}Li energy differences respectively. Both L and ΔH^c are given in keV

J^π	$\tilde{j}$	$\sum_s \mathscr{S}_{1\tilde{j}s}(J^\pi)$	$L(J^\pi, M_T, \tilde{c})$		$\Delta H^c(J^\pi, M_T)$	
			$M_T = 0$	$M_T = -1$	$M_T = 0$	$M_T = -1$
2^+	3/2	1·016	−73	−180		
2^+	1/2	0·231	−14	−34		
2^+	7/2	0·224	−4	−9		
2^+	5/2	0·032	0	−1		
2^+	5/2*	0·703	−8	−18		
2^+	7/2*	0·256	−2	−5		
2^+	3/2*	0·060	0	−1		
2^+	3/2($T = 3/2$)	1·059	−6	−14		
2^+	J	0·419	−1	−3		
		Total:	−108	−265	1574	3625
1^+	3/2	0·454	−54	−132		
1^+	1/2	0·858	−68	−176		
1^+	5/2	0·271	−2	−5		
1^+	5/2*	0·302	−3	−8		
1^+	3/2*	0·173	−1	−3		
1^+	3/2($T = 3/2$)	0·930	−5	−12		
1^+	J	1·012	−2	−6		
		Total:	−135	−342	1586	3628
3^+	3/2	0·300	−35	−86		
3^+	7/2	1·143	−37	−89		
3^+	5/2	0·155	−2	−6		
3^+	5/2*	0·646	−12	−29		
3^+	7/2*	0·241	−3	−7		
3^+	3/2*	0·087	−1	−2		
3^+	3/2($T = 3/2$)	1·007	−9	−21		
3^+	J	0·421	−2	−4		
		Total:	−101	−244	1546	3632

The experimental values shown in Fig. 2 are obtained from the level energies given by Ajzenberg-Selove and Lauritsen (1974), allowance being made for isospin mixing in the ^{8}Be levels as in Barker (1966). For the values of the channel radius $a_{\tilde{c}} \approx$ 5–6 fm favoured for the cases considered in subsections (*a*)–(*c*) above, it is seen that there is better agreement for $\Delta(2^+, M_T)$ with $b = 1{\cdot}60$ fm rather than $b = 1{\cdot}65$ fm. Also there is best agreement in the values of $\Delta_x(J^\pi, M_T)$ for $a_{\tilde{c}} \approx$ 5–6 fm, although exact agreement is not obtained for any value of $a_{\tilde{c}}$; in particular, the calculations predict only one half of the observed difference in the excitation energies of the first excited states of ^{8}Li and ^{8}B. This could be taken as evidence for a charge-dependent interaction other than the Coulomb interaction, or it may merely indicate the inaccuracy of some of the approximations made in the perturbation treatment, such as the assumption that $u_{1\tilde{c}}$ is independent of M_T.

(e) 2^- Levels of ^{8}Be

The case of the 2^- levels of ^{8}Be has been considered separately by Barker (1977). The Dalton and Robson (1966) model was used to discuss the isospin mixing, except that the contribution H^c_{01} was calculated on a one-body model with $H^c = \frac{1}{2}\{V_c(r) - \Delta_c\}$, where $V_c(r)$ is the one-body Coulomb interaction and Δ_c is the Coulomb displacement energy. In this case the surface contribution is crucial, because the lower 2^- level is observed to have a neutron reduced width larger than the proton reduced width, contrary to the situation in all other cases.

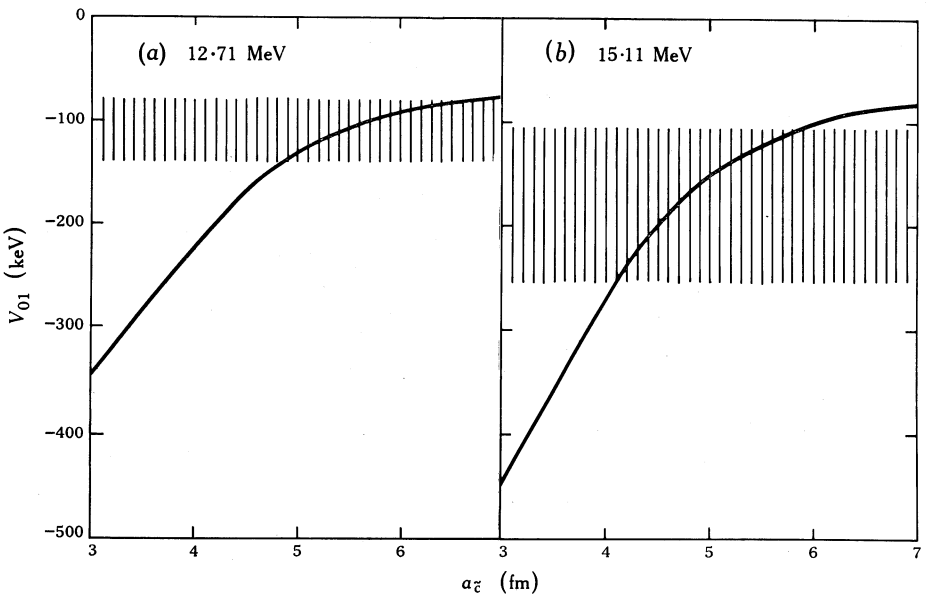

Fig. 3. Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the isospin mixing matrix element V_{01} for the indicated 1^+ levels of ^{12}C. The calculations are based on the shell model interaction of Cohen and Kurath (1965). Experimental results with estimated uncertainties are denoted by the hatching.

(f) 1^+ Levels of ^{12}C at 12·71 and 15·11 MeV

For the 1^+ levels of ^{12}C at 12·71 and 15·11 MeV we adopt an approach similar to that used above for the positive parity levels of ^{8}Be, except that here the two 1^+ levels of ^{12}C are so far apart that degeneracy is not assumed. Thus we have $V^a_{01} \neq V^b_{01}$, and the formulation (13) should be used to describe the levels. The wavefunctions Ψ_T are taken as the shell model wavefunctions of Cohen and Kurath (1965) for their POT interaction. With harmonic oscillator single-particle wavefunctions and $b = 1{\cdot}67$ fm from elastic electron scattering on ^{12}C (Ajzenberg-Selove 1975) we obtain $H^c_{01} = -65$ keV.† Because of the large separation of the two levels, the isospin mixing is small and $u_{T\tilde{c}}(r)$ is calculated at the energy of level a for $T = 0$ and level b for $T = 1$. The channels included correspond to the four lowest levels of ^{11}B and ^{11}C ($\tilde{J} = 3/2$, $1/2$, $5/2$ and $3/2^*$), with excitation energies $\lesssim 5$ MeV (Ajzen-

† Our sign convention is that the largest components of Ψ_0 and Ψ_1 in the LS coupling representation have the same sign. For 1^+ states of the configuration $(1p_{3/2})^7(1p_{1/2})$, we calculate $H^c_{01} = -29$ keV rather than the -50 keV quoted by Reisman *et al.* (1970).

berg-Selove). These are well identified as states of the lowest configuration (Cohen and Kurath) and give p-wave nucleon channels. Higher excited states do not give p-wave nucleon channels, or are not known to belong to the lowest configuration.

Table 3. Values of quantities related to isospin mixing in ^{16}O

Quantity	Value for J^π equal to:				Notes†
	0^-	1^-	2^-	3^-	
E_a (MeV)	10·95	12·44	12·53	13·13	
E_b (MeV)	12·80	13·09	12·97	13·25	
α	0·95 (0·98)	0·88	0·92	0·95	
β	0·31 (0·22)	0·48	0·40	0·32	
V_{01} (keV)	−540 (−400)	−270	−160	−40	
H^c_{01} (keV)	−104	−75	−101	−113	A
	−104	−38	−51	−57	B
	−104	−41	−29	−69	C
l_p	0	0	2	2	
$\mathscr{S}_a$	1·59 (1·44)	0·97	1·16	0·60	A
	1·59 (1·44)	0·53	0·51	0·29	B
	1·59 (1·44)	0·66	0·97	0·40	C
	0·76	0·40	0·72	0·32	D
		0·32	0·51	0·34	E
$\mathscr{S}_b$	0·41 (0·56)	0·33	0·48	0·60	A
	0·41 (0·56)	0·52	0·62	0·73	B
	0·41 (0·56)	0·48	0·80	0·71	C
	0·44	0·58	0·40	0·46	D
	0·38	0·35	0·60	0·61	E

† Notes on entries:

A, calculated values using Elliott and Flowers (1957) shell model wavefunctions;
B, calculated values using modified matrix elements (see text);
C, calculated values using Gillet and Vinh Mau (1964) shell model wavefunctions;
D, experimental values of Bohne *et al.* (1972) from stripping reactions;
E, experimental values from $^{15}N+p$ resonance reactions.

The resulting values of V^a_{01} and V^b_{01} are shown in Fig. 3 as functions of channel radius. The two most reliable experimental values of the isospin mixing matrix elements, with our sign convention, are -179 ± 75 keV (Lind *et al.* 1977), which is essentially a value of V^b_{01} since it depends strongly on the $^{13}C(d,t)$ spectroscopic factor for the 15·11 MeV state and only weakly on that for the 12·71 MeV state, and -110 ± 30 keV (Adelberger *et al.* 1977), which is essentially a value of V^a_{01} since it depends mainly on the $T = 1$ admixture to the 12·71 MeV state. Each group has criticized the work of the other, presumably on the assumption that the different types of experiment should necessarily lead to the same value of the mixing matrix element. These experimental values are also shown in Fig. 3 and it is seen that the calculated values agree with them for $a_c \approx$ 5–6 fm. The conventional channel radius in this case is 4·77 fm.

(g) 2^+ Levels of ^{12}C

In their study of the isospin mixing of the 1^+ levels of ^{12}C, Lind *et al.* (1977) assumed that the 16·11 MeV 2^+ level of ^{12}C has pure isospin $T = 1$, because they

argued that the most likely $T = 0$ admixtures would come from two distant states—the 4·43 MeV state and the giant quadrupole resonance at ~26 MeV. Of the 2^+, $T = 0$ states predicted by Cohen and Kurath (1965), the next above the 4·43 MeV state are at 15·73 and 18·13 MeV. The calculated values of H^c_{01} coupling these three $T = 0$ states to the lowest 2^+, $T = 1$ state are 31, −27 and −1·7 keV respectively, suggesting that $T = 0$ admixtures from the 15·73 MeV state would be more important than those from the 4·43 MeV state. Also surface contributions to the mixing from the 15·73 MeV state might be appreciable. Thus it is not clear why the 16·11 MeV state should have small $T = 0$ admixtures. Quantitative calculations are difficult, however, because the 15·73 MeV state has not been located experimentally and because a two-state mixing model may not be appropriate.

(*h*) *Negative Parity Levels of* ^{16}O

The analogues of the four lowest $T = 1$ states of ^{16}N occur in ^{16}O at ~13 MeV excitation, and each appears to have appreciable $T = 0$ admixture. In each case it seems reasonable to assume that just one $T = 0$ state is contributing, so that we have the pairs of isospin mixed levels given in Table 3 (experimental values in this subsection are taken from Ajzenberg-Selove (1977) unless another reference is given). With sufficient accuracy, we use the description (7) for each pair. Such descriptions of these levels have been used before, in particular by Brochard (1972) who obtained values of α and β for each pair, and these are given in Table 3. These are based mainly on relative values of α-particle and γ-ray partial widths, but also on the selection rules $\Delta T = 0$ for α-particle decay and $\Delta T = \pm 1$ for El γ-ray decay in self-conjugate nuclei. For the 0^- levels, however, only Ml γ-ray decays are available, and Brochard obtained his values of α and β on the assumption that $\Delta T = 0$ contributions are negligible. We consider the accuracy of this assumption below. There is additional uncertainty in the α and β values for the 0^- case, due to the large uncertainty in the lifetime of the 10·95 MeV level, $\tau = 8 \pm 5$ fs.

Values of the mixing matrix element V_{01} given in Table 3 are calculated from $V_{01} = -\alpha\beta(E_b - E_a)$. These agree with previously given magnitudes for the 1^- and 3^- levels of 270 and 45 keV respectively (Bray *et al.* 1977), and for the 2^- levels of 156 ± 30 keV (Jackson and Rolfs 1973) and $\gtrsim 155 \pm 30$ keV (Wagner 1977). We wish to compare these experimental values of V_{01} with values calculated from the formula (14). We first consider shell model values of H^c_{01}. Numerous shell model calculations have been made for the negative parity levels of ^{16}O, with varying degrees of complexity. It seems fair to say that none gives a really adequate description. For this reason, we consider initially only a simple model ($1p$–$1h$ states relative to a closed shell ^{16}O ground state) and allow in a qualitative way for more complicated configurations involving many particle–hole excitations. Consequently, detailed agreement with experiment should not be expected. We start with the $1p$–$1h$ wavefunctions of Elliott and Flowers (1957) for a Rosenfeld interaction with $V_c = 40$ MeV, and with those of Gillet and Vinh Mau (1964). These give similar and probably fairly accurate descriptions of the relevant $T = 1$ states. More elaborate calculations involving $1p_{1/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits (Zuker *et al.* 1968; Reehal and Wildenthal 1973) give coefficients of 0·8–0·9 for the $1p$–$1h$ components of these states, and a similar value for the $1p$–$2h$ component of the ^{15}N ground state. Likewise $1p$–$1h$ coefficients of about 0·8 are given for the lowest 0^-, 1^-, 2^- and 3^-, $T = 0$ states (and these include the 10·95 MeV 0^- state); however, the coefficients for the second

lowest 1^-, 2^- and 3^-, $T = 0$ states, which we identify with the states of interest here, are $\sim 0{\cdot}4$. Other calculations including $1p$–$1h$ and $3p$–$3h$ excitations and also including the $1p_{3/2}$ and $1d_{3/2}$ orbits (Ellis and Engeland 1970) obtained at most 38% $3p$–$3h$ intensity in these states. As an approximate way of obtaining an upper limit on the effect of higher configurations, we therefore reduce the $1p$–$1h$ values of matrix elements (such as H^c_{01} and $\mathscr{S}^{\frac{1}{2}}_{0\tilde{c}}$) involving the 1^-, 2^- or 3^-, $T = 0$ states by a factor of $0{\cdot}5$, on the assumption that the many particle–hole components do not contribute appreciably, and refer to these as modified values. Calculated values of H^c_{01} are given in Table 3 for $b = 1{\cdot}83$ fm, obtained from electron scattering and muonic atoms (Dubler *et al.* 1974). Before considering the calculation of L^{λ}_{01} in equation (14), we complete the discussion of other quantities in Table 3.

Brochard (1972) obtained values of α and β for the 0^- levels by neglecting $\Delta T = 0$ contributions to the M1 decay from these levels to the 1^-, $T = 0$ level at $7{\cdot}12$ MeV. From the shell model wavefunctions, we may calculate the Ml matrix elements for the states of pure isospin. For the lowest $T = 0$ and 1 states, the ratio of the matrix elements is very close to the value

$$R \equiv (\mu_p + \mu_n - 0{\cdot}5)/(\mu_p - \mu_n - 0{\cdot}5) = 0{\cdot}0903,$$

where $\mu_p = 2{\cdot}793$ and $\mu_n = -1{\cdot}913$ are the proton and neutron magnetic moments (in nuclear magnetons). Then from

$$\Gamma_{a\gamma}/\Gamma_{b\gamma} = (E_{a\gamma}/E_{b\gamma})^3 \,|\, (\alpha R + \beta)/(\beta R - \alpha) \,|^2,$$

we obtain the values of α and β for the 0^- levels given in parentheses in Table 3. We note that the partial width of the $12{\cdot}80 \rightarrow 7{\cdot}12$ Ml transition, which is measured to be $2{\cdot}5 \pm 0{\cdot}2$ eV, is calculated to be $2{\cdot}0$ eV from the Elliott and Flowers (1957) wavefunctions and $4{\cdot}1$ eV from those of Gillet and Vinh Mau (1964).

For each J value, values of the spectroscopic amplitudes $\mathscr{S}^{\frac{1}{2}}_{Ts}$ for $^{16}\mathrm{O} \rightarrow {}^{15}\mathrm{N}(\mathrm{g.s.}) + \mathrm{p}$ may be obtained from the shell model wavefunctions for the pure $T = 0$ and 1 states. Then the spectroscopic factors for the mixed states are

$$\mathscr{S}_{\lambda} = \sum_s \mathscr{S}_{\lambda s},$$

with

$$\mathscr{S}_{as} = (\alpha \mathscr{S}^{\frac{1}{2}}_{0s} + \beta \mathscr{S}^{\frac{1}{2}}_{1s})^2, \qquad \mathscr{S}_{bs} = (\beta \mathscr{S}^{\frac{1}{2}}_{0s} - \alpha \mathscr{S}^{\frac{1}{2}}_{1s})^2, \tag{23}$$

and these may be compared with experimental values from stripping and resonance reactions. The calculated values of $\mathscr{S}_{\lambda}$ are given in Table 3. As the values of $\mathscr{S}_{\lambda}$ from stripping reactions on $^{15}\mathrm{N}$, we give those that Bohne *et al.* (1972) selected as the best values, although individual experiments give widely different results. The values of $\mathscr{S}_{\lambda}$ from $^{15}\mathrm{N} + \mathrm{p}$ resonance reactions are obtained from $\gamma^2_{\lambda \mathrm{p}} = \frac{1}{2}\mathscr{S}_{\lambda}\gamma^2_0$, where the factor $\frac{1}{2}$ comes from the isospin Clebsch–Gordan coefficient, and $\gamma^2_0 = \theta^2_0 \hbar^2/m_c a_c^2$, where

$$\theta^2_0 = \tfrac{1}{2} a_c u^2(a_c) \Big/ \int_0^{a_c} u^2(r)\,\mathrm{d}r.$$

We use the conventional value of the channel radius $a_c = 5{\cdot}03$ fm, and take $u(r)$ as a 2s or 1d proton wavefunction in a Woods–Saxon potential with a resonance at the observed energy. For the 1^- and 3^- levels, we have taken values of $\gamma^2_{\lambda \mathrm{p}}$ from the

analysis of Bray *et al.* (1977). For the 0^- and 2^- levels, we obtain $\gamma^2_{\lambda p}$ from the observed proton partial width $\Gamma^0_{\lambda p}$ using the one-level formula with the Thomas approximation (neglecting all but the ground state proton channel)

$$\Gamma^0_{\lambda p} = 2\gamma^2_{\lambda p} P_p/(1 + \gamma^2_{\lambda p} \mathrm{d}S_p/\mathrm{d}E),$$

where P_p and S_p are the penetration and shift factors for the proton channel. The experimental values are $\Gamma^0_{bp} = 38$ keV for the 0^- level, and $\Gamma^0_{ap} = 0{\cdot}019$ keV and $\Gamma^0_{bp} = 1{\cdot}12$ keV for the 2^- levels, all in the c.m. system.

For the 2^- levels, comparison may also be made between the calculated and experimental values of the channel spin ratio in the $^{15}\mathrm{N}+\mathrm{p}$ channel. Measured values (Barnes *et al.* 1952; Kraus *et al.* 1953; Brochard 1972; Bray *et al.* 1977) of the fraction of channel spin 0, namely $\mathscr{S}_{\lambda 0}/\mathscr{S}_\lambda$, are $0{\cdot}85 \pm 0{\cdot}03$ for the 12·53 MeV level and $0{\cdot}58 \pm 0{\cdot}03$ for the 12·97 MeV level. Calculated values for the lower and upper levels respectively are 0·99 and 0·40 (Elliott and Flowers 1957), 0·96 and 0·58 (modified values), and 0·94 and 0·21 (Gillet and Vinh Mau 1964). Comparison of these values of $\mathscr{S}_{\lambda 0}/\mathscr{S}_\lambda$ for the 2^- levels and of the calculated and experimental values of $\mathscr{S}_\lambda$ in Table 3 suggests moderate agreement, possibly best for the modified values.

We now return to the calculation of $L_{01} \equiv L^\lambda_{01}$, which occurs in equation (14). In this case it is convenient to use the channel labels $\tilde{c} \equiv \tilde{J}lj$, where l and j are the orbital and total angular momentum of the nucleon in the channel, and equation (18) becomes

$$L_{01} = \frac{\hbar^2}{4ma_{\tilde{c}}} \sum_{\tilde{J}l} u^2_{\tilde{J}l}(a_{\tilde{c}}) \left(\sum_j \mathscr{S}^{\frac{1}{2}}_{0\tilde{J}lj} \mathscr{S}^{\frac{1}{2}}_{1\tilde{J}lj} \right) \{S(\tilde{J}l, \tfrac{1}{2}) - S(\tilde{J}l, -\tfrac{1}{2})\}. \qquad (24)$$

The levels $\tilde{J}$ should include positive parity as well as negative parity levels of $^{15}\mathrm{N}$ and $^{15}\mathrm{O}$, that is, $1p$–$2h$ states as well as $1h$ states. The $1h$ states are the $1/2^-$ ground states and the $3/2^-$ states at ~ 6 MeV excitation. The $\mathscr{S}^{\frac{1}{2}}_{T\tilde{J}lj}$ for these states are obtained from the wavefunctions of Elliott and Flowers (1957) or Gillet and Vinh Mau (1964). Since the $1/2^-$ ground states give the dominant contribution to L_{01}, the only positive parity states we include are the $1/2^+$ and $5/2^+$ states at ~ 5 MeV excitation, which are expected to be the most important. We approximate them by assuming that their only $A = 14$ parent state is the lowest $T = 1$, $J = 0$ state (such as the $^{14}\mathrm{C}$ ground state); this is supported by the large spectroscopic factors for producing these states in reactions such as $^{14}\mathrm{C}(^3\mathrm{He}, \mathrm{d})^{15}\mathrm{N}$, obtained both experimentally and from calculations (see Reehal and Wildenthal 1973). Then for these states $\mathscr{S}^{\frac{1}{2}}_{0\tilde{J}lj} \mathscr{S}^{\frac{1}{2}}_{1\tilde{J}lj}$ ($j = 1/2$ or $3/2$) can be written simply in terms of the spectroscopic factor of the $1/2^-$ or $3/2^-$ $A = 15$ state for the $T = 1$, $J = 0$, $A = 14$ channel, and this we obtain from Cohen and Kurath (1967).

The values of the different contributions to L_{01} for each J value are shown in Table 4 for the particular case of the Elliott and Flowers (1957) wavefunctions and for a channel radius $a_{\tilde{c}} = 5{\cdot}0$ fm. Values of V_{01} are shown in Fig. 4 as functions of channel radius for these wavefunctions and for those of Gillet and Vinh Mau (1964), and also for the modified values. It is seen that agreement between experimental and calculated values can be obtained for $a_{\tilde{c}} \approx 4$–6 fm, except for the 3^- levels, where the experimental magnitude of V_{01} is much smaller than the calculated magnitudes,

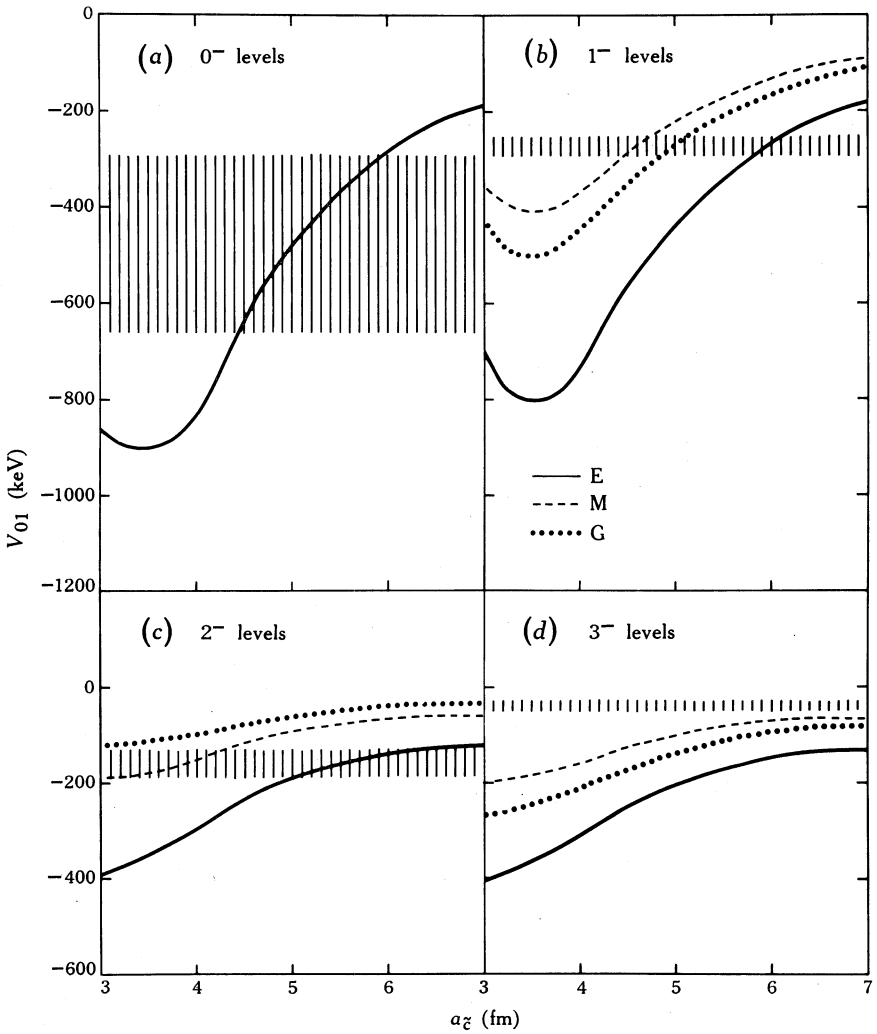

Fig. 4. Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the isospin mixing matrix element V_{01} for the indicated pairs of negative parity levels of ^{16}O. The calculations are based on the shell model interactions of E, Elliott and Flowers (1957); M, modified matrix elements (see text); G, Gillet and Vinh Mau (1964). The theoretical results are indistinguishable for the 0^- levels. Experimental results with estimated uncertainties are denoted by the hatching.

even for the modified values. The experimental magnitude is insensitive to the values of α and β, and cannot be greater than half the separation of the 3^- states, that is, 60 keV. Agreement could be obtained if the 3^-, $T = 0$ state contained much less $1p$–$1h$ component than we have assumed even for the modified values, but this would also reduce the proton spectroscopic factor $\mathcal{S}_a$ for the 13·13 MeV level. It seems likely that the two-state isospin mixing assumption is upset by the presence of other 3^- ($T = 0$) states of ^{16}O at 11·6, 14·1 and 15·4 MeV.

Table 4. Contributions to L_{01} for negative parity levels of ^{16}O

The calculations use the shell model wavefunctions of Elliott and Flowers (1957) and are for $a_{\tilde{c}} = 5{\cdot}0$ fm

J^π	$\tilde{J}$	l	$\sum_j \mathscr{S}^{\frac{1}{2}}_{0\tilde{J}lj}\mathscr{S}^{\frac{1}{2}}_{1\tilde{J}lj}$	$\frac{\hbar^2}{4ma_{\tilde{c}}} u^2_{\tilde{J}l}(a_{\tilde{c}})$ (keV)	$S(\tilde{J}l,\frac{1}{2}) - S(\tilde{J}l,-\frac{1}{2})$		$L_{01}(\tilde{c})$ (keV)
0^-	$1/2^-$	0	0·998	511	−0·645		−329
0^-	$1/2^+$	1	1·251	112	−0·301		−42
0^-	$3/2^-$	2	0·002	101	−0·241		0
						Total:	−371
1^-	$1/2^-$	0	0·574	599	−0·927		−319
1^-	$1/2^-$	2	0·004	239	−0·477		0
1^-	$1/2^+$	1	0·736	125	−0·319		−29
1^-	$5/2^+$	1	0·000	125	−0·319		0
1^-	$3/2^-$	0	0·138	254	−0·306		−11
1^-	$3/2^-$	2	0·003	111	−0·252		0
						Total:	−359
2^-	$1/2^-$	2	0·593	238	−0·475		−67
2^-	$1/2^+$	1	0·000	125	−0·319		0
2^-	$5/2^+$	1	0·644	125	−0·319		−26
2^-	$3/2^-$	0	0·003	253	−0·306		0
2^-	$3/2^-$	2	−0·064	110	−0·252		2
						Total:	−91
3^-	$1/2^-$	2	0·478	255	−0·524		−64
3^-	$5/2^+$	1	0·619	132	−0·329		−27
3^-	$3/2^-$	2	0·179	116	−0·258		−5
						Total:	−96

4. Summary

In all but one of the cases of isospin mixing considered here, we have used a two-state mixing model with one free parameter, the channel radius, to fit one piece of experimental data, the mixing matrix element. In most cases this has been possible and has been obtained with a reasonable value of the channel radius. For the 1^+ levels of ^{12}C, which are separated sufficiently for two different matrix elements to be involved, the experimental values are satisfactorily fitted with the same value of the channel radius. The one case where a reasonable fit does not seem possible is for the 3^- levels of ^{16}O at 13·13 and 13·25 MeV, but here the two-state mixing model is probably inappropriate.

It has been assumed that the isospin mixing is due to the Coulomb interaction alone, with contributions coming from both the internal region and the nucleon channel region. Thus the reasonable fits obtained suggest that these cases of isospin mixing provide no evidence for contributions from a specifically nuclear charge-dependent interaction.

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