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Evolution of the Radio Emission from Young SNRs according to Observational Data

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Abstract

Results are given from a still-continuing series of absolute flux density measurements for the SNR Cas A. The measurements have been taken regularly over the past 16 years at 24 wavelengths in the range 3–60 cm. New data are presented for the rate of flux density decrease, and for time and frequency variations in the spectral index. Intensity increases that are localized in frequency are described. The results are generalized in a model for the radio emission from a young SNR, and this is used to analyse experimental data for flux density decreases in the emission from 3C 10 and 58.

Introduction

Absolute flux density measurements for the SNR Cas A have been carried out at NIRFI over the past 16 years. Antenna temperature calibration is performed by reference to the radio emission from absolute black discs located within the antenna Fraunhofer zone, which provides a high accuracy of measurement (errors not larger than 3%). Utilization of the same radiotelescopes and standards throughout tends to conserve the permanence of systematic errors, and so offers advantages when results obtained at different epochs are intercompared. An additional control over the antenna aperture characteristics was provided by flux density measurements for Cyg A made over the same time span. No time variations in the intensity of this source were observed.

In order to investigate the evolution of the radio emission from the SNR Cas A, absolute flux density measurements were made regularly at two-yearly intervals (1967, '69, '71, '73, '75 and '77) at 24 wavelengths in the range 3–60 cm. The results obtained are described below.

Spectral Distribution of Radio Emission

Average spectral indices for Cas A over portions of the 3–60 cm band are listed in Table 1 for each of the two-yearly intervals described above. The spectral indices α (where $S \propto \nu^{-\alpha}$) were calculated by the method of least squares from absolute flux density measurements. It is clear from the table that the spectral index α_{3-60} (see column 2) for the spectrum as a whole is not representative of all parts of the band at each epoch. This spectral index also undergoes variations with epoch.

If we compare the data for α_{3-60} in Table 1 at separate epochs, we see that the index averaged over the entire source spectrum changes non-monotonically with time; relative to its mean value, the oscillations amount to $\Delta\alpha_{3-60} = \pm 0{\cdot}03$. Thus, in the period 1969–73 the spectral index decreased by $\Delta\alpha_{3-60} = -0{\cdot}064$, amounting to $-2 \cdot 4\%$ per annum. In 1973–75 it increased by $\Delta\alpha_{3-60} = +0 \cdot 035$. Superimposed upon the overall spectrum are local variations observed in the range 3–4 GHz (7·5–10 cm); these are exemplified by a flux density increase in the period 1971–73, with a subsequent rapid decrease to the level of the undisturbed spectrum (Barabanov *et al.* 1977). The corrected spectral index $\alpha_{3-60}^{\text{corr}}$ listed in column 6 of Table 1 has been calculated by disregarding points at which the local flux density increases were observed.

Fig. 1*a* gives a detailed view of the spectrum in the 3–4 GHz band ($\sim$0·5–0·6 in $\log_{10} \nu$), showing the local flux density variations. Here the upper and lower straight lines represent the source spectrum plotted according to the 1969 and 1975 data respectively. For these years the spectrum does not contain any peculiarities, and the spectral index over the 3–4 GHz band corresponds to that over the broader 0·5–10 GHz band. The difference between the flux densities for the two epochs characterizes a secular decrease in the flux density over six years with an average rate $S^{-1}\,\mathrm{d}S/\mathrm{d}t = -0 \cdot 7\% \pm 0 \cdot 15\%$ per annum.

Table 1. Variations in spectral index of Cas A

α_{a-b} is the spectral index averaged over the band *a–b* cm, while $\alpha_{3-60}^{\text{corr}}$ is the average spectral index across the 3–60 cm band corrected for local flux density variations in the 3–4 GHz range

(1) Epoch	(2) α_{3-60}	(3) α_{3-20}	(4) α_{20-60}	(5) α_{17-35}	(6) $\alpha_{3-60}^{\text{corr}}$
1967	—	—	$0 \cdot 855 \pm 0 \cdot 02$	—	—
1969	$0 \cdot 857 \pm 0 \cdot 008$	$0 \cdot 843 \pm 0 \cdot 012$	$0 \cdot 860 \pm 0 \cdot 022$	$0 \cdot 927\,^{+0 \cdot 028}_{-0 \cdot 038}$	$0 \cdot 857 \pm 0 \cdot 008$
1971	$0 \cdot 8362^{+0 \cdot 0056}_{-0 \cdot 0033}$	$0 \cdot 799 \pm 0 \cdot 011$	$0 \cdot 782 \pm 0 \cdot 017$	$0 \cdot 987\,^{+0 \cdot 006}_{-0 \cdot 012}$	$0 \cdot 8407 \pm 0 \cdot 005$
1973	$0 \cdot 7931 \pm 0 \cdot 0056$	$0 \cdot 796 \pm 0 \cdot 007$	$0 \cdot 795 \pm 0 \cdot 014$	$0 \cdot 778 \pm 0 \cdot 007$	$0 \cdot 8106 \pm 0 \cdot 005$
1975	$0 \cdot 8286 \pm 0 \cdot 0032$	$0 \cdot 8052 \pm 0 \cdot 005$	$0 \cdot 8278 \pm 0 \cdot 011$	$0 \cdot 828 \pm 0 \cdot 023$	$0 \cdot 8286 \pm 0 \cdot 0032$
1977	$0 \cdot 8154 \pm 0 \cdot 0052$	$0 \cdot 7963 \pm 0 \cdot 0115$	$0 \cdot 8368 \pm 0 \cdot 0122$	$0 \cdot 8302 \pm 0 \cdot 0313$	$0 \cdot 8154 \pm 0 \cdot 0052$

During the period 1969–73 a gradual frequency-dependent intensity increase, with a maximum at $\lambda = 8$ cm (3·8 GHz), was observed to be superimposed on the undisturbed spectrum. At this wavelength, the flux density increment above the overall secular decrease amounted to 140 Jy or 14·5%; the increase occurred uniformly at a rate $S^{-1}\,\mathrm{d}S/\mathrm{d}t = 3 \cdot 6\% \pm 0 \cdot 7\%$ per annum. During the subsequent two-year period (1973–75) the flux density decreased to the level of the undisturbed spectrum, at a rate accounted for by linear approximation as being $S^{-1}\,\mathrm{d}S/\mathrm{d}t = -(6\text{–}7)\%$ per annum. A diagram of the flux density variation at $\lambda = 8$ cm is shown in Fig. 1*b*. Thus the following picture emerges: within the radio emission spectrum of the SNR in the 3–4 GHz range (and probably at some higher frequencies) an additional component was present, the lifetime of which was 5–6 years.

An estimate of the spectral index of the additional component at wavelengths shorter than 8 cm was made using the 3·2 cm data for the corresponding epochs. At 3·2 cm the contribution of the variable component has fallen to no more than 20 Jy. The indices obtained for the intensity spectrum and for the energetic power spectrum of the relativistic electrons amount to $\alpha \geqslant 2$ and $\gamma \geqslant 5$ respectively. These values are essentially uncharacteristic of the Cas A radio emission spectrum as a whole ($\alpha = 0 \cdot 8$ and $\gamma = 2 \cdot 6$) and evidently describe a spectral component independent of the source envelope.

Should a transient optical phenomenon be observed having a similar duration to the additional radio burst component then attention would focus on the decay time for moving condensations within the SNR. According to the observations (Van den Bergh and Dodd 1970) this decay time amounts to no more than 10 years.

When plotting a new map of Cas A with the 5 km Cambridge radiotelescope at 5 GHz in 1974, Bell *et al.* (1975) detected some very compact regions of strong radio emission within the envelope. The strongest emission was observed to come from a compact region at R.A. $23^{h}21^{m}06^{s}\cdot 3$ and Dec. $58^{\circ}29'47''$ (of angular dimensions $4''$) which coincides with an optical filament observed in 1974. It is significant that a typical optical survey of the SNR in 1969 revealed no visible objects at this location. Thus there are good reasons to believe that a burst seen above the background of the overall Cas A spectrum corresponds to radio emission from a large condensation during the period of its activity. That is why the calculation of the intensity and spectrum of the radio burst component observed here is of interest.

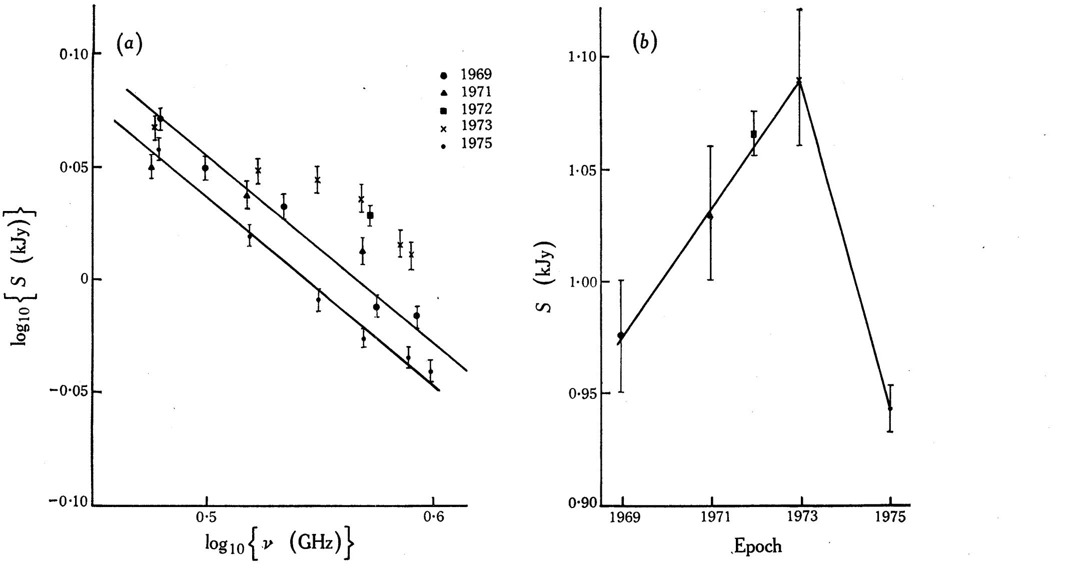

Fig. 1. Intensity variations of Cas A during the period 1969–75:

(*a*) logarithmic plot of the flux density S as a function of frequency ν over the 3–4 GHz band ($\log_{10}\nu \approx 0\cdot 5$–$0\cdot 6$), with average source spectra for the years 1969 and 1975 represented by the upper and lower straight lines respectively;

(*b*) linear plot of the flux density S as a function of time at wavelength 8 cm ($3\cdot 8$ GHz).

According to Searle (1971) the electron temperature in the moving condensations is $T_e = 2\times 10^4$ K and the density of the thermal electrons is $n_e = 3\times 10^4\,\mathrm{cm}^{-3}$. With typical dimensions for the condensations of $l = 10^{17}$ cm (Van den Bergh 1971), corresponding to an angular dimension of $\theta \approx 2''$, the radio spectrum for the condensation would have a turnover due to absorption by free–free transitions at frequencies $\nu \leqslant 4$ GHz, which corresponds to the maximum frequency at which the radio burst was observed. When choosing values for the magnetic field H we use the data of Bell *et al.* (1975) who found that, to interpret the radiation power of the compact regions, a value of $H = 3\times 10^{-3}$ G was necessary.

A synchrotron mechanism may generate electromagnetic waves if the energy ε of the relativistic electrons is sufficiently large that $\varepsilon/mc^2 \gg \omega_{pe}/\omega_{He}$, where ω_{pe} and ω_{He} are the plasma frequency and the gyrofrequency of the ambient electrons. For the condensation conditions given above, we have $\varepsilon \gg 10^8$ eV, which exceeds by an order of magnitude the energy of the relativistic electrons in the envelope of the SNR. Using the known formulae for synchrotron radiation and imposing the condition of equality between the volume energy densities of the relativistic particles and the magnetic field, we obtain the following expression for the radio emission flux density of a condensation:

$$S \approx 5\times 10^{20}\,\varepsilon_*^{\gamma-2}\,H^{(\gamma+5)/2}\,(6{\cdot}26\times 10^{18}/\nu)^{(\gamma-1)/2}\,\theta^3 \quad \mathrm{Jy}, \tag{1}$$

with ε_* in ergs, ν in hertz and θ in radians.

For $\gamma = 5$, a cutoff energy $\varepsilon_* = 2\times 10^8$ eV for the relativistic electrons, and $\nu = 4$ GHz, we find that a flux density $S = 140$ Jy may be radiated by an inhomogeneity of size $\theta \approx 5''$. The large condensation observed by Bell *et al.* (1975) had dimensions $\theta \approx 3$–$4''$ but, in principle, the radio emission may be generated by several condensations. Bell *et al.* noted large regions with increased activity within condensations, especially in the SW. section of the envelope.

Under conditions of dense condensations, plasma mechanisms of radiation may be more effective than the synchrotron mechanism. For the synchrotron mechanism to generate Langmuir waves, the relativistic electrons must have relatively low energies:

$$\left(\frac{\omega_{pe}}{\omega_{He}}\right)^{\frac{1}{2}} < \frac{\varepsilon_*}{mc^2} < \frac{\omega_{pe}}{\omega_{He}} \qquad \text{or} \qquad 10 < \varepsilon_* < 2\times 10^8 \quad \mathrm{eV}. \tag{2}$$

Compton scattering of Langmuir and electromagnetic waves by relativistic electrons leads to a radio emission power spectrum with index $(\gamma-1)/2$ and a maximum at $\omega_{max} = 2\omega_{pe}(\varepsilon_*/mc^2)^2$ (Kaplan and Tsytovich 1972). For condensations we have $\omega_{pe} = 10$ MHz so that, for $\varepsilon_* = 2\times 10^7$ eV, we have ω_{max} decreasing in the 3–4 GHz range, i.e. in the vicinity of the maximum of the observed radio burst. It may be shown that the ratio of the radiating capabilities I_ω^t due to Compton scattering and I_ω due to synchrotron radiation is given by

$$\frac{I_\omega^t}{I_\omega} = \frac{8\pi W_e}{H^2}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^{(\gamma-3)/2}, \tag{3}$$

where W_e is the energy of the plasma waves. To obtain an estimate of this ratio we take $W_e \approx H^2/8\pi$; it then follows that at $\gamma > 3$ the plasma mechanism is energetically more advantageous. In our case $I_\omega^t/I_\omega \approx 10^2$, and hence the angular dimensions of the radiating region decrease to $\theta = 1$–$2''$, that is, they correspond to the average dimensions of rapidly moving condensations.

In the envelope of Cas A there are nearly 100 moving inhomogeneities, and they account for 10% of the power radiated by the source. However, in individual cases, conditions may be realized where the radiation from single condensations is very intense. The characteristic feature of such radiation is a steep spectrum. Thus, as early as 1964, an increase in flux density at long wavelengths (25–30 cm) was detected in the spectrum of Cas A, and this was interpreted as being additional radiation generated inside the condensations by relativistic electrons with $\gamma = 4 \cdot 6$ (Stankevich 1964). This implies that generation of inherently relativistic particles must occur in such condensations. It might be expected that decreases in optical depth are responsible for the variations in radio emission. However, if the period of the activity coincides in both the optical and radio ranges, a more probable cause is variations in the relativistic particle density inside the condensation: the buildup of the relativistic particles from the moment the burst commences until it attains its maximum, and their subsequent ejection into the envelope.

The radio emission from condensations influences the behaviour of the source spectrum. As a whole, the spectrum of the integrated radio emission from the SNR forms an envelope on which the emission from condensations is imposed. The latter has a nonstationary character and is a partial cause of variations in the spectral index with time and with frequency for a given epoch.

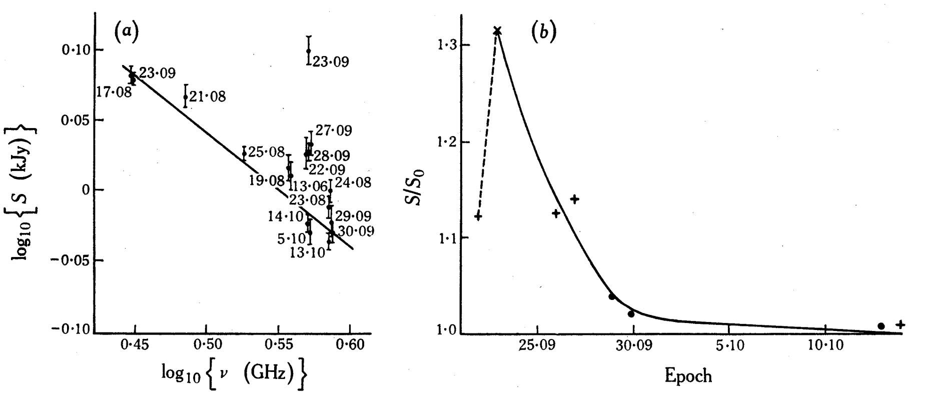

Fig. 2. Intensity variations of Cas A during the period Aug.–Oct. 1977, with dates indicated numerically by the day followed by the month:

(*a*) logarithmic plot of the flux density S as a function of frequency ν over the 2·8–4 GHz band ($\log_{10}\nu \approx 0 \cdot 45$–$0 \cdot 6$), with the quiescent spectrum represented by the straight line;

(*b*) linear plot of the flux density S relative to the quiescent value S_0 as a function of time at wavelength 8 cm (3·8 GHz).

Short Bursts within Spectrum

During our extended series of observations, several sets of measurements were made continuously for many months at separate frequencies in a search for effects of the burst type, i.e. short-lived variations of the intensity. In the period August–October 1977, during observations at frequencies in the range 3·7–3·9 GHz, the detected increase and subsequent decrease in intensity accounted for nearly 10% of the average flux density of the source. Fig. 2*a* shows the behaviour of the Cas A spectrum during this period over the 2·8–4 GHz band (0·45–0·6 in $\log_{10}\nu$), while the development of the process with time at 3·8 GHz is shown in Fig. 2*b*.

Most probably, phenomena of the type shown in Fig. 2 may be referred to the radio emission of condensations, but another interpretation is possible. It was noted above that a condensation is a plasma region with a strongly developed turbulence. If both radiation and relativistic particles are 'locked' inside the condensation then conditions are realized for a turbulent plasma reactor, where stochastic acceleration of particles is possible due to radiation absorption. But conditions for a reactor with a high boundary frequency of $\omega_R \approx 10$ GHz may be realized only in filaments of large dimension. Bell *et al.* (1975) noted that the envelope contains regions of increased activity which decay in a number of filaments as a result of strong internal turbulent motions. If the whole region of activity is a turbulent plasma reactor then, during its decay (or in a localized region), it may give rise to radiation within a limited frequency interval. Estimates show that conditions for a synchrotron reactor are more advantageous than those for a Compton reactor, and parameters relevant to a synchrotron reactor are not inconsistent with those pertaining to the surrounding medium.

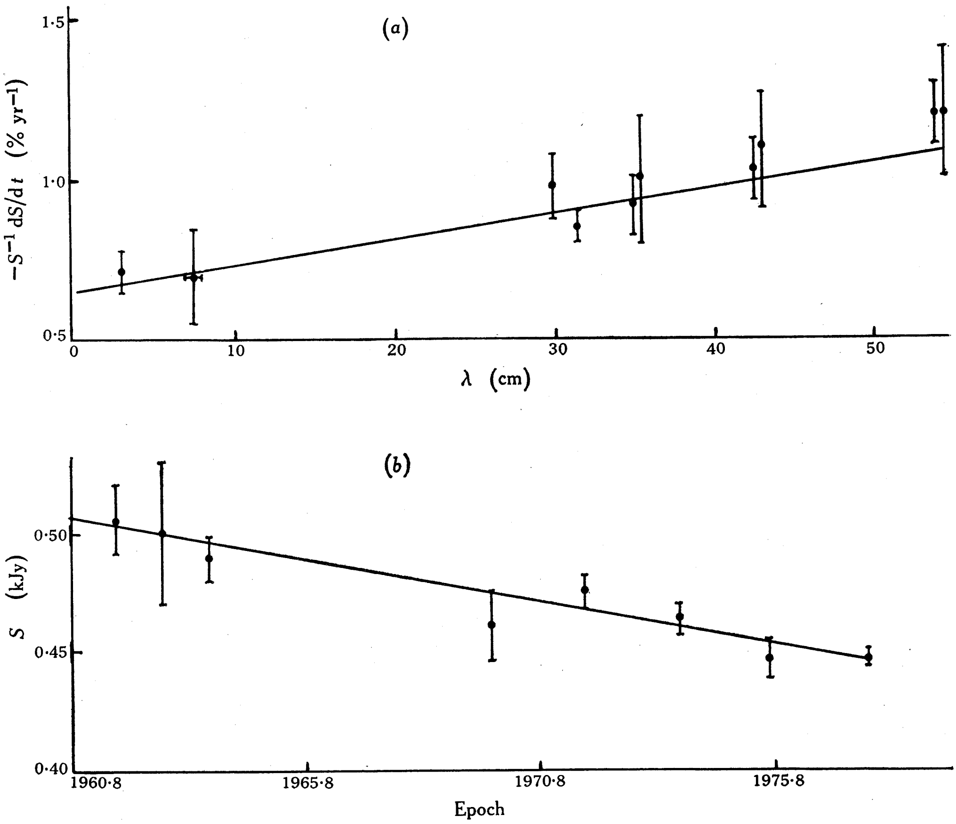

Fig. 3. Intensity variations of Cas A:

(*a*) average annual rate $-S^{-1}\,dS/dt$ of flux density decrease as a function of wavelength λ over the 3–54 cm band;

(*b*) flux density S at 3·2 cm as a function of time, showing the decrease from 1960·8 to 1977·8.

Secular Decrease in Flux Density

The secular decrease in the flux density of the Cas A SNR was investigated at a number of wavelengths using data from absolute measurements and from a relative comparison with the intensities of the Cyg A and Tau A sources. Some results have been published (Stankevich *et al.* 1973*a*, 1973*b*; Stankevich 1977). Both sets of these data are used in Fig. 3*a*, which shows the average annual rate of flux density decrease as a function of wavelength in the 3–54 cm band. On the other hand, Fig. 3*b* shows the decrease in flux density at 3·2 cm over the much longer period of 17 years. Experimental values for the average annual rate of decrease Ω in the flux density depend on the wavelength, and are well approximated by

$$\Omega = -1\cdot 25\times 10^{-3}(29 - \ln\nu), \tag{4}$$

where ν is in hertz.

A secular decrease in flux density was predicted by Shklovsky (1960) in his theory of the adiabatic expansion of SNRs. In subsequent papers, different variants of the evolution of such objects have been considered, for which corresponding rates of the flux density decrease have been calculated. However, the observed frequency dependence of the decrease does not follow from any of these models. It is known that a supernova envelope is not uniform in its radiating capability. In a two-component model of a source, consisting of radiation from an expanding envelope and from a stationary component, the frequency dependence of the annual flux density decrease may be explained in general terms. But then the integrated spectrum should possess a break followed by a decrease in the spectral index at high frequencies. Detailed experimental investigations of the spectrum of Cas A exclude such a possibility.

Again, difficulties occur due to the very large rate observed for the flux density decrease of the variable component. It may be shown in general that, if the spectrum of the source radio emission is described by a function of the form $S = A(t)\nu^{-\alpha}$ and there is also a frequency dependence for the rate $S^{-1}\,\mathrm{d}S/\mathrm{d}t$ of flux density decrease, the spectral index $\alpha(t)$ must depend on time.

Thus, to interpret the $\Omega(\nu)$ dependence we consider the radio emission from a homogeneous envelope of a source with varying spectral index. For a synchrotron radiation mechanism the flux density S, being proportional to the radiating capability J_ν and the volume R^2d of the generation region (where R and d are the external radius and depth of the envelope), is well described by the known relation

$$S(\nu,t) \approx J_\nu R^2 d = 13\cdot 5\,a(\gamma)\,\kappa_{\mathrm{e}}\,H^{(\gamma+1)/2}(6\cdot 26\times 10^{18}/\nu)^{(\gamma-1)/2}R^2 d \quad \mathrm{Jy}, \tag{5}$$

where the electron energy spectrum is given by $\mathrm{d}n(\varepsilon) = \kappa_{\mathrm{e}}\,\varepsilon^{-\gamma}\,\mathrm{d}\varepsilon$. Using the condition that the energy density of the relativistic electrons equals that of the magnetic field we obtain the relations

$$\kappa_{\mathrm{e}} = \frac{H^2}{8\pi}(\gamma-2)\varepsilon_*^{\gamma-2} \qquad \text{and} \qquad n = \frac{\gamma-2}{\gamma-1}\frac{H^2}{8\pi}\frac{1}{\varepsilon_*}, \tag{6}$$

where n is the total number density of the relativistic electrons.

At $\gamma = 2\cdot 6$ the rate of change in flux density due to variations in γ and to expansion losses $S^{-1}(\mathrm{d}S/\mathrm{d}t)_R$ has the form

$$\frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t} = \left(47\cdot 3 + \ln(H\varepsilon_*^2) - \ln\nu\right)\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \frac{1}{S}\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_R. \tag{7}$$

If we consider that the decrease in the source radio spectrum at low frequencies is a consequence of the distribution function for the relativistic electrons, $f_\varepsilon \propto \varepsilon^2$ in the energy region $\varepsilon < \varepsilon_*$, then the product $H\varepsilon_*^2$ defines the location of the maximum to be at frequency $\nu_* = 4\cdot2\times10^{18}\,H\varepsilon_*^2$. For Cas A we have $\nu_* = 20$ MHz, and hence $H\varepsilon_*^2 = 5\times10^{-12}$. Then from equation (7) it follows that

$$\frac{1}{S}\frac{dS}{dt} = \left(21\cdot3 - \ln\nu\right)\frac{d\alpha}{dt} + \frac{1}{S}\left(\frac{dS}{dt}\right)_R. \tag{8}$$

Comparing equation (8) for $S^{-1}(dS/dt)_R$ with the experimental dependence Ω (equation 4) we obtain annual rates of decrease in the spectral index and in the flux density due to expansion of the envelope to be $d\alpha/dt = -1\cdot25\times10^{-3}$ and $S^{-1}(dS/dt)_R = 7\cdot7\,d\alpha/dt = -0\cdot9\,\%\,\mathrm{yr}^{-1}$ respectively. The estimate for $S^{-1}(dS/dt)_R$ depends considerably on the values of H and ε_*. At 1 GHz, the measured flux density is 2700 Jy, and so for $H\varepsilon_*^2 = 5\times10^{-12}$ and $d = 0\cdot2\,R$, we obtain $H = 7\times10^{-4}$ G and $\varepsilon_* = 50$ MeV. These values are somewhat larger than the ordinarily accepted ones for Cas A, namely $H = 2\cdot5\times10^{-4}$ G and $\varepsilon_* = 10$ MeV (Shklovsky 1976*a*), for which $S^{-1}(dS/dt)_R = -1\cdot5\,\%$ per annum—a value close to the limit given by the theory of adiabatic expansion. However, the decrease in the spectral index of the radio emission must be a consequence of those processes of relativistic particle acceleration that exclude the adiabatic expansion of the envelope. That is why a value of $-0\cdot9\,\%$ per annum for the flux density decrease is the most probable value. With this interpretation, attention focuses on the hydrodynamic model of a supernova since it permits the possibility of relativistic particle acceleration in the convective zone. Using this model, Gull (1973*b*) obtained a value for the flux density decrease in Cas A of $S^{-1}(dS/dt)_R = -0\cdot9\,\%$ per annum. However, it is of interest to analyse Gull's calculation in detail.

The expression for the radio emission flux density of the source is conveniently given in the form

$$S \propto R^3 n H^{\alpha+1}\varepsilon_*^{2\alpha},$$

where the width of the convective zone is taken to be proportional to the radius, that is, $d \propto R$. This occurs when the ratio of the mass trapped by the matter envelope as it expands in the interstellar medium to the mass ejected at the supernova explosion lies in the range 0·3–3. When investigating the time dependence, Gull (1973*a*) made three assumptions: (i) the energy in the convective zone is distributed equally between the magnetic field, the relativistic particles and the turbulence of the medium, and hence we have $n \propto H^2\varepsilon_*^{-1}$; (ii) the number of radiating relativistic electrons is constant, that is, $R^3 n = $ const.; (iii) the energy of each electron changes with the total energy E_R of the convective zone. On the basis of Gull's (1973*b*) diagrams for Cas A we may calculate the density of the turbulence energy ε_R in the convective zone. It turns out that over a large interval of the mass ratio from 0·3–10 we have $\varepsilon_R \propto R^{-2}$. Then from assumption (i) we obtain $H \propto R^{-1}$. If, in the process of expansion, the total number of particles is not changed then from assumption (ii) and the number density formula (assumption i) we obtain $\varepsilon_* \propto R$. Here $S \propto R^{\alpha-1}$ and $|S^{-1}(dS/dt)_R| < 0\cdot1\,\%$ per annum, i.e. a rather small value in comparison with the observed one.

We can make different assumptions. In principle, the conditions given above define the magnetic field to be $H \propto R^{-3/2}E_R$. According to Gull (1973*b*), within the mass ratio range 0·3–3, we have $E_R \propto R$ and hence $H \propto R^{-1}$. Again we arrive at a small value for the annual flux density decrease.

The conditions $H \propto R^{-1}$ and $E_R = \text{const.}$ correspond in Gull's model to the empirically determined rate $S^{-1}(\mathrm{d}S/\mathrm{d}t)_R = -0{\cdot}9\%$ per annum. But, as shown above, for the assumptions we have made, H and E_R cannot be given independently. We may conclude that, in the supernova envelope, conditions are realized which differ from those calculated by Gull. To interpret the observations, Shklovsky's (1976*b*) hypothesis of energy conservation becomes attractive: the energy of the relativistic electrons within the convective zone is conserved throughout the expansion. This leads to another law for the magnetic field variation, namely $H \propto R^{-3/2}$, for which

$$S^{-1}(\mathrm{d}S/\mathrm{d}t)_R = -\tfrac{3}{2}(\alpha+1)/T. \tag{9}$$

Equation (9) yields the required rate of $-0{\cdot}9\%$ per annum for an age $T = 300$ years. However, to be consistent with the decrease of the spectral index of the radiation, energy conservation for relativistic particles in the convective zone is only possible under rather strict constraints on the acceleration mechanism that pumps the relativistic electrons from low to high energies.

In general, the rate of decrease due to losses by expansion is given by

$$S^{-1}(\mathrm{d}S/\mathrm{d}t)_R = -\tfrac{3}{2}(\alpha+1)\,R^{-1}\,\mathrm{d}R/\mathrm{d}t = -\tfrac{3}{2}(\alpha+1)\eta/T, \tag{10}$$

where T is the age of the supernova and $R \propto t^{\eta}$. For young supernova remnants we have $0{\cdot}4 \leqslant \eta \leqslant 1$. The general expression for the rate of flux density decrease has the form

$$\frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t} = \left(\frac{2}{2\alpha-1} + \ln(10\,\nu_*/\nu)\right)\frac{\mathrm{d}\alpha}{\mathrm{d}t} - \tfrac{3}{2}(\alpha+1)\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}. \tag{11}$$

It is of interest to consider the experimental data obtained for the SNRs 3C 10 (Stankevich *et al.* 1973*b*) and 3C 58 (V. P. Ivanov, personal communication) on the basis of the expression (11). The main characteristics of these sources are:

Source	α	T	$S^{-1}(\mathrm{d}S/\mathrm{d}t)_R$	Ω	$\mathrm{d}\alpha/\mathrm{d}t$
3C 10	0·74	400	$-0{\cdot}4\ \%\mathrm{yr}^{-1}$	$-0{\cdot}8\ \%\mathrm{yr}^{-1}$	-10^{-3}
3C 58	0·1	800	$-0{\cdot}2\ \%\mathrm{yr}^{-1}$	$-0{\cdot}4\ \%\mathrm{yr}^{-1}$	10^{-3}

The measured values for the flux density decrease are given at $\nu = 980$ MHz. It is seen from the tabulation that the rate of decrease in spectral index appears to decrease with time after the explosion. Thus, the mechanism of loss of relativistic electrons prevails in the envelope of 3C 58, leading to the fact that the spectral index at later stages of the expansion has a tendency to increase.

Equation (11) allows us to calculate the dependence of the average surface brightness Σ on the radius of the SNR through the relation

$$\Sigma = AR^{\beta}, \tag{12}$$

where the index β is given by

$$\beta+2 = \frac{S^{-1}\,\mathrm{d}S/\mathrm{d}t}{R^{-1}\,\mathrm{d}R/\mathrm{d}t} = -\tfrac{3}{2}(\alpha+1) + \frac{T}{\eta}\left(\frac{2}{2\alpha-1} + \ln(10\,\nu_*/\nu)\right)\frac{\mathrm{d}\alpha}{\mathrm{d}t}. \tag{13}$$

Experimental values previously obtained for β are $-4{\cdot}46$ (Milne 1970), $-3{\cdot}6\pm0{\cdot}5$ (Downes 1971), $-4{\cdot}01\pm0{\cdot}2$ (Ilovaiskij and Lequeux 1972) and -3 (Clark and Caswell

1976). It should be noted from equation (13) that the dependence given by $\beta = -4\cdot 2$ takes place at the average spectral index of the supernovae $\bar{\alpha} = 0\cdot 5$ for $d\alpha/dt = 0$. Thus, deviations observed in the value of $\beta - 2$ for separate supernovae in a Σ–R plot from the value $-\frac{3}{2}(\alpha+1)$ may be considered to be consequences of spectral index variations with time. Hence we have the possibility of obtaining independent estimates of $d\alpha/dt$, a most important parameter in the theory of the evolution of the radiation from an SNR.

Conclusions

As a result of continuing absolute measurements of radio emission from Cas A, interesting peculiarities have been detected. These mainly refer to the nonstationariness of the energy distribution of the spectrum, the frequency dependence of the rate of flux density decrease, and the time and frequency variations in the spectral index. Separate bursts have been detected in a limited frequency range, with intensity variations on monthly and 5–6 yearly time scales. It is possible that similar phenomena take place in other objects of this class; a decrease of flux density with time has been detected in some of them. This shows that within SNRs, even those which have no active pulsar, processes occur which lead to the generation and acceleration of relativistic particles. To improve the general picture regarding the evolution of the radio emission from SNRs it is most important to carry out detailed spectral measurements for young objects, which have envelopes in different stages of expansion.

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