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3^+ States of ^{8}Be

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Abstract

Level parameters for the 3^+ doublet at about 19 MeV in ^{8}Be are obtained from a two-level R-matrix fit to the ^{7}Li+p $^5P_3$ phase shift, using the restrictions of the two-state isospin mixing model. The energy of the lower level is appreciably below the accepted value. Predicted contributions from the 3^+ levels to the cross sections for the reactions ^{7}Li(p, n), ^{7}Li(p, γ), ^{10}B(d, α) and ^{9}Be(d, t) are compared with measured values, and qualitative agreement is obtained. The new values of the level parameters give agreement with values of the isospin mixing matrix element and of the excitation energy difference of the 3^+, $T = 1$ states in ^{8}Li and ^{8}Be, calculated assuming that the charge-dependent interaction is purely Coulomb.

1. Introduction

The most recent compilation of energy levels of ^{8}Be (Ajzenberg-Selove and Lauritsen 1974) gives a pair of 3^+ levels at $19{\cdot}06 \pm 0{\cdot}02$ and $19{\cdot}22$ MeV, and a possible 3^+ level at $21{\cdot}5$ MeV. The pair of levels has been interpreted as an isospin-mixed doublet (Barker 1966). The observed properties of the levels provide values for the matrix element of the charge-dependent interaction causing the isospin mixing, and the expectation energy of the pure $T = 1$, 3^+ state of ^{8}Be. From the latter value one may obtain the energy difference of the analogue 3^+ states of ^{8}Li and ^{8}Be, relative to the analogue 2^+ states. These values of the mixing matrix element and of the energy difference have been compared with calculated values (Barker 1978), and some disagreement was obtained with values calculated on the assumption that the charge-dependent interaction was purely Coulomb. Here we investigate whether these discrepancies could be due to inaccurate values for some of the measured properties of the levels, in particular their energies and the ratios of neutron and proton reduced widths. Population of the $19{\cdot}06$ and $19{\cdot}22$ MeV levels has been reported for many reactions (Ajzenberg-Selove and Lauritsen 1974), but most quantitative information has come from the three resonance reactions ^{7}Li(p, γ), ^{7}Li(p, n) and ^{7}Li(p, p).

The ^{7}Li(p, γ_1) reaction to the first excited state of ^{8}Be has shown a resonance at $E_p = 2{\cdot}06 \pm 0{\cdot}02$ MeV (Riech 1963), corresponding to $E_x = 19{\cdot}06 \pm 0{\cdot}02$ MeV. Other groups have also observed a peak at about $2{\cdot}1$ MeV, either in the γ_1 transition (Fisher *et al.* 1976) or in the summed $\gamma_0 + \gamma_1$ transitions (Newson *et al.* 1957; Perry *et al.* 1963). It has been assumed that this peak is due to the lower 3^+ level.

Only the upper level appears to contribute to the ^{7}Li(p, n) reaction, with the peak energy well determined as $E_p = 2{\cdot}25$ MeV, giving $E_x = 19{\cdot}22$ MeV (probably within about $0{\cdot}01$ MeV). The data can be fitted with any value of the reduced width ratio γ_n^2/γ_p^2 between 3 and 10 (Macklin and Gibbons 1958).

Early measurements of the ^{7}Li(p, p) excitation function at $\theta_{lab} = 164°$ showed a sharp peak at $E_p = 2{\cdot}06$ MeV (Bashkin and Richards 1951). Later measurements at several angles suggested the presence of two interfering levels of ^{8}Be in this energy region, one corresponding to the resonance seen in ^{7}Li(p, γ), the other to the resonance in ^{7}Li(p, n), but no parametric fit to the data was given (Malmberg 1956). These measurements showed that the peak observed by Bashkin and Richards moved to lower energies at smaller angles. Recently Brown *et al.* (1973) have extracted phase shifts from ^{7}Li(p, p) measurements with both polarized and unpolarized protons. The imaginary parts of the phase shifts were chosen to fit the ^{7}Li(p, n) and other data. They obtained trial values for their $^5P_3$ phase shift, which contains information about the 3^+ levels of ^{8}Be, on the assumption that two such levels occur in the region, but they did not obtain values of the level parameters by fitting the resultant phase shift.

Among the reactions that populate ^{8}Be as a product nucleus, rather than as a compound nucleus, only ^{7}Li(d, n)^{8}Be has been reported to excite all three of the levels at 18·93, 19·05 and 19·24 MeV (Kerr 1967), but details of this work are unpublished. The 18·9 MeV level is 2^- (Ajzenberg-Selove and Lauritsen 1974). In ^{10}B(d, α)^{8}Be, peaks observed at about 18·9 and 19·2 MeV were attributed to the 2^- level and the upper 3^+ level (Callender and Browne 1970). In all other reactions, only a single peak appeared in the spectrum in this energy region; in the cases where the resolution was sufficient to separate the levels, the peak corresponded to a level at about 19·2 MeV in each of ^{9}Be(p, d)^{8}Be (Kull 1967), ^{9}Be(d, t)^{8}Be (Oothoudt and Garvey 1977) and ^{9}Be(^{3}He, α)^{8}Be (Ajzenberg-Selove *et al.* 1976).

The most promising method of obtaining more accurate values of the level parameters for these 3^+ levels of ^{8}Be would seem to be a two-level fit to the $^5P_3$ phase shift of Brown *et al.* (1973), and this is performed in the next section. In Section 3 we check if these parameter values are consistent with the cross sections observed in other reactions involving the 3^+ levels. Then in Section 4 the derived values of the isospin mixing matrix element and of the energy difference of analogue states are compared with calculated values.

2. Two-level Fit to ^{7}Li+p $^5P_3$ Phase Shift

The energy dependence of the complex $^5P_3$ phase shift $\delta \equiv \delta^R + i\delta^I$ is given by the formulae of *R*-matrix theory (Lane and Thomas 1958) in terms of the eigenenergies E_λ and reduced width amplitudes $\gamma_{\lambda c}$ for the level λ and channel c. With the ^{7}Li+p channel denoted by p, we have

$$\exp(2i\delta) \equiv \eta \exp(2i\delta^R) = \exp(-2i\phi_p)\left(1 + 2i P_p \sum_{\lambda\mu} \gamma_{\lambda p}\gamma_{\mu p} A_{\lambda\mu}\right), \qquad (1)$$

where $\eta = \exp(-2\delta^I)$ is the absorption coefficient and the $A_{\lambda\mu}$ are elements of a matrix in level space, defined by its inverse

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_c L_c^0 \gamma_{\lambda c}\gamma_{\mu c}. \qquad (2)$$

Here $L_c^0 = S_c^0 + iP_c$ and $S_c^0 = S_c - B_c$, where S_c, P_c and $-\phi_c$ are the shift factor, penetration factor and hard-sphere phase shift, evaluated at the channel radius a_c, and B_c is the boundary condition parameter.

In the two-level approximation, the level labels λ and μ have the values a for the lower level and b for the upper level (to avoid confusion with isospin labels). The

sum over c in equation (2) is in principle over all channels, open and closed. We approximate by neglecting all closed channels, and also the proton channel to the first excited state of ^{7}Li, which would require f-wave protons. Then c takes on the values p and n only, corresponding respectively to the ^{7}Li and ^{7}Be ground state channels with p-wave nucleons. We assume the two-state isospin mixing model (Barker 1966), in which

$$\Psi_a = \alpha\Psi_0 + \beta\Psi_1, \qquad \Psi_b = \beta\Psi_0 - \alpha\Psi_1, \qquad (\alpha^2+\beta^2 = 1). \tag{3}$$

This gives for the reduced width amplitudes

$$\gamma_{ap} = 2^{-\frac{1}{2}}(\alpha\gamma_0 + \beta\gamma_1), \qquad \gamma_{bp} = 2^{-\frac{1}{2}}(\beta\gamma_0 - \alpha\gamma_1), \tag{4a}$$

$$\gamma_{an} = 2^{-\frac{1}{2}}(-\alpha\gamma_0 + \beta\gamma_1), \qquad \gamma_{bn} = 2^{-\frac{1}{2}}(-\beta\gamma_0 - \alpha\gamma_1), \tag{4b}$$

where γ_T ($T = 0, 1$) is the reduced width amplitude for the pure T state (excluding the isospin Clebsch–Gordan coefficient). The relations (4) restrict the values of the four quantities $\gamma_{\lambda c}$ by expressing them in terms of three independent parameters, say γ_0, γ_1 and α. These, with E_a and E_b, provide five adjustable parameters, for given values of a_c and B_c.

As reasonable values of the channel radii, we take $a_p = a_n = 5{\cdot}0$ fm (Barker 1978). The effect of changing this value is discussed in Section 5. Equally good fits to the phase shift can be obtained for any choice of the B_c provided that the values of the E_λ and $\gamma_{\lambda c}$ are freely adjustable (Barker 1972); since the summation over λ of $\gamma_{\lambda c}^2$ remains invariant under such changes of B_c, and since the restriction implied by equations (4) is that this summation should be independent of c, one can also get equally good fits for any B_c with only the five adjustable parameters. For convenience, we choose each B_c equal to the average value of the shift factor S_c in the energy region fitted, giving $B_p = -0{\cdot}53$ and $B_n = -0{\cdot}81$.

Experimental values of the $^5P_3$ phase shift $\delta_{exp} \equiv \delta^R_{exp} + i\delta^I_{exp}$ have been given by Brown *et al.* (1973), and these values of δ^R_{exp} and values of $\eta_{exp} = \exp(-2\delta^I_{exp})$ are shown in Fig. 1 for $E_p > 1{\cdot}5$ MeV. A least squares fit to these is made by minimizing the quantity

$$X = \frac{1}{N^R+N^I}\left(\sum_{i=1}^{N^R}\left|\frac{\delta^R_{exp}(E_i)-\delta^R(E_i)}{\varepsilon^R(E_i)}\right|^2 + \sum_{i=1}^{N^I}\left|\frac{\eta_{exp}(E_i)-\eta(E_i)}{\varepsilon^I(E_i)}\right|^2\right), \tag{5}$$

where the E_i are the energies at which measurements have been made and ε^R and ε^I are the errors in the real part of the phase shift and in the absorption coefficient respectively. We fit the real phase shift for $E_p > 1{\cdot}5$ MeV ($N^R = 30$) and the absorption coefficient for $E_p > 2{\cdot}0$ MeV ($N^I = 12$), since η is necessarily unity below the neutron threshold at $E_p = 1{\cdot}881$ MeV. We assign, rather arbitrarily, equal errors $\varepsilon^R = 5°$ and $\varepsilon^I = 0{\cdot}02$ at each of these energies.

The best fit is obtained with $\alpha = 0{\cdot}53$ (for $\beta > 0$) and is illustrated in Fig. 1. If α is changed from this value and the other parameter values are optimized, the fit to δ^R remains good but the fit to η worsens, particularly in the region of the wings. Fits regarded as acceptable are obtained for α between about $0{\cdot}4$ and $0{\cdot}7$. The solid curves in Fig. 2 show these optimum values of X, E_λ and γ_T as functions of α. Corresponding values of various derived quantities are shown by the solid curves in Fig. 3.

Of these, the values of $\gamma_{\lambda n}^2/\gamma_{\lambda p}^2$ are obtained from equations (4). Widths of the levels are calculated in an approximate way by using the parameter values E_λ and $\gamma_{\lambda c}$ in an independent one-level approximation for each level, so that the observed width in the Thomas approximation (Lane and Thomas 1958) is given by

$$\Gamma_\lambda^0 = \sum_c \Gamma_{\lambda c}^0, \qquad \Gamma_{\lambda c}^0 = 2\gamma_{\lambda c}^2 P_c \Big/ \left(1 + \sum_{c'} \gamma_{\lambda c'}^2 \frac{dS_{c'}}{dE}\right), \tag{6}$$

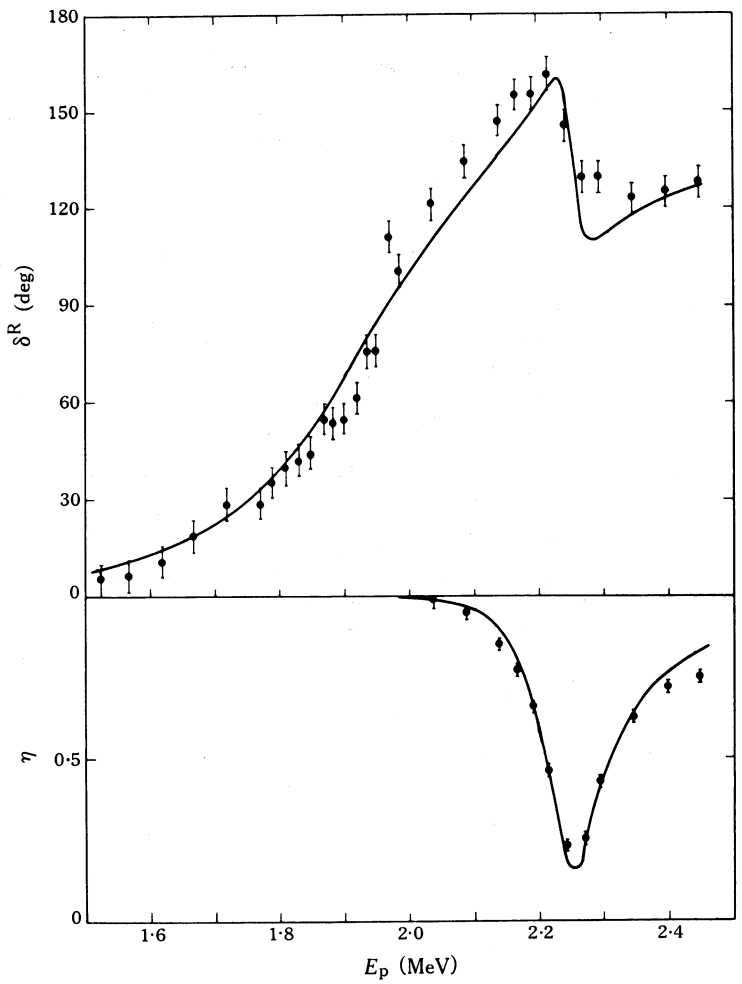

Fig. 1. Real part of the $^5P_3$ phase shift δ^R for ^7Li+p elastic scattering and the corresponding absorption coefficient η as functions of proton energy E_p. The experimental points are from Brown *et al.* (1973) and the error bars show the assumed errors. The curves are best fits from a two-level *R*-matrix approximation with parameters restricted by the two-state isospin mixing model.

evaluated at the peak energy. The energies E_T^0 of the states of pure isospin T and the isospin mixing matrix element V_{01} are given by

$$E_0^0 = \alpha^2 E_a + \beta^2 E_b, \quad E_1^0 = \beta^2 E_a + \alpha^2 E_b, \quad V_{01} = -\alpha\beta(E_b - E_a). \tag{7}$$

The above parameter values all correspond to a particular choice of B_c values. The properties of the level λ ($\lambda = a$ or b) are probably best described by the parameter values for $B_c = S_c(E_\lambda)$; we write these values as $B_c^{(\lambda)}$ and the corresponding parameter values that give exactly the same fit to the data (Barker 1972) as $E_\mu^{(\lambda)}$ and $\gamma_{\mu c}^{(\lambda)}$, with $\mu = a, b$ and $c = \mathrm{p}, \mathrm{n}$. Values of $E_a^{(a)}$ and $E_b^{(b)}$ are shown by the dashed curves in Fig. 2. This procedure is not quite consistent, since the values of $\gamma_{\lambda c}^{(\lambda)}$ do not satisfy

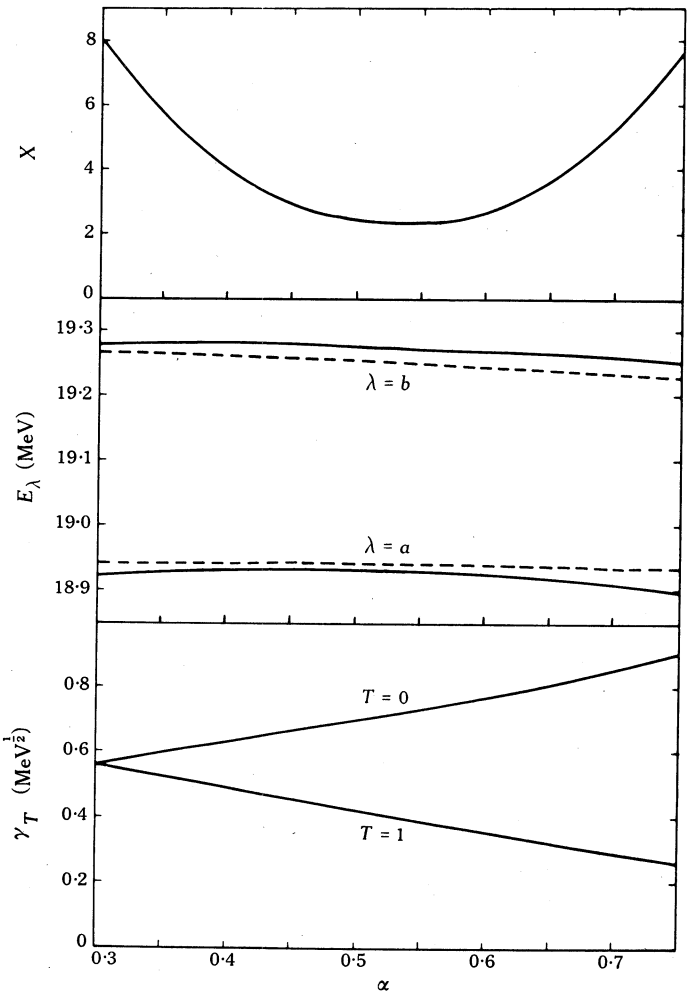

Fig. 2. Minimum values of X (equation 5) and corresponding values of the level parameters E_λ ($\lambda = a, b$) and γ_T ($T = 0, 1$) as functions of the isospin mixing parameter α. The solid curves are for $B_\mathrm{p} = -0{\cdot}53$ and $B_\mathrm{n} = -0{\cdot}81$. The dashed curves are values of $E_\lambda^{(\lambda)}$ ($\lambda = a, b$) corresponding to $B_c = B_c^{(\lambda)} = S_c(E_\lambda^{(\lambda)})$ ($c = \mathrm{p}, \mathrm{n}$).

exactly equations like (4), but they satisfy them approximately with the original value of α. The dashed curves in Fig. 3 show the corresponding values of $(\gamma_{\lambda\mathrm{n}}^{(\lambda)}/\gamma_{\lambda\mathrm{p}}^{(\lambda)})^2$ and of Γ_λ^0, E_T^0 and V_{01} obtained by using values of $E_\lambda^{(\lambda)}$ and $\gamma_{\lambda c}^{(\lambda)}$ in equations (6) and (7).

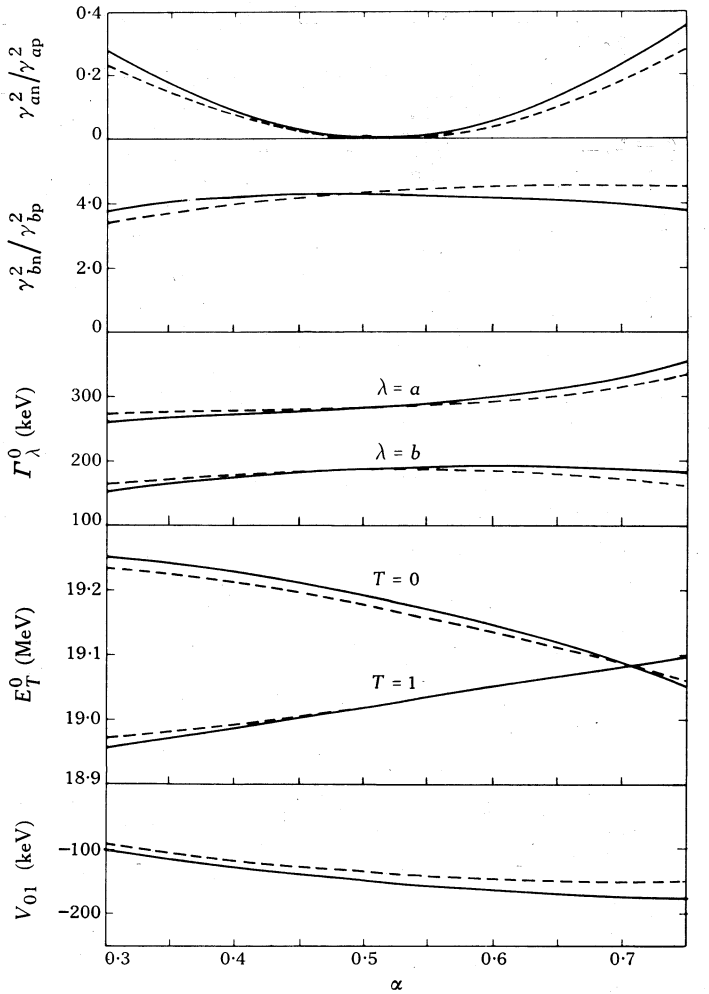

Fig. 3. Values of quantities derived from the parameter values of Fig. 2 using equations (4), (6) and (7).

3. Predictions of Cross Sections for Other Reactions

Before comparing the derived values of V_{01} and E^0_1 with shell model values, we calculate the cross sections for various reactions involving the 3^+ levels, using the new values of the level parameters obtained from fitting the $^5P_3$ phase shift in ^{7}Li+p scattering, to check that consistent fits are possible and to see if the range of allowed values of α may be reduced. For some reactions, values of additional parameters are required, and these are obtained from shell model calculations or from isospin conservation.

(*a*) ^{7}Li(p, n) *Reaction*

The method used by Brown *et al.* (1973) to determine the imaginary part of the $^5P_3$ phase shift ensures that the predicted 3^+ contribution to the ^{7}Li(p, n) cross section will be consistent with the measured values, that the values of E_b (or $E_b^{(b)}$) in Fig. 2 and of $\gamma^2_{bn}/\gamma^2_{bp}$ in Fig. 3 will agree with previous values, and that $\gamma^2_{an}/\gamma^2_{ap}$ will be small,

supporting the view that only the upper 3^+ level contributes appreciably to the ^{7}Li(p, n) cross section.

(*b*) ^{7}Li(p, γ) *Reaction*

It is not immediately obvious why the ^{7}Li(p, γ_1) cross section should have only a single peak in the energy region of the 3^+ levels, as is observed, nor why this peak should be attributed to the lower level alone. One expects the 3^+ contribution to the cross section to be incoherent with the background, since the former requires channel spin 2 in the ^{7}Li+p channel, while the background should be mainly channel spin 1 (this assumes that the background is due to direct capture of s- and d-wave protons with E1 radiation, and that the ^{7}Li ground state and ^{8}Be first excited state are adequately described as the *LS* coupled states [3] $^{22}P_{3/2}$ and [4] $^{11}D_2$ respectively (Barker 1966)).

Since the angular distribution of the 3^+ contribution to the ^{7}Li(p, γ_1) cross section is unique for p-wave protons ($\propto 1-\frac{9}{28}\cos^2\theta_\gamma$), the shapes of the excitation functions measured at 0° and 90° may be compared directly with the calculated integrated cross section, which is given by

$$\sigma = (7\pi/2k_p^2)\, P_p E_\gamma^3 \left| \sum_{\lambda\mu} \gamma_{\lambda p}\gamma_{\mu\gamma} A_{\lambda\mu} \right|^2 . \qquad (8)$$

Here the M1 radiation width of the level μ is taken as $\Gamma_{\mu\gamma} = E_\gamma^3 \gamma_{\mu\lambda}^2$, and $A_{\lambda\mu}$ is the same as in equation (1). From the two-state isospin mixing model, the reduced width amplitudes $\gamma_{\mu\gamma}$ are given by

$$\gamma_{a\gamma} = \alpha\gamma_{0\gamma} + \beta\gamma_{1\gamma}, \qquad \gamma_{b\gamma} = \beta\gamma_{0\gamma} - \alpha\gamma_{1\gamma}, \qquad (9)$$

where shell model values may be used for the $\gamma_{T\gamma}$. Since we do not consider absolute values of σ, we require only values of the ratio $\gamma_{0\gamma}/\gamma_{1\gamma}$, for which shell model calculations give 0·057 (Cohen and Kurath 1965), 0·059 (Barker 1966) and 0·058 (Kumar 1974).

Fig. 4 shows calculated values of σ, normalized to unity at the highest point, for $\gamma_{0\gamma}/\gamma_{1\gamma} = 0{\cdot}058$ and for three values of α, the optimum value of 0·53 and values of 0·4 and 0·7. Even though both levels are contributing appreciably, the cross section does not show two distinct peaks, owing to the constructive interference in the region between the two levels (since $\gamma_{ap}/\gamma_{bp} > 0$ and $\gamma_{a\gamma}/\gamma_{b\gamma} < 0$). The experimental points in Fig. 4 are the 0° and 90° excitation functions of Newson *et al.* (1957) and the 90° excitation function of Riech (1963), after subtraction of linear backgrounds chosen to make the resonant contribution resemble the calculated curves for E_p near 1·7 and 2·4 MeV and normalization to make the resonant contribution approximately unity at the peak. From the scatter of points in the region of the tails, it is clear that there are problems with this procedure; in fact Newson *et al.* comment on an interference dip at $E_p \approx 2{\cdot}35$ MeV. However, the ratio of the normalization factors required for the 0° and 90° excitation functions of Newson *et al.* is 1·39, which agrees with the ratio of 28/19 = 1·47 expected for a pure 3^+ contribution. No allowance for experimental energy resolution is necessary since target thicknesses of 5 keV (Newson *et al.* 1957) and 20 keV (Riech 1963) were used.

It is seen that there is qualitative agreement between the calculated and experimental results in Fig. 4. The main discrepancy, independent of the value of α, is that the calculated cross section is too large on the low-energy side of the peak. Changing the value of $\gamma_{0\gamma}/\gamma_{1\gamma}$ does not significantly affect this; in fact changes of $\pm 0{\cdot}03$ in

$\gamma_{0\gamma}/\gamma_{1\gamma}$ can be almost exactly compensated by simultaneous changes of $\mp 0{\cdot}02$ in α. The shape of the peak at higher energies probably limits α to between about $0{\cdot}4$ and $0{\cdot}65$.

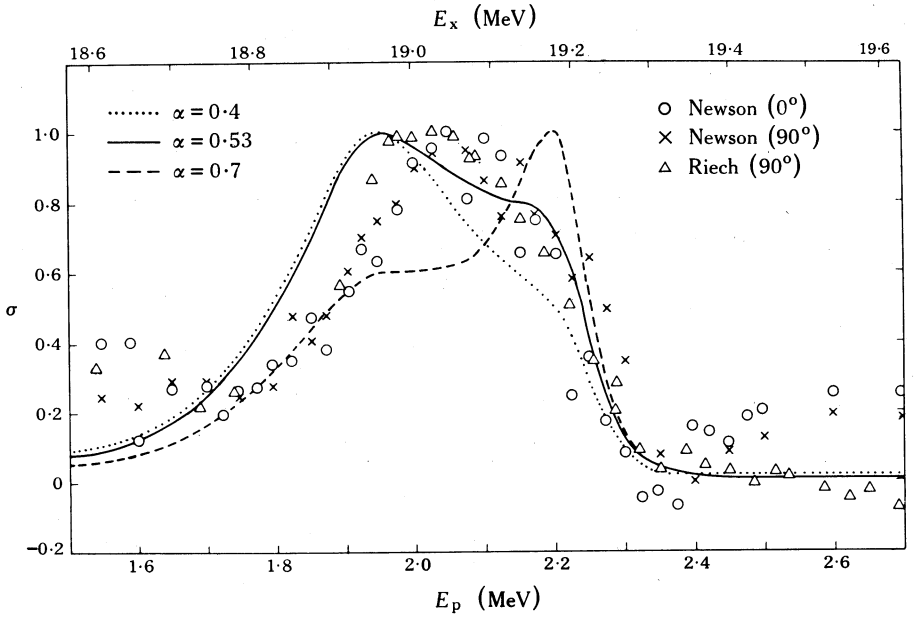

Fig. 4. Normalized contribution to the ^{7}Li(p, γ_1) cross section σ due to 3^+ levels of ^{8}Be as a function of proton energy. The experimental points are the 0° and 90° excitation functions of Newson *et al.* (1957) and the 90° excitation function of Riech (1963), with backgrounds subtracted. The curves are calculated using parameter values obtained from fits to the ^{7}Li+p $^5P_3$ phase shift and shell model values of the γ-ray reduced width amplitudes, for the three different values of α indicated.

(*c*) ^{10}B(d, α)^{8}Be *Reaction*

For reactions of this type, in which ^{8}Be appears as a product nucleus, the dependence of the cross section on ^{8}Be excitation energy should be given by a formula similar to (8), but with the reduced width amplitudes for the γ channel replaced by feeding amplitudes, which are dependent on the particular reaction (Barker 1967). More precisely, the contribution of the 3^+ levels to the cross section is taken to be

$$\sigma_x \propto \sum_c P_c \left| \sum_{\lambda\mu} \gamma_{\lambda c} g_{\mu x} A_{\lambda\mu} \right|^2, \tag{10}$$

where x specifies the producing reaction and the sum is over both p and n channels, since the decay of the ^{8}Be is not observed. A possible weak dependence on the energy of the emitted particle (the α particle) has been omitted. The feeding amplitudes $g_{\mu x}$ are given by

$$g_{ax} = \alpha g_{0x} + \beta g_{1x}, \quad g_{bx} = \beta g_{0x} - \alpha g_{1x}, \tag{11}$$

in terms of the feeding amplitudes g_{Tx} for the pure T states.

For the ^{10}B(d, α)^{8}Be reaction, isospin conservation requires that only the $T = 0$ parts of the ^{8}Be states are fed, so that $g_{1x} = 0$. The calculated cross section is shown by the curves in Fig. 5 for the same α values as in Fig. 4. In this case $g_{ax}/g_{bx} = \alpha/\beta > 0$,

so that there is destructive interference in the region between the levels, which therefore appear as two distinct peaks. The experimental points are from Callender and Browne (1970). No background has been subtracted, because the 3^+ contribution could be coherent with background contributions coming from other levels of ^{8}Be. Comparable normalization has been used for the calculated and experimental values. The energy resolution was about 14 keV. It seems reasonable to interpret the peak observed at 18·9 MeV as being due to the lower 3^+ level instead of attributing it, as did Callender and Browne, to the 2^- level of ^{8}Be known to exist at this energy. The 2^- level should not be populated if the ^{10}B(d, α) reaction proceeds as a direct transition and the ^{10}B ground state belongs to the lowest shell model configuration. The size of the 18·9 MeV peak is consistent with $\alpha \approx 0{\cdot}5$.

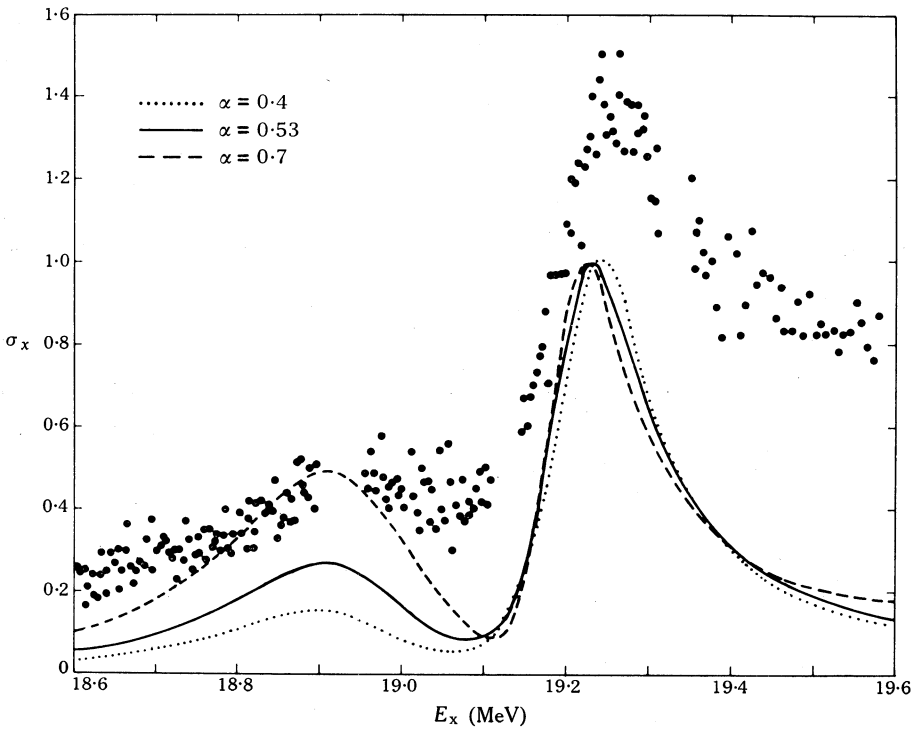

Fig. 5. Normalized contribution to the ^{10}B(d, α)^{8}Be cross section σ_x due to 3^+ levels of ^{8}Be as a function of ^{8}Be excitation energy. The experimental points are from Callender and Browne (1970) and include a background contribution (with a constant value of 0·3 subtracted from the ordinate). The curves are calculated using parameter values obtained from fits to the ^{7}Li+p $^5P_3$ phase shift and from isospin conservation, for three α values as in Fig. 4.

(*d*) ^{9}Be(d, t)^{8}Be *Reaction*

If the ^{9}Be(d, t) reaction proceeds by neutron pickup, then the feeding amplitude g_{Tx} is proportional to the spectroscopic amplitude (including the isospin Clebsch–Gordan coefficient) of the ^{9}Be ground state for the p-wave neutron channel with ^{8}Be in its 3^+ state with isospin T. Shell model values of these spectroscopic amplitudes give $g_{0x}/g_{1x} = -0{\cdot}88$ (Barker 1966), $-0{\cdot}78$ (Cohen and Kurath 1967) and $-0{\cdot}95$ (Kumar 1974).

Fig. 6 shows calculated values of σ_x for $g_{0x}/g_{1x} = -0{\cdot}9$, for the same three values of α. For $\alpha \approx 0{\cdot}53$ one has $g_{ax}/g_{bx} \approx -0{\cdot}3$; this small negative value implies constructive interference between the levels and a single peak in the region of the upper level. The experimental points in Fig. 6 are from Oothoudt and Garvey (1977), without background subtraction because of possible coherence, and with suitable normalization. Again there is qualitative agreement, but here the width of the calculated peak ($\approx$150 keV) is less than the measured width ($\approx$200 keV), the difference being too great to attribute to the experimental energy resolution ($\lesssim$40 keV). Reasonable changes in the value of g_{0x}/g_{1x} have little effect on the shape of the cross section. The large width observed for the peak favours smaller magnitudes of g_{0x}/g_{1x} and also the smaller allowed values of α.

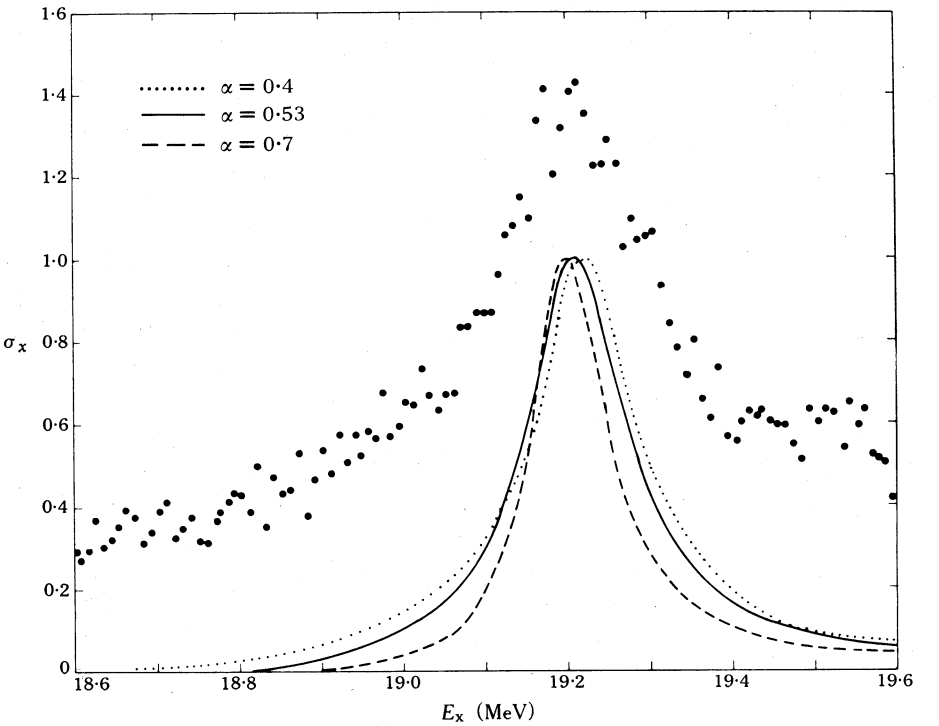

Fig. 6. Normalized contribution to the ^{9}Be(d, t)^{8}Be cross section σ_x due to 3^+ levels of ^{8}Be as a function of ^{8}Be excitation energy. The experimental points are from Oothoudt and Garvey (1977) and include a background contribution. The curves are calculated using parameter values obtained from fits to the ^{7}Li+p $^5P_3$ phase shift and shell model values of the feeding amplitudes, for three α values as in Fig. 4.

(*e*) ^{9}Be(p, d)^{8}Be *and* ^{9}Be(^{3}He, α)^{8}Be *Reactions*

If these reactions also proceed by neutron pickup, then the formulae and value of g_{0x}/g_{1x} are the same as for ^{9}Be(d, t). Thus the calculated curves of Fig. 6 should also be valid for these reactions. In the ^{9}Be(p, d) reaction, with $E = 33{\cdot}6$ MeV and an energy resolution in the deuteron spectrum of 100–130 keV, Kull (1967) observed a single peak corresponding to a level at 19·21 MeV with a width of 208 ± 30 keV. With $E_p = 185$ MeV, Sundberg and Källne (1969) observed a peak at 19·16 MeV with

an intrinsic width of 500 keV but their energy resolution was about 350 keV. Ajzenberg-Selove *et al.* (1976) studied the ^{9}Be(^{3}He, α)^{8}Be reaction, with a bombarding energy of 49·3 MeV and an energy resolution of about 50 keV, and observed a peak at 19·22 ± 0·03 MeV with a width of 265 ± 30 keV (Ajzenberg-Selove, personal communication).

As for the ^{9}Be(d, t) reaction, the measured values of the width are much greater than the calculated value, although the peak positions agree. Fits to the phase shift, in which the level parameters were restricted so that they would give a larger width for the 19·2 MeV peak in these cross sections, were acceptable only for widths less than about 160 keV.

(f) ^{7}Li(d, n)^{8}Be *Reaction*

If the ^{7}Li(d, n)^{8}Be reaction proceeds by stripping, then the feeding amplitudes g_{Tx} are proportional to the reduced width amplitudes γ_T used as parameters in the phase shift fit. Therefore additional parameters are not required in calculating the cross section. Since the proton decay channel gives the main contribution, there is destructive interference in the region between the levels, which produce a peak at about 18·93 MeV and a weaker peak at about 19·26 MeV. The former of these peaks would not be resolvable from a peak due to the 2^- level of ^{8}Be at 18·9 MeV, so the origin of the peak reported by Kerr (1967) at 19·05 MeV is not apparent.

4. Comparison with Values from Model Calculations

In this section we compare the parameter values obtained from fits to the $^5P_3$ phase shift, or quantities derived from them, with values obtained from model calculations.

The reduced width amplitudes γ_T of the 3^+ states of ^{8}Be for the $A = 7$ ground state channels may be written

$$\gamma_T = \mathscr{S}_T^{\frac{1}{2}}\{\theta_0^2(1\mathrm{p})\hbar^2/m_c a_c^2\}^{\frac{1}{2}}. \tag{12}$$

We take values of the spectroscopic amplitudes $\mathscr{S}_T^{\frac{1}{2}}$ from shell model calculations, and calculate the single-particle dimensionless reduced width

$$\theta_0^2(1\mathrm{p}) = \tfrac{1}{2}a_c u^2(a_c)\Big/\int_0^{a_c} u^2(r)\,\mathrm{d}r,$$

using radial wavefunctions $u(r)$ in a Woods–Saxon potential (Barker 1978). With $a_c = 5{\cdot}0$ fm, calculated values of γ_0, γ_1 (in MeV$^{\frac{1}{2}}$) are 0·46, 0·40 (Barker 1966), 0·42, 0·42 (Cohen and Kurath 1967) and 0·43, 0·40 (Kumar 1974). To avoid the uncertainty in the value of $\theta_0^2(1\mathrm{p})$, we may consider only values of γ_0/γ_1, which are 1·16, 1·01 and 1·08 respectively. Comparison with the values of γ_0 and γ_1 in Fig. 2 favours the smaller allowed values of α.

The value of E_a (or $E_a^{(a)}$) from Fig. 2 is about 18·94 MeV, which is 120 keV below the accepted value for the lower 3^+ level (Ajzenberg-Selove and Lauritsen 1974). This changed value of E_a leads to considerably different values of E_1^0 and V_{01} from those obtained or used previously, namely $E_1^0 = 19{\cdot}07$ MeV and $V_{01} = -63$ keV (Barker 1966), and $E_1^0 = 19{\cdot}09 \pm 0{\cdot}03$ MeV and $V_{01} = -60 \pm 12$ keV (Barker 1978).

Since our calculated values for the cross sections and γ_T favoured values of α somewhat lower than the 0·53 obtained in the best fit to the phase shift, we take the acceptable range of α as 0·4 to 0·6; then from Fig. 3 the range of E_1^0 is 18·99 to 19·05 MeV and the range of V_{01} is -115 to -145 keV. Fig. 7 is a redrawing of Figs 1*c* and 2*e* of Barker (1978) using these new experimental values. Agreement between the experimental and calculated values of V_{01} is now obtained for a smaller value of the channel radius $a_{\tilde{c}}$, about 4 fm, while the previous discrepancy for $\Delta_x(3^+,0)$ is now removed, without the requirement of a charge-dependent interaction other than the Coulomb interaction.

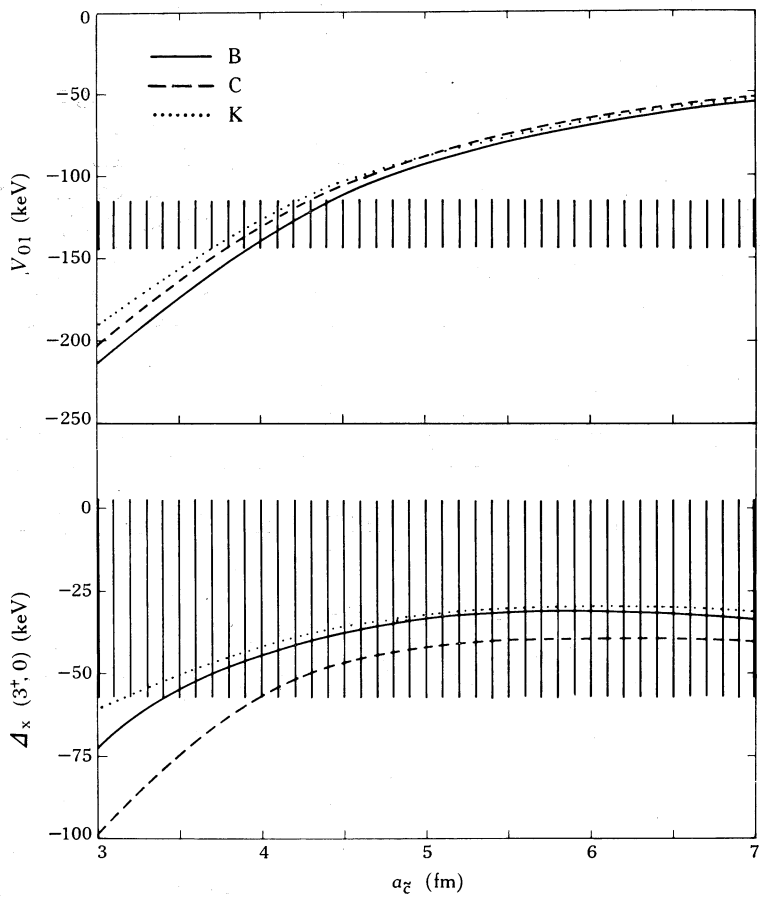

Fig. 7. Comparison of theoretical and experimental results for the variation with channel radius $a_{\tilde{c}}$ of the isospin mixing matrix element V_{01} for the 3^+ levels of ^{8}Be, and of the excitation energy difference $\Delta_x(3^+,0)$ for the 3^+ levels of ^{8}Li and ^{8}Be. The calculated values are taken from Barker (1978); they are for the interactions of B, Barker (1966); C, Cohen and Kurath (1965); K, Kumar (1974). The experimental results with estimated uncertainties as obtained in the present paper are denoted by the hatching.

5. Discussion

From a two-level *R*-matrix fit to the ^{7}Li+p $^5P_3$ phase shift, we have obtained level parameter values for the 3^+ doublet of ^{8}Be that should be more accurate than previous values. This has the effect of allowing qualitative agreement to be obtained

with the measured cross sections for the ^{7}Li(p, γ_1), ^{10}B(d, α)^{8}Be and ^{9}Be(d, t)^{8}Be reactions, involving reinterpretation of one of the peaks seen in the (d, α) reaction. It also improves the agreement with calculated values of the excitation energy difference for the 3^+, $T = 1$ levels of ^{8}Li and ^{8}Be and possibly of the isospin mixing matrix element in ^{8}Be.

The fits and predictions are by no means perfect. The discrepancy in the ^{7}Li(p, γ) cross section can be reduced by using a smaller channel radius, say $a_c = 4{\cdot}0$ fm, which increases the energy of the lower level, but the effect is too small to remove the discrepancy altogether. This change in the channel radius does not produce significant changes in the fits to the phase shift, the cross sections for the other reactions and the values of E_1^0 and V_{01}. Part of the discrepancies may be due to the data, and part may be attributed to the use of the two-level approximation and of the two-state isospin mixing model for describing the 3^+ doublet, thus ignoring effects of other 3^+ levels. If the restrictions of this model, as embodied in equations (4), were not imposed then one could probably improve the agreement in most quantities by reducing the width of the lower level and increasing the width of the upper level. Without this model, however, we could not have calculated the various reaction cross sections, since the γ-ray reduced width amplitudes and the feeding amplitudes would not be obtainable in a simple way.

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