Aust. J. Phys., 1978, **31**, 205–7

Gravitons from a Spinning Rod*

George L. Murphy

Department of Physics, Luther College, Decorah, Iowa 52101, U.S.A.

Abstract

It is shown that the gravitational radiation from a spinning rod cannot be calculated classically unless the mass of the rod exceeds several tonnes. If laboratory sources of gravitational waves ever become feasible, they will have to be described quantum mechanically, and should make possible the detection of individual gravitons.

The spinning rod was the first source for which emission of gravitational waves was calculated by Einstein (1916, 1918) and Eddington (1924), and the formula for emitted power has often been used for illustrative purposes. All discussions of the problem of which I am aware use non-quantum techniques. It might seem that classical concepts would suffice for any rod which might be contemplated as a laboratory source, but the weakness of gravitational interactions and the consequent low probability of graviton emission make this assumption questionable. Radiation from a source which must move through many cycles before there is a significant probability for one quantum to be emitted cannot be described classically. We shall see here that a rod must be fairly large before classical methods are valid.

If a mechanical system rotates with frequency ν, it may emit an energy E during a cycle in the form of gravitational waves with frequencies of the order of ν, as long as we are considering a regime in which the classical approximation is good. The frequency of quadrupole radiation will be 2ν here. The change in the classical action of the mechanical system during the entire cycle in which the waves must be considered to be emitted is E/ν, and a classical description is valid only if this change is much greater than h. Thus the criterion for validity of a classical description is $E/\nu \gg h$. The number N of quanta emitted in one period, $N \approx E/h\nu$, must be much greater than unity.

The classical formula for the power emitted by a rod with moment of inertia I, rotating with angular frequency Ω, is (Eddington 1924)

$$\mathrm{d}E/\mathrm{d}t = (32G/5c^5)I^2\Omega^6\,. \tag{1}$$

The rate of emission of energy will be related to the rate $\mathrm{d}n/\mathrm{d}t$ at which gravitons with angular frequency 2Ω are emitted by $\mathrm{d}E/\mathrm{d}t = 2\hbar\Omega\,\mathrm{d}n/\mathrm{d}t$, and the number

* This paper is a revised version of one which was awarded an honourable mention in the 1977 Gravity Research Foundation Essay Contest.

of gravitons emitted in one period will be $N = (2\pi/\Omega)\,\mathrm{d}n/\mathrm{d}t$. Therefore

$$N = \tfrac{32}{5}\,\pi(G/\hbar c^5)I^2\Omega^4 = \tfrac{32}{45}\pi(G/\hbar c)M^2(V/c)^4\,, \tag{2}$$

where M is the mass of the rod and V the speed of an end.

The rate at which we can spin such a rod will depend on the elastic properties of the material. It could be spun at such a high rate that it would be ready to explode, but this is neither safe nor practical. The maximum elastic strain for a metal is usually less than $0\cdot5\%$ (Hayden *et al.* 1965), and it seems reasonable to limit the strain to 1%. Since the stress required to supply the centripetal acceleration, and thus the strain, is proportional to V^2, V cannot be greater than one-tenth of the speed at which unit strain would occur, approximately the speed of sound s in the material. Thus

$$N \approx \tfrac{32}{45}\pi\times10^{-4}(G/\hbar c)M^2(s/c)^4\,. \tag{3}$$

The value of s will be less than $6\times10^3\ \mathrm{m\,s^{-1}} = 2\times10^{-5}\,c$. This gives $N \approx 7\cdot5\times10^{-8}M^2$, with M in kilograms. For this to be much greater than unity, we must have

$$M \gg 4\times10^3\ \mathrm{kg}\,. \tag{4}$$

Thus the mass of the rod must be many tonnes before a classical description of the radiation process will be valid. For masses not satisfying the condition (4), classical calculations are incorrect in principle, though they may give good estimates.

It is easy to develop a semiclassical theory of gravitational radiation which will suffice for any reasonable laboratory source. The necessary formulae are given by Weinberg (1972). We use the wavefunctions for a rigid rotator

$$\psi_n = (2\pi)^{-\frac{1}{2}}\exp(\mathrm{i}n\theta)\,,$$

for integral n, in the quadrupole moment matrix elements

$$D_{11}(a\to b) = I\int_0^{2\pi}\psi_b^*\cos^2\theta\,\psi_a\,\mathrm{d}\theta\,,$$

$$D_{22}(a\to b) = I\int_0^{2\pi}\psi_b^*\sin^2\theta\,\psi_a\,\mathrm{d}\theta\,,$$

$$D_{12}(a\to b) = I\int_0^{2\pi}\psi_b^*\cos\theta\sin\theta\,\psi_a\,\mathrm{d}\theta\,.$$

These are inserted into Weinberg's equation (10.8.6) for the transition rate

$$\Gamma(a\to b) = (2G\omega^5/5\hbar c^5)\{D_{ij}^*(a\to b)\,D_{ij}(a\to b) - \tfrac{1}{3}\,|D_{ii}(a\to b)|^2\}\,.$$

Here $\omega = 2\Omega$ is the angular frequency of the radiation. (I have inserted the appropriate power of c and have corrected a subscript j to i in the last term.) The result is

$$\Gamma(a\to b) = 16GI^2\Omega^5/5\hbar c^5\,.$$

If this is multiplied by the period of the rod, we recover equation (2). This is not surprising, for the semiclassical formulae can be obtained from correspondence principle arguments.

These estimates suggest that the construction of laboratory sources of gravitational waves and the instrumentation to detect their radiation—admittedly not a very immediate prospect—should reveal the quantized character of the gravitational field; it would be a small step from there to observation of the effects of individual gravitons. The prospect of observing the gravitational analogue of the photoelectric effect is sufficient to justify further thought being devoted to the difficult problem of building sources of gravitational radiation.

References

Eddington, A. S. (1924). 'The Mathematical Theory of Relativity', Note 8 (Cambridge Univ. Press).
Einstein, A. (1916). *Sb. Preuss. Akad. Wiss.* p. 688.
Einstein, A. (1918). *Sb. Preuss. Akad. Wiss.* p. 154.
Hayden, H. W., Moffat, W. G., and Wulff, J. (1965). 'The Structure and Properties of Materials', Vol. 3, Sect. 1.2 (Wiley: New York).
Weinberg, S. (1972). 'Gravitation and Cosmology', Sect. 10.8 (Wiley: New York).

Manuscript received 15 July 1977