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Quantum Theory of Energy Loss by Test Ions to Plasma Electrons

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Abstract

Full quantum-theoretical calculations are carried out for the rate of energy loss by energetic (but nonrelativistic) test ions to Maxwellian plasma electrons. The calculations are based on the kinetic theory of Kihara and Honda. For a test ion of speed V and charge Ze, the rate is given in terms of the quantum-theoretical Coulomb logarithm and tabulated universal functions of V, Z and the electron temperature T. It is shown that the loss rate formula derived here contains all known formulae as appropriate limiting cases. Several numerical examples are discussed.

1. Introduction

Consider the Coulomb collision of an ion of mass M, charge Ze and velocity $\boldsymbol{V}$ with an electron of mass m and velocity $\boldsymbol{v}$. In the classical context, the minimum impact parameter is given by (Jackson 1962; Kihara and Aono 1963)

$$b_c = Ze^2/mg^2, \tag{1}$$

where $\boldsymbol{g} = \boldsymbol{V}-\boldsymbol{v}$ and the reduced mass has been approximated by m. The de Broglie wavelength associated with this collision process is

$$b_q = \hbar/mg, \tag{2}$$

so that

$$b_c/b_q = Ze^2/\hbar g. \tag{3}$$

Under the condition $b_c/b_q \gg 1$, quantum diffraction effects are negligible and collisions may be studied within the framework of classical mechanics. If, on the contrary, we have $b_c/b_q \lesssim 1$ then quantum diffraction effects are important for close collisions and hence the collisions must be studied quantum theoretically. In the quantum limit we have $b_c/b_q \ll 1$ and the Born approximation is valid.

Let us next turn to collisions of the test ion with plasma electrons characterized by the Maxwellian velocity distribution

$$f(\boldsymbol{v}) = n(m/2\pi T)^{3/2}\exp(-mv^2/2T), \tag{4}$$

where n is the electron number density and T is the electron temperature in energy units. Depending on the magnitudes of V and T, some of the collisions may be described classically while others may require a quantum description. The overall importance of the quantum diffraction effect, however, may be judged from the appropriate average value $\bar{g}$ of g in the ratio (3). The energy loss by a test ion to plasma electrons in the two limiting cases $b_c/b_q \approx Ze^2/\hbar g \gg 1$ (classical limit) and $b_c/b_q \ll 1$ (quantum limit) has been studied in a recent paper by one of us (Hamada 1978; hereinafter referred to as paper H). The present work aims at deriving an energy loss formula that is valid for general values of the parameter b_c/b_q. In the appropriate limits the formula must reduce, of course, to the limiting forms obtained in paper H.

The necessity for such a general formula arises from the following considerations. The average $\bar{g}$ is characterized by the two parameters

$$\nu = Ze^2/\hbar v^* \qquad \text{and} \qquad x = V/v^*, \tag{5}$$

where $v^* = (2T/m)^{\frac{1}{2}}$ is the most probable thermal speed of plasma electrons. For $Z = 2$, $\nu \approx 1$ corresponds to $T \approx 50$ eV. Consider first a fast test ion such that $x > 1$. Since $g \approx V$, equation (3) implies

$$b_c/b_q \approx \nu/x \qquad (x > 1). \tag{6}$$

Thus even in a low temperature plasma with $\nu > 1$ the energy loss of a sufficiently fast ion must be described in the quantum limit. As the ion slows down, x decreases and, for $x < 1$, we have $g \approx v^*$ so that

$$b_c/b_q \approx \nu \qquad (x < 1). \tag{7}$$

Hence for $\nu \gg 1$ the energy loss formula in the classical limit applies in this case. In short, the energy loss by a sufficiently fast ion injected into a plasma with $\nu \gg 1$ will first be described by the quantum limit formula and finally by the classical limit formula, both of which are given in paper H. In between there is an intermediate region $b_c/b_q \approx 1$ where neither the quantum nor the classical limit applies. Thus in order to have a unified description of the energy loss process, a full quantum-theoretical formula that is valid for arbitrary values of the ratio b_c/b_q is clearly required. Such a formula will also be required to find the energy loss in a plasma of nonuniform or time-dependent temperature distribution.

Kihara (1964) and Honda (1964*a*, 1964*b*) give a full quantum-theoretical convergent kinetic equation which forms the basis of the present work. This kinetic equation is equivalent to the one of Gould and DeWitt (1967), according to Williams and DeWitt (1969). However, Honda (1964*b*) gives the loss rate only for $x \gg 1$ (arbitrary ν) and for $x \ll 1$ ($\nu \gg 1$ and $\nu \ll 1$). The loss rate for $x \gg 1$ and arbitrary ν has also been given by Gould (1971).

In Section 2 below we describe calculations which lead to an expression for the energy loss rate by a test ion to plasma electrons in terms of the quantum-theoretical Coulomb logarithm and tabulated universal functions of x and ν. In Section 3 we show that this expression contains all known loss rate formulae as appropriate limiting cases. Section 4 is devoted to a discussion of several consequences of interest which follow from the result obtained in Section 2. Further generalization of the present work to include the loss to plasma ions is not difficult. However, the result cannot be given in terms of universal functions alone, and separate numerical calculations are required for each given plasma species. Such calculations will be reported soon elsewhere.

2. Energy Loss Rate to Plasma Electrons

According to Honda (1964*b*), the full quantum-theoretical energy loss rate can be evaluated in two parts:

$$\mathrm{d}E/\mathrm{d}t = (\mathrm{d}E/\mathrm{d}t)_{\mathrm{I}} + (\mathrm{d}E/\mathrm{d}t)_{\mathrm{II}}. \tag{8}$$

The first part is the loss rate in the Born approximation and is given by

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{I}} = \frac{4(Ze^2)^2}{\hbar}\int \mathrm{d}\boldsymbol{v}\, f(\boldsymbol{v}) \int \mathrm{d}\boldsymbol{k}\, \frac{\boldsymbol{k}\,.\,\boldsymbol{V}}{|\,k^2\varepsilon(\boldsymbol{k},\omega)\,|^2}\, \delta(\boldsymbol{k}\,.\,\boldsymbol{g} + \hbar k^2/2m), \tag{9}$$

where

$$\varepsilon(\boldsymbol{k},\omega) = 1 + \frac{\omega_{\mathrm{p}}^2}{nk^2}\int \mathrm{d}\boldsymbol{u}\, \frac{\boldsymbol{k}\,.\,\{\partial f(\boldsymbol{u})/\partial \boldsymbol{u}\}}{\omega - \boldsymbol{k}\,.\,\boldsymbol{u} + \mathrm{i}\delta} \tag{10}$$

is the dielectric response function (with $\omega = \boldsymbol{k}\,.\,\boldsymbol{V}$) and the plasma frequency $\omega_{\mathrm{p}} = (4\pi ne^2/m)^{\frac{1}{2}}$. Terms of order m/M have been neglected. The dielectric response function $\varepsilon(\boldsymbol{k},\omega)$ takes care of the Debye shielding and the collective excitation for small k. The δ function, on the other hand, introduces a cutoff at large k corresponding to the impact parameter (2).

The second part in equation (8) gives the correction to the Born approximation and can be written in the form (Honda 1964*b*)

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{II}} = \frac{4\pi(Ze^2)^2}{m}\int \mathrm{d}\boldsymbol{v}\, \frac{\boldsymbol{v}\,.\,\boldsymbol{g}\, f(\boldsymbol{v})}{g^3}\left[\gamma + \mathrm{Re}\left\{\psi\left(\frac{\mathrm{i}Ze^2}{\hbar g}\right)\right\}\right]. \tag{11}$$

Here $\gamma = 0\cdot 577\,21\ldots$ is Euler's constant and $\psi(z)$ is the digamma function (Abramowitz and Stegun 1972).

The loss rate given by equation (9) is exactly the one in the quantum limit evaluated in paper H. This should be so since the Born approximation is valid in this limit. Thus we have

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{I}} = -\frac{(Ze\omega_{\mathrm{p}})^2}{V}\,\Psi(x)\left\{\ln\left(\frac{4T}{\hbar\omega_{\mathrm{p}}}\right) + \Delta_1(x) + \tfrac{1}{2}\Delta_2(x) + \tfrac{1}{2}\right\}, \tag{12}$$

where

$$\Psi(x) = \operatorname{erf} x - 2\pi^{-\frac{1}{2}}\, x \exp(-x^2) \tag{13}$$

and $\Delta_1(x)$ and $\Delta_2(x)$ are the slowly varying functions of x defined and tabulated by May (1969).

Let us next turn to the second part of the loss rate (8) as given by equation (11). Defining

$$\boldsymbol{x} = (m/2T)^{\frac{1}{2}}\boldsymbol{V}, \qquad \boldsymbol{s} = (m/2T)^{\frac{1}{2}}\boldsymbol{g}, \tag{14a, b}$$

we find

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{II}} = \frac{(Ze\omega_{\mathrm{p}})^2 x}{\pi^{3/2}V}\int \mathrm{d}\boldsymbol{s}\, s^{-3}\exp\{-(\boldsymbol{x}-\boldsymbol{s})^2\}\,(\boldsymbol{x}\,.\,\boldsymbol{s})\,[\gamma + \mathrm{Re}\{\psi(\mathrm{i}\nu/s)\}], \tag{15}$$

where ν/s is just the ratio (3). In passing, we note the relation

$$\frac{x}{\pi^{3/2}}\int \mathrm{d}\boldsymbol{s}\, s^{-3}\exp\{-(\boldsymbol{x}-\boldsymbol{s})^2\}\,(\boldsymbol{x}\,.\,\boldsymbol{s}) = \frac{4}{\sqrt{\pi}}\int \mathrm{d}s\, s^2\exp(-s^2) = \Psi(x). \tag{16}$$

We now define a new function Δ_3 by

$$\Delta_3(x,v) = -\gamma - \frac{x}{\pi^{3/2}\Psi(x)}\int \mathrm{d}\boldsymbol{s}\, s^{-3}\exp\{-(\boldsymbol{x}-\boldsymbol{s})^2\}\,(\boldsymbol{x}\,.\,\boldsymbol{s})\,\mathrm{Re}\{\psi(\mathrm{i}v/s)\}$$

$$= -\gamma - \frac{1}{2\sqrt{\pi}\,\Psi(x)}\left(\int_{-\infty}^{x} - \int_{x}^{\infty}\right)\mathrm{d}t\,\exp(-t^2)\,\frac{2x-2xt-1}{(t-x)^2}\,\mathrm{Re}\{\psi(\mathrm{i}v/t-x)\}\,. \tag{17}$$

Equations (8), (12) and (15) then lead to

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{(Ze\omega_{\mathrm{p}})^2}{V}\,\Psi(x)\left\{\ln\left(\frac{4T}{\hbar\omega_{\mathrm{p}}}\right) + \Delta_1(x) + \tfrac{1}{2}\Delta_2(x) + \Delta_3(x,v) + \tfrac{1}{2}\right\}, \tag{18}$$

which is the formula sought. This formula is valid for arbitrary values of x and v. It is subject, however, to the condition that the values of b_{c} and b_{q}, as defined in the Introduction, are both much less than the Debye length $(4\pi ne^2/T)^{-\frac{1}{2}}$ (Kihara 1964). The function $\Delta_3(x,v)$ is tabulated in Table 1. It is reassuring to note that the entries conform to the limiting expressions given in Section 3 below.

3. Limiting Cases

In the quantum limit $v/s \ll 1$ we have

$$\mathrm{Re}\{\psi(\mathrm{i}v/s)\} = -\gamma + O(v^2/s^2)\,.$$

Let $\bar{s}$ be an appropriate average of the variable s defined by equation (14b). Note that we have $\bar{s} \approx 1$ for $x \lesssim 1$, and $\bar{s} \approx x$ for $x \gtrsim 1$. Then, in view of the relation (16), we have in the limit $v/\bar{s} \ll 1$

$$\Delta_3(x,\, v \ll \bar{s}) = O(v^2/\bar{s}^2)\,, \tag{19}$$

so that equation (18) reduces, as it should, to the quantum limit obtained in paper H, which is equation (12) of the present paper.

In the opposite (classical) limit $v/s \gg 1$, we have

$$\mathrm{Re}\{\psi(\mathrm{i}v/s)\} = \ln(v/s) + O(s^2/v^2)\,.$$

According to the definition (17) and May (1969), this implies that

$$\Delta_3(x,\, v \gg \bar{s}) = -\ln v + \tfrac{1}{2}\{\Delta_2(x) + \ln 2 + 1\} - \gamma + O(\bar{s}^2/v^2)\,, \tag{20}$$

so that equation (18) reduces to

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{(Ze\omega_{\mathrm{p}})^2}{V}\,\Psi(x)\left\{\ln\left(\frac{8T^{3/2}}{Ze^2\omega_{\mathrm{p}}\,m^{\frac{1}{2}}}\right) + \Delta_1(x) + \Delta_2(x) + 1 - \gamma\right\}, \tag{21}$$

which is exactly the classical limit obtained in paper H.

Another interesting limiting case is $x \gg 1$, for which we have

$$\Delta_3(x \gg 1,\, v) = -\gamma - \mathrm{Re}\{\psi(\mathrm{i}v/x)\}\,. \tag{22}$$

Table 1. Values of the function $\Delta_3(x, \nu)$
Note that all entries are $-\Delta_3(x, \nu)$

Values of $-\Delta_3(x, \nu)$ for :

x	$\nu = 0·1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·2	1·4
0·1	0·042	0·136	0·246	0·359	0·470	0·576	0·677	0·772	0·862	0·946	1·099	1·236
0·5	0·039	0·127	0·232	0·339	0·445	0·548	0·645	0·738	0·825	0·908	1·059	1·193
0·8	0·035	0·114	0·208	0·307	0·406	0·502	0·595	0·683	0·767	0·846	0·993	1·125
1·0	0·031	0·102	0·188	0·279	0·371	0·461	0·549	0·633	0·714	0·790	0·933	1·061
1·2	0·026	0·088	0·164	0·246	0·330	0·413	0·495	0·574	0·651	0·724	0·860	0·985
1·4	0·022	0·074	0·139	0·211	0·286	0·361	0·436	0·509	0·580	0·649	0·779	0·898
1·6	0·017	0·059	0·114	0·175	0·240	0·306	0·373	0·440	0·505	0·569	0·690	0·804
1·8	0·013	0·046	0·090	0·141	0·196	0·253	0·312	0·371	0·430	0·488	0·601	0·707
2·0	0·010	0·035	0·069	0·110	0·156	0·204	0·255	0·307	0·359	0·411	0·514	0·613
2·5	0·004	0·016	0·034	0·057	0·084	0·114	0·147	0·182	0·219	0·257	0·335	0·414
3·0	0·002	0·008	0·018	0·031	0·048	0·067	0·089	0·112	0·138	0·165	0·223	0·284
3·5	0·001	0·005	0·011	0·020	0·031	0·044	0·059	0·076	0·094	0·114	0·158	0·205
4·0	0·001	0·004	0·008	0·014	0·022	0·032	0·043	0·055	0·069	0·084	0·118	0·155
4·5	0·001	0·003	0·006	0·011	0·017	0·024	0·033	0·042	0·053	0·065	0·091	0·121
5·0	0·001	0·002	0·005	0·009	0·013	0·019	0·026	0·033	0·042	0·052	0·073	0·097
6·0	0·000	0·001	0·003	0·006	0·009	0·013	0·017	0·023	0·029	0·035	0·050	0·067
7·0	0·000	0·001	0·002	0·004	0·006	0·009	0·013	0·016	0·021	0·025	0·036	0·049

x	$\nu = 1·6$	1·8	2·0	3·0	4·0	5·0	6·0	7·0	8·0	9·0	10·0
0·1	1·358	1·467	1·567	1·960	2·243	2·465	2·646	2·799	2·933	3·050	3·155
0·5	1·314	1·423	1·522	1·914	2·197	2·418	2·599	2·752	2·886	3·003	3·108
0·8	1·243	1·351	1·449	1·838	2·120	2·341	2·522	2·675	2·808	2·925	3·031
1·0	1·178	1·284	1·381	1·767	2·048	2·269	2·450	2·603	2·736	2·853	2·958
1·2	1·099	1·203	1·298	1·681	1·961	2·181	2·362	2·515	2·648	2·765	2·870
1·4	1·008	1·110	1·203	1·581	1·860	2·079	2·259	2·412	2·545	2·662	2·767
1·6	0·910	1·008	1·099	1·471	1·748	1·966	2·146	2·298	2·431	2·548	2·653
1·8	0·808	0·902	0·990	1·355	1·629	1·846	2·025	2·177	2·309	2·426	2·531
2·0	0·708	0·797	0·882	1·237	1·508	1·724	1·902	2·054	2·186	2·303	2·407
2·5	0·493	0·569	0·643	0·970	1·230	1·441	1·617	1·768	1·899	2·015	2·119
3·0	0·347	0·410	0·473	0·766	1·012	1·217	1·390	1·539	1·669	1·784	1·888
3·5	0·255	0·306	0·359	0·618	0·848	1·045	1·214	1·360	1·489	1·603	1·706
4·0	0·195	0·237	0·281	0·507	0·720	0·908	1·072	1·216	1·342	1·456	1·558
4·5	0·154	0·189	0·225	0·423	0·618	0·796	0·954	1·094	1·219	1·331	1·432
5·0	0·124	0·153	0·185	0·356	0·534	0·702	0·854	0·990	1·113	1·223	1·323
6·0	0·086	0·107	0·130	0·262	0·408	0·554	0·692	0·820	0·937	1·043	1·141
7·0	0·063	0·079	0·096	0·199	0·319	0·445	0·569	0·687	0·797	0·899	0·993

Using the asymptotic forms of $\Delta_1(x)$ and $\Delta_2(x)$ for $x \gg 1$, as given by May (1969), we find

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{x>>1} = \frac{(Ze\omega_{\mathrm{p}})^2}{V}\left[\ln\left(\frac{2mV^2}{\hbar\omega_{\mathrm{p}}}\right) - \gamma - \mathrm{Re}\left\{\psi\left(\frac{\mathrm{i}Ze^2}{\hbar V}\right)\right\}\right], \tag{23}$$

in complete agreement with Honda (1964*b*; note that his $\ln\gamma$ is equal to our γ). The limiting form (23) has also been obtained by Gould (1971).

It is very interesting to compare the loss rate (23) to plasma electrons with the loss rate to atomic electrons in ordinary matter as obtained by Bloch (1933). Let ω_n be the frequency corresponding to an atomic transition from state n to the ground state;

let also f_n be the oscillator strength of the transition. The loss rate (23) can then be obtained from the Bloch formula by replacing $\sum f_n \ln \omega_n$ by $\ln \omega_p$. This correspondence in the classical and quantum limits has been pointed out in paper H. It now turns out that the correspondence is of more general nature.

Finally, it is easy to show that

$$\Delta_3(x \ll 1, v) = -\gamma - \Lambda(v) + O(x^2), \tag{24}$$

where

$$\Lambda(v) = 2\int_0^\infty \mathrm{d}s\, s \exp(-s^2)\, \mathrm{Re}\{\psi(\mathrm{i}v/s)\}. \tag{25}$$

Table 2. Values of the function $\Lambda(v)$

v	$\Lambda(v)$	v	$\Lambda(v)$	v	$\Lambda(v)$
0·02	−0·577	0·9	0·286	3·5	1·536
0·05	−0·566	1·0	0·370	4·0	1·668
0·10	−0·535	1·2	0·524	4·5	1·785
0·20	−0·441	1·4	0·660	5·0	1·889
0·30	−0·330	1·6	0·782	5·5	1·984
0·40	−0·217	1·8	0·892	6·0	2·071
0·50	−0·106	2·0	0·992	7·0	2·224
0·60	0·000	2·3	1·126	8·0	2·357
0·70	0·101	2·5	1·207	9·0	2·475
0·80	0·197	3·0	1·385	10·0	2·580

The function $\Lambda(v)$ is tabulated in Table 2. Using values of $\Delta_1(x)$ and $\Delta_2(x)$ for $x \ll 1$ as given by May (1969), we find that

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{x<<1} = \frac{4(Ze\omega_p)^2x^3}{3\sqrt{\pi}\,V}\left\{\ln\left(\frac{4T}{\hbar\omega_p}\right) - \tfrac{1}{2} - \tfrac{3}{2}\gamma - \tfrac{1}{2}\ln 2 - \Lambda(v)\right\}. \tag{26}$$

In the classical limit $v \gg 1$, we have

$$\Lambda(v \gg 1) = \ln v + \tfrac{1}{2}\gamma, \tag{27}$$

so that

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{c},x<<1} = \frac{4(Ze\omega_p)^2x^3}{3\sqrt{\pi}\,V}\left\{\ln\left(\frac{4T^{3/2}}{Ze^2m^{\frac{1}{2}}\omega_p}\right) - 2\gamma - \tfrac{1}{2}\right\}, \tag{28}$$

which is in complete agreement with equation (32) in paper H. At the value $v = 10$ given in Table 2, the asymptotic form (27) is already accurate to within 1%. In the opposite quantum limit $v \ll 1$, on the other hand, we have

$$\Lambda(v \ll 1) = -\gamma, \tag{29}$$

so that

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{q},x<<1} = \frac{4(Ze\omega_p)^2x^3}{3\sqrt{\pi}\,V}\left\{\ln\left(\frac{2^{3/2}T}{\hbar\omega_p}\right) - \frac{1+\gamma}{2}\right\}, \tag{30}$$

in agreement with equation (33) in paper H.

We have thus shown that the formula (18) contains all known expressions for the stopping power as appropriate limiting cases. As far as the energy loss by a non-relativistic ion to quiescent Maxwellian plasma electrons is concerned, we thus believe that equation (18) constitutes the most general solution of the problem.

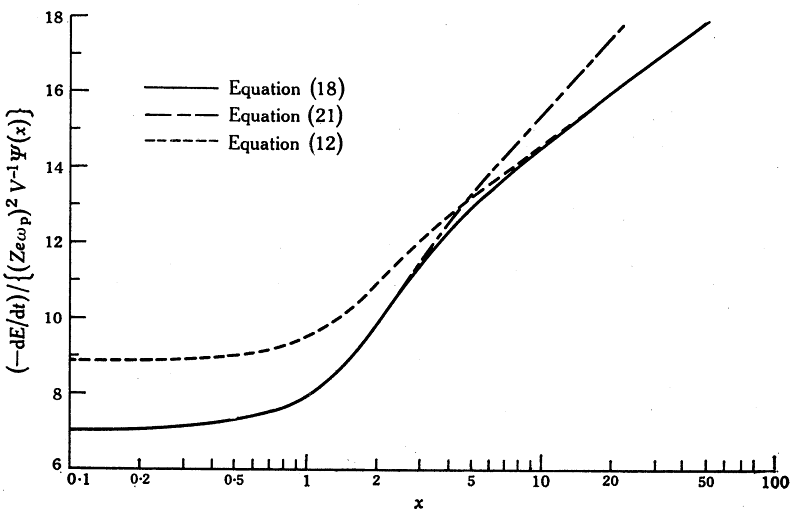

Fig. 1. Full quantum-theoretical energy loss rate dE/dt derived here (equation 18) compared with the classical (equation 21) and quantum (equation 12) limits. The graphs shown are for the parameter values $Z = 1$, $T = 2$ eV and $n = 10^{14}$ cm^{-3}.

4. Discussion

In the Introduction we argued that the energy loss of an ion to plasma electrons with $\nu \gg 1$ will be described by the quantum limit formula (12) for $\nu/x \ll 1$. As the ion slows down it will pass through the region $\nu/x \approx 1$, where the full quantum-theoretical equation (18) must be used to describe its behaviour, and finally it will settle down in the classical region with $x < 1$, where the classical equation (21) is applicable. These features are illustrated in Fig. 1 for $Z = 1$ (proton), $T = 2$ eV ($\nu = 2\cdot6$) and $n = 10^{14}$ cm^{-3}, parameters that are appropriate to the experiment by Burke and Post (1974). At $x \approx 1$, where the energy loss has been actually measured, the quantum effect is entirely negligible so that the analysis by Burke and Post of their experimental results is justified, except for their use of May's (1969) formula which is in error, as discussed in paper H.

The quantum effect in Caby-Eyraud's (1970) experiment for 5 keV protons at $T = 1\cdot5$ eV ($x = 1\cdot3$, $\nu = 3\cdot0$) is also negligible, the difference between equations (18) and (21) above being less than 1%.

Let us finally consider typical fusion plasmas. For $T > 10$ keV and fusion-produced alpha particles, the quantum limit (12) is practically exact since then $\nu < 0\cdot07$. We have evaluated the energy loss to electrons of an alpha particle, of energy $E \leqslant 3\cdot5$ MeV in a plasma with $n = 10^{14}$ cm^{-3} and $T = 20$ and 80 keV, using equation (12) and found excellent agreement with the results obtained by Sigmar and Joyce (1971). The Lenard–Balescu kinetic theory used by Sigmar and Joyce contains a divergence for large momentum transfers so that they had to introduce cutoffs in their calculation.

In view of this unsatisfactory feature of their calculations, it is somewhat surprising that such an excellent agreement has been found. This point will be discussed further in a forthcoming paper in which contributions of plasma ions will be included.

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References

Abramowitz, M., and Stegun, I. A. (1972). 'Handbook of Mathematical Functions' (Dover: New York).
Bloch, F. (1933). *Ann. Phys. (Leipzig)* **16**, 285.
Burke, R. J., and Post, R. F. (1974). *Phys. Fluids* **17**, 1422.
Caby-Eyraud, M. (1970). *J. Phys. (Paris)* **31**, 445.
Gould, H. A., and DeWitt, H. E. (1967). *Phys. Rev.* **155**, 68.
Gould, R. J. (1971). *Phys. Lett.* A **36**, 485.
Hamada, T. (1978). *Aust. J. Phys.* **31**, 291.
Honda, N. (1964*a*). *J. Phys. Soc. Jpn* **19**, 1201.
Honda, N. (1964*b*). *J. Phys. Soc. Jpn* **19**, 1935.
Jackson, J. D. (1962). 'Classical Electrodynamics', Ch. 13 (Wiley: New York).
Kihara, T. (1964). *J. Phys. Soc. Jpn* **19**, 108.
Kihara, T., and Aono, O. (1963). *J. Phys. Soc. Jpn* **18**, 837.
May, R. M. (1969). *Aust. J. Phys.* **22**, 687.
Sigmar, D. J., and Joyce, G. (1971). *Nucl. Fusion* **11**, 447.
Williams, R. H., and DeWitt, H. E. (1969). *Phys. Fluids* **12**, 2326.

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