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Partial Wave Analysis of the $^{18}O(p, \alpha_0)^{15}N$ Reaction

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Abstract

A partial wave analysis of the differential cross sections for the $^{18}O(p, \alpha_0)^{15}N$ reaction has been carried out by applying the formalism of Blatt and Biedenharn (1952), made specific for this reaction. The differential cross sections, measured at 200 keV intervals from 6·6 to 10·4 MeV bombarding energy, were subjected to least-squares fitting to this specific analytic expression. Two resonances were given by the analysis, the ^{19}F states being at $14\cdot71 \pm 0\cdot07$ MeV ($1/2^-$) and $14\cdot80 \pm 0\cdot07$ MeV ($1/2^+$).

1. Introduction

Jolivette and Richards (1969) pointed out that an explicit and simple analytic form for the differential cross section of the reaction $^{16}O(d, \alpha_1)^{14}N^*$ (2·31 MeV) could be obtained from the general equations of Blatt and Biedenharn (1952). In the case quoted a pair of spin zero even-parity particles emerge from a single state J^π of a compound nucleus formed initially by a 1^+ and 0^+ pair. The simple analytic form takes account of interference between all partial waves involved in both the incident and exit channels.

This paper reports that the same general approach of Jolivette and Richards (1969) is possible for the case where a spin zero and spin 1/2 pair form a single state of angular momentum J^π which then decays into a spin zero and spin 1/2 pair. This simple situation is exemplified by the $^{18}O(p, \alpha_0)^{15}N$ reaction, which is discussed here. The present measurement was made as part of an overall program to study the charged particles from ^{18}O plus proton reactions; complete details will be published later.

In Section 2, the analytic form for the differential cross section is given and discussed. Sections 3 and 4 present the experimental data and the results of the analysis respectively.

2. Form of Differential Cross Section

The $^{18}O(p, \alpha_0)^{15}N$ reaction has both entrance and exit channel spins of 1/2; the entrance channel spin s is $1/2^+$ while the exit channel spin s' is $1/2^-$. Thus, for a compound state of angular momentum and parity J^π, the angular momentum conditions are

$$\boldsymbol{s}+\boldsymbol{l} = \boldsymbol{J}^\pi = \boldsymbol{s}'+\boldsymbol{l}', \qquad s, s' = 1/2^+, 1/2^- . \tag{1}$$

In this case, both l and l' are uniquely determined by the angular momentum and parity of the compound nucleus state, such that

$$l = J-\tfrac{1}{2}(-1)^{\pi+J-\frac{1}{2}}, \qquad l' = J+\tfrac{1}{2}(-1)^{\pi+J+\frac{1}{2}}, \tag{2}$$

where π is the parity of the compound state. As a consequence it is meaningful to attempt to express the analytic form of the differential cross section in terms of the angular momentum J of the compound state.

Blatt and Biedenharn (1952) give the cross section formula

$$\mathrm{d}\sigma_{\alpha's',\alpha s} = \frac{1}{\hat{s}^2}\sum_{m_s=-s}^{+s}\ \sum_{m_s'=-s'}^{+s'} \mathrm{d}\sigma_{\alpha's'm_{s'},\alpha s m_s}, \tag{3a}$$

where

$$\frac{\mathrm{d}\sigma_{\alpha's'm_{s'},\alpha s m_s}}{\mathrm{d}\Omega} = \lambda\!\!\!^{-2}_{\alpha}\Bigg|\sum_{J=0}^{\infty}\ \sum_{M=-J}^{+J}\ \sum_{l=|J-s|}^{J+s}\ \sum_{l'=|J-s'|}^{J+s'}\ \sum_{\mu'=-l'}^{+l'} \mathrm{i}^{l-l'}\pi^{\frac{1}{2}}\hat{l}$$

$$\times\Big(-S^{J}_{\alpha's'l',\alpha sl}\Big)(l0sm_s\,|\,JM)(l'\mu's'm_{s'}\,|\,JM)\,\mathrm{Y}_{l'\mu'}(\theta,\phi)\Bigg|^2. \tag{3b}$$

Here $S^{J}_{\alpha's'l',\alpha sl}$ is the probability amplitude for a collision with total angular momentum J from channel αsl into channel $\alpha's'l'$ (S is in fact an element of the scattering matrix), α (α') is used to denote all quantum numbers needed to specify the channel other than s and l (s' and l'), and the notation $\hat{x} \equiv (2x+1)^{\frac{1}{2}}$ has been used.

The simple relationships (2) between the angular momenta enable that part of the expression (3b) within the modulus signs to be written explicitly in terms of J and the sums to be carried out. Then, introducing explicit forms for the Clebsch–Gordan coefficients (Condon and Shortley 1935), and using the fact that

$$\mathrm{Y}_{l,1}(\theta,\phi) = \left(\frac{2l+1}{4\pi}\frac{(l-1)!}{(l+1)!}\right)^{\frac{1}{2}}\frac{\mathrm{dP}_l(\cos\theta)}{\theta\mathrm{d}},$$

we find

$$\frac{\mathrm{d}\sigma_{\alpha's',\alpha s}}{\mathrm{d}\Omega}$$

$$= \tfrac{1}{4}\lambda\!\!\!^{-2}_{\alpha}\Bigg[\Bigg|\sum_{J=\frac{1}{2}}^{\infty}\Big\{\Big(S_{J-\frac{1}{2},J+\frac{1}{2},J}\Big)(J+\tfrac{1}{2})\,\mathrm{P}_{J-\frac{1}{2}}(\cos\theta) - \Big(S_{J+\frac{1}{2},J-\frac{1}{2},J}\Big)(J+\tfrac{1}{2})\,\mathrm{P}_{J+\frac{1}{2}}(\cos\theta)\Big\}\Bigg|^2$$

$$+\Bigg|\sum_{J=\frac{1}{2}}^{\infty}\Big\{\Big(S_{J-\frac{1}{2},J+\frac{1}{2},J}\Big)\frac{\mathrm{dP}_{J-\frac{1}{2}}}{\mathrm{d}\theta} + \Big(S_{J+\frac{1}{2},J-\frac{1}{2},J}\Big)\frac{\mathrm{dP}_{J+\frac{1}{2}}}{\mathrm{d}\theta}\Bigg|^2\Big\}\Bigg]. \tag{4}$$

Here we have written $S_{ll'J}$ for the complex amplitude of the pair of partial waves that give a particular J^{π} for the compound state. Each sum contains contributions from both even and odd parity states of a given J. The angle θ is measured in the centre of mass system, and $\lambda\!\!\!^{-}_{\alpha}$ is the reduced wavelength of relative motion in the $^{18}O+p$ channel.

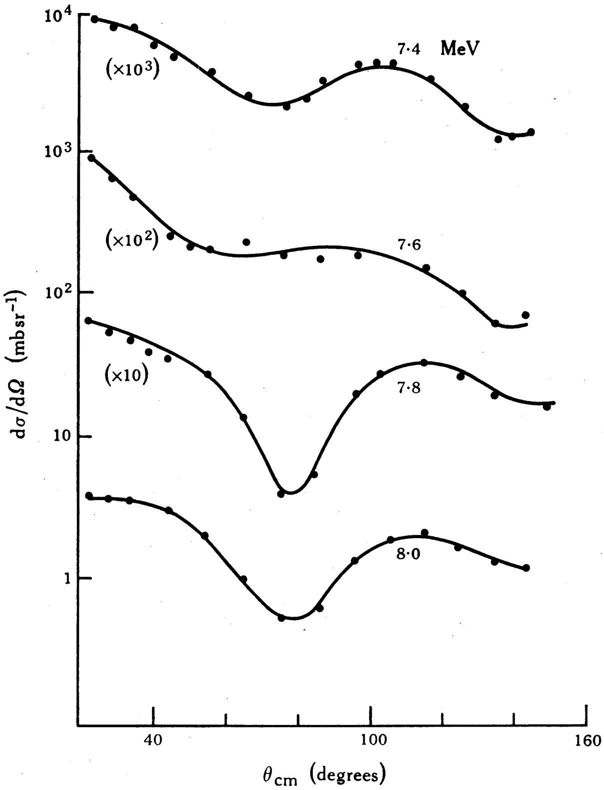

Fig. 1. Differential cross sections of α particles leaving ^{15}N in its ground state, for proton bombarding energies between 7·4 and 8·0 MeV. These results are presented as a typical set of differential cross sections. The curves represent least-squares fits to equation (4). Statistical errors are about the size of the data points.

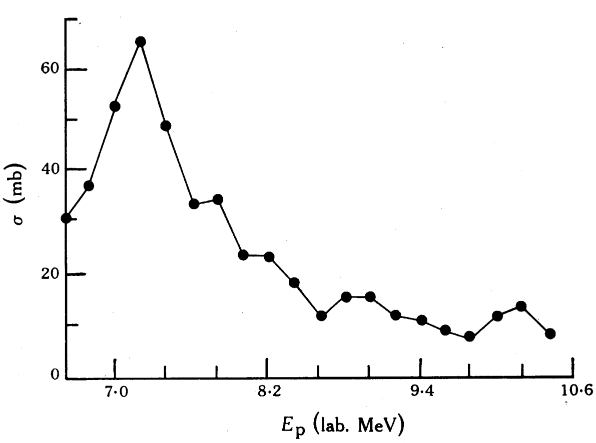

Fig. 2. Total cross section σ for the $^{18}O(p, \alpha_0)^{15}N$ reaction as a function of the proton energy E_p between 6·6 and 10·4 MeV.

The trajectory of the complex amplitude $S_{ll'J} = R_{ll'J}\exp(i\phi_{ll'J})$ corresponding to an isolated resonance is a circle traced in the counter-clockwise direction. The radius of this resonance circle is $(\Gamma_{\alpha l}\Gamma_{\alpha' l'})^{\frac{1}{2}}/\Gamma$, and its centre is determined by the nonresonant background, as discussed by McVoy (1967); Γ is the total width of the resonance, $\Gamma_{\alpha l}$ and $\Gamma_{\alpha' l'}$ being the partial widths appropriate to the entrance and exit channels respectively.

3. Experiment

Differential cross sections were measured for the $^{18}O(p, \alpha_0)^{15}N$ reaction at 200 keV intervals from 6·6 to 10·4 MeV proton energy in the laboratory frame. The target system consisted of a small volume gas cell; its construction and the manner of obtaining absolute cross sections using it have been described previously (Crinean *et al.* 1975). The particle identification system, which was necessary to separate the α particles from scattered protons, deuterons and tritons, has also been described previously (Hudson *et al.* 1972). The protons were obtained from the Melbourne University variable energy cyclotron. Measurements were also made of the excitation functions at 35° and 90° laboratory angles, in 50 keV steps. Excitation functions for this reaction have not been measured previously in the present energy range.

The most significant feature of the angular distributions is the rapid change of shape with change in bombarding energy. Indeed, there is no indication of any single consistent feature which might be taken as an indicator of a direct reaction process; this lack, in fact, encouraged the attempt to analyse the reaction data within the compound nucleus framework.

The differential cross sections are shown in Fig. 1. The total cross section was obtained by fitting the data with a Legendre polynomial series, and Fig. 2 shows a plot of this total cross section ($= 4\pi a_0$) as a function of proton energy. The probable error associated with the absolute cross section values is estimated to be about $\pm 10\%$. The dominant feature of the total cross section is the large peak centred at 7·2 MeV incident proton energy.

4. Analysis and Results

The measured differential cross sections were least-squares fitted to equation (4) and the magnitudes and phases of the reaction amplitudes $S_{ll'J}$ were determined as a function of energy. The number of data points in the measured angular distributions limited the number of partial waves that could be considered. The compound nucleus states able to be considered were thereby limited to J^{π} up to $5/2^{\pm}$. No satisfactory fit was obtained for the 6·6 MeV angular distribution; however, all other cases considered were fitted well. This is consistent with the finding of the Legendre polynomial series fit which indicated that the inclusion of terms higher than $P_6(\cos\theta)$ gave no improvement in the χ^2 of the fit to angular distributions above 6·6 MeV.

Two methods of searching for the reaction channel amplitudes were employed. The first method made an independent fit to each differential cross section, while the second method used the parameters found in the fit at one energy as starting parameters in the search for the best fit at the next energy. The energy dependence of the magnitudes of all the reaction channel amplitudes determined by these methods is shown in Fig. 3.

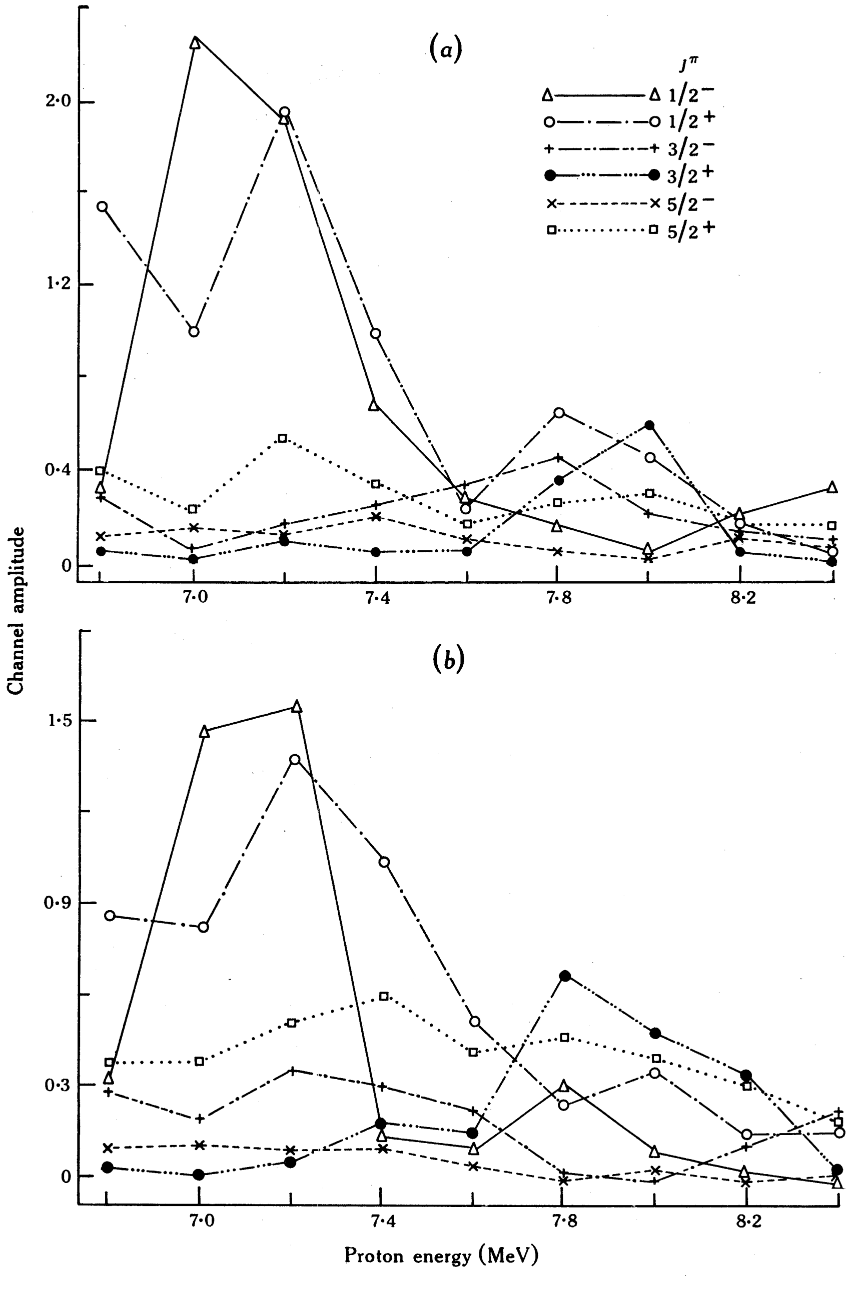

Fig. 3. Energy dependence of the reaction channel amplitudes $S_{ll'J}$, for the indicated J^π values, as a function of proton bombarding energy. The results were determined by two methods: (*a*) an independent fit to each differential cross section, and (*b*) parameters found at one energy being used as starting parameters for the search at the next energy.

The two fitting methods produced differences in detail in the reaction amplitudes $S_{ll'J}$; however, the dominant features are the substantial peaks in the $J^{\pi} = 1/2^{+}$ and $1/2^{-}$ reaction amplitudes near 7·2 MeV. The differences in detail are due to inherent ambiguities in this method of analysis (Jolivette and Richards 1969). The results indicate that the peak observed in the excitation function is due to two overlapping states which interfere. The energy dependences of the $1/2^{+}$ and $1/2^{-}$ reaction amplitudes were fitted with Breit–Wigner resonance shapes, and these fits resulted in the resonance parameters given in Table 1.

Table 1. Resonance parameters in $^{18}O(p, \alpha_0)^{15}N$ cross section

Resonance J^{π}	Proton energy (c.m.) E_p (MeV)	^{19}F excitation energy (MeV)	Width (lab.) (keV)
$1/2^{-}$	$6 \cdot 72 \pm 0 \cdot 07$	$14 \cdot 71 \pm 0 \cdot 07$	270 ± 70
$1/2^{+}$	$6 \cdot 80 \pm 0 \cdot 07$	$14 \cdot 80 \pm 0 \cdot 07$	400 ± 100

To attempt to confirm the resonance nature of the two peaks in the reaction amplitudes, attention was turned to the phases deduced for the $S_{ll'J}$ channel amplitudes from the least-squares fitting of the data to equation (4). As noted in Section 2 above, the work of McVoy (1967) indicates that the trajectory of the complex amplitude, for an isolated resonance, is a circle traced in the counter-clockwise direction. The plots of the two peaks are shown in Fig. 4.

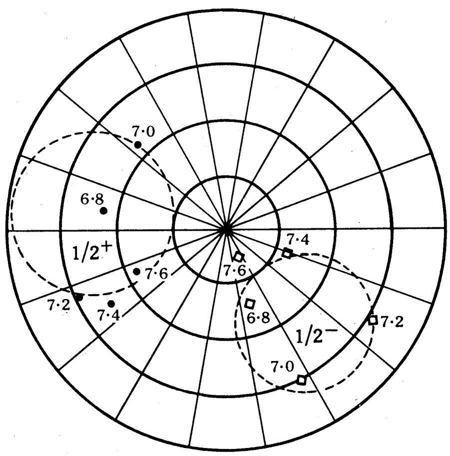

Fig. 4. Resonance 'circles' for the $1/2^{+}$ and $1/2^{-}$ states of ^{19}F (following McVoy 1967).

The present attempt to demonstrate the resonance circles from actual data suffers from several disadvantages. Firstly, the differential cross sections were measured at intervals of 200 keV, which are larger than is desirable for such resonances. Secondly, Fig. 3 shows substantial background beneath these two states. The lack of data below 6·6 MeV bombarding energy precludes any estimate of the energy dependence of this nonresonant background. Thirdly, there is known to be an inherent ambiguity in the phase determination in this method of resonance study (Jolivette and Richards 1969), and the present data do not contain enough information to remove this. We have attempted to minimize the ambiguity problem by plotting the phases determined in the second (sequential) method; it was felt that this procedure was more likely to lead to a consistent set of amplitudes and phases. Fig. 4 shows that this expectation was in fact borne out.

In spite of the above problems, Fig. 4 does show that resonance 'circles' are obtained, and this is taken as confirmation of the resonance nature of the peaks. Analysis of the differential cross sections therefore leads to the identification of two states in the compound nucleus ^{19}F, whose parameters are given in Table 1. The $1/2^-$ state appears to be a new state of ^{19}F, provided that the 14·80 MeV state ($1/2^+$) is identified with the resonance observed in the $^{18}O(p, n)^{18}F$ reaction by Bair *et al.* (1964), who give the excitation energy as $14 \cdot 78 \pm 0 \cdot 02$ MeV. The widths (laboratory system) also compare well: Bair *et al.* give 300 keV and our value is 400 ± 100 keV.

References

Bair, J. K., Jones, C. M., and Willard, H. B. (1964). *Nucl. Phys.* **53**, 209.
Blatt, J. M., and Biedenharn, L. C. (1952). *Rev. Mod. Phys.* **24**, 258.
Condon, E. U., and Shortley, G. H. (1935). 'Theory of Atomic Spectra', pp. 76–7 (Cambridge Univ. Press).
Crinean, G. B., Wild, L. W. J., and Spicer, B. M. (1975). *Nucl. Phys.* A **244**, 67.
Hudson, G. M., Crinean, G. B., Kelly, D. T., and Spicer, B. M. (1972). *Nucl. Phys.* A **184**, 175.
Jolivette, P. L., and Richards, H. T. (1969). *Phys. Rev.* **188**, 1660.
McVoy, K. (1967). *In* 'Fundamentals of Nuclear Theory' (Eds C. Villi and A. de Shalit), p. 478 (IAEA: Vienna).

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