Aust. J. Phys., 1979, **32**, 295–306

Photon Fission in Intense Laser Fields and in the Coulomb Field of a Nucleus

J. F. Lindsey, H. S. Perlman and G. J. Troup

Department of Physics, Monash University, Clayton, Vic. 3168.

Abstract

A calculation of the photon fission cross section in the Coulomb field of a nucleus reveals that the real part of the transition amplitude is the predominant contributor for photon energies up to 2 MeV. Since it is just this part that is associated with the fourth-order vacuum polarization process, it is suggested, given the present developmental state of laser technology, that coincidence experiments with photon fission might well afford a test of higher order quantum electrodynamics.

Introduction

While the excellent agreement of the Lamb shift calculations with the experimental values (Triebwasser *et al.* 1953; Layzer 1960) is good evidence for the vacuum polarization predicted by quantum electrodynamics, it would be fair to say that the predictions concerning the scattering processes, namely photon–photon scattering (Delbruck 1933), photon fission and photon fusion, have barely been tested. The photon–photon scattering cross section is too small to be measured even with current laser techniques, and suggestions for enhancing it (e.g. Varfolomeev 1966) have not yet been performed experimentally. The imaginary part contribution in Delbruck scattering has been observed, but evidence for the real-part contribution is marginal (Ehlotsky and Sheppey 1964; Jackson and Wetzel 1969; Papatzacos and Mork 1975) and there are unexplained discrepancies between theory and experiment. The two experiments in which photon fission has been observed (Adler and Cohen 1966; Jarlskog *et al.* 1973) are not in good agreement with theoretical predictions, and the theoretical results to date have given no separate values for the real and imaginary part contributions.

While the photon fission cross section, whether in an external field or the Coulomb field of a nucleus, is smaller than the photon–photon cross section, experiments to observe photon fission do offer the possibility of coincidence measurements with all the consequent data processing advantages. For this reason, and to determine separate values for the real and imaginary part contributions to the cross section, we decided to study photon fission in the Coulomb field of a nucleus.

Laser powers continue to increase, and Hughes (1978*a*, 1978*b*) has proposed a possible collective photon effect at high powers. Some evidence for an energy-density dependent photon energy effect in focused laser beams has been presented by Panarella (1977) and discussed by Allen (1977). If we are to decide whether quantum electrodynamics is indeed correct, we must know its predictions so that we can check the theory. Thus a non-null experiment for photon fission using a laser, in a situation where fission is predicted to be unobservable by quantum electrodynamics, might well be of significance.

Finally, the splitting of photons in an external slowly varying electromagnetic field has been studied by a number of authors. Their results as applied to laser–laser interaction, or laser–γ-ray interaction, are summarized in the next section.

Photon Fission in a Slowly Varying External Field

Any electromagnetic radiation field can be categorized by the two constants $(\boldsymbol{E}.\boldsymbol{H})^2$ and $(\boldsymbol{E}^2-\boldsymbol{H}^2)$. From these a third constant $\Gamma = (\boldsymbol{E}^2-\boldsymbol{H}^2)^2+4(\boldsymbol{E}.\boldsymbol{H})^4$ can be formed; when this constant is zero the field is said to be 'null' (Synge 1958). Plane electromagnetic wave fields are not only null but they are also 'wrenchless' (Synge 1958); that is, there is no Lorentz frame in which the fields transform to single component electric and magnetic fields having a common line. Null wrenchless fields remain so in all Lorentz frames, and therefore it is desirable to be able to treat the laser field at the focus of a lens as null and wrenchless. This will be so for f numbers greater than or equal to $f/4$; below this value the vector nature of the electromagnetic fields becomes evident, and there are regions of the Airy cylinder where the fields are non-null (Richards and Wolf 1959; Boivin and Wolf 1965).

In the optical region, the electromagnetic field $\boldsymbol{F}(\boldsymbol{E},\boldsymbol{H})$ is 'slowly varying'; that is, $(\hbar/mc)|\operatorname{grad}\boldsymbol{F}| \ll |\boldsymbol{F}|$ and $(\hbar/mc^2)|\partial\boldsymbol{F}/\partial t| \ll |\boldsymbol{F}|$. This means that the field Lagrangian depends on the values of the field invariants at a given moment in time and coincides with the Lagrangian for a constant field with the instantaneous values for the field strength (Toll 1952; Bunkin and Tugov 1970). We can therefore treat the laser field at the focus of a lens of $f/4$ or greater as a constant external field and can apply the results of Bialynicka-Birula and Bialynicki-Birula (1970). Papanyan and Ritus (1972) comment that only this work and one other (Adler *et al.* 1970) correctly preserve Lorentz, charge-conjugation and gauge invariance.

Although it would seem that the fourth-order diagram (light–light scattering with external field at one vertex) would give the main contribution to the photon splitting cross section, it does not, and the process is essentially a sixth-order one, that is, it is proportional to α^6, where $\alpha = 1/137$ is the fine structure constant. Thus it will be of the order of 10^{-4} times the light–light scattering cross section.

The Bialynicka-Birula and Bialynicki-Birula (1970) result for the transition rate W is

$$W = 0\cdot 07\,\alpha^3(|\boldsymbol{k}|/m)^4(\alpha\,|\boldsymbol{Q}|^2/m^4)^3\,|\boldsymbol{k}|, \tag{1}$$

where $\boldsymbol{k}$ is the wave vector for the incoming radiation and

$$\boldsymbol{Q} = \boldsymbol{n}\times\boldsymbol{E}+\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{B}), \tag{2}$$

with $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic fields for the 'constant' external electromagnetic field and $\boldsymbol{n} = \boldsymbol{k}/|\boldsymbol{k}|$. Their calculated results are exemplified in terms of a mean free path of 10^{16} cm for 50 keV γ rays ($k/m \approx 10^{-1}$) and a magnetic field of $10^{12}/\sqrt{(4\pi)}$ G (1 G $\equiv 10^{-4}$ T).

Consider now a laser field whose power density is $10^{20}\ \mathrm{W\,cm^{-2}}$ (Hora *et al.* 1978*a*, 1978*b*). Lorrain and Corson (1970) relate the average power flow S_{av} (in $\mathrm{W\,m^{-2}}$) to the r.m.s. field E_{rms} by

$$S_{\mathrm{av}} = 2\cdot 66\times 10^{-3}\,E_{\mathrm{rms}}^2\,.$$

From this relation and $B_0 = E_0/c$ (for peak values), we have $B_0 \approx 10^9$ G. Next assume $|k/m| = 1$, that is, that $0 \cdot 5$ MeV radiation, say from electron–positron annihilation, is available. Substitution of these values in equation (1) indicates that the mean free path for this case is of the order of 10^{10} times that for the previous case. Thus the $|Q|^6$ dependence of equation (1) is a very serious one.

It should be commented here that the expression (1) used above is essentially for low energy incident photons. The more detailed work of Papanyan and Ritus (1972) gives a reciprocal photon lifetime with respect to splitting of 6×10^9 s for a 25 GeV photon in a field of 4×10^8 G. This is much more hopeful. However, the two-fold increase in power density implied by the $\sim 10^9$ G field of a 10^{20} W cm^{-2} laser would only increase this reciprocal lifetime by at most a factor of 2, according to their theory.

What of the possibility of observing photon fission in two opposed laser beams? If we apply standard quantum electrodynamical techniques to the problem, the lowest order process again involves a sixth-order diagram and is such that, even with a laser field of power density 10^{20} W cm^{-2}, the process would be unobservable. The collective effects proposed by Hughes (1978*a*, 1978*b*) are ignored in such a calculation, and it may be objected that single-photon calculations such as are used in quantum electrodynamics do not apply to lasers. However, we argue as follows. Suppose during the laser pulse that the field is in a pure coherent state $|\alpha\rangle$ (Glauber 1963). This implies a Poisson distribution over the number states $|n\rangle$. As the mean $\langle n\rangle$ of the distribution increases, $\langle n\rangle^{\frac{1}{2}}/\langle n\rangle$ decreases so that, for very large $\langle n\rangle$, as in high power lasers, the distribution is essentially $\delta(n-\langle n\rangle)$, a Dirac delta function. Thus we can, to a good approximation, treat the laser field as being in the pure state $|\langle n\rangle\rangle$. The success of semiclassical (non-quantized field) coherence theory in predicting such effects as the splitting of resonance fluorescence on heavily pumping the levels (Grove *et al.* 1977), and the agreement in the low energy limit of quantum electrodynamics, the Euler (1936) approach and the work of McKenna and Platzman (1963), tend to support the above argument.

Photon Fission in the External Field of a Nucleus

It is assumed in these calculations that the rest energy of the nucleus is much greater than the energies of the photons. There is therefore no nuclear recoil and the external field is static and Coulomb. The process is inelastic in the sense that the number of photons present changes, but because the external field is static no energy transfer to or from the field is possible, and the energy in the initial one-photon state must equal the energy in the final two-photon state. The transition amplitude for such a three-photon scattering process (see Fig. 1 below) has both a real and imaginary part. The imaginary part corresponds to the production and annihilation of real electron–positron pairs in the intermediate state (and is thus zero below the energy threshold for pair production) and the real part corresponds to the production and annihilation of virtual electron–positron pairs in the fourth-order vacuum polarization process that constitutes the lowest order nonlinear interaction between electromagnetic fields. And since it is just this interaction that is germane to the adequacy of quantum electrodynamics, it is necessary to obtain the separate contributions to the photon fission cross section from the real and imaginary parts of the transition amplitude (and this without recourse to the method of the analytic continuation of the pair-production cross section). The previous numerical results (other than those in the low and high incident photon energy limits where the cross section decreases and so is of limited interest) are due to Shima (1966) and he gives only the total cross sections.

The transition amplitude for photon fission is, by analogy with Delbruck scattering (Jauch and Rohrlich 1976),

$$\langle k_3, k_4 | S^{(4)} | k_1 \rangle = \frac{1}{(2k_{10})^{\frac{1}{2}}} \frac{1}{(2k_{30})^{\frac{1}{2}}} \frac{1}{(2k_{40})^{\frac{1}{2}}} e^{\mu}_{\lambda_1}(k_1)\, e^{\lambda *}_{\lambda_3}(k_3)\, e^{\sigma *}_{\lambda_4}(k_4)$$

$$\times \int d^3q\, A^{(e)\nu}(\boldsymbol{q})\, \pi_{\mu\nu\lambda\sigma}(k_1, q, k_3, k_4)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{q} + \boldsymbol{k}_3 + \boldsymbol{k}_4)\, \delta(k_{10} + k_{30} + k_{40}), \quad (3)$$

where $\pi_{\mu\nu\lambda\sigma}$ (k_1, q, k_3, k_4) is the fourth-rank vacuum polarization tensor, with its arguments k_1, k_3, k_4 and q denoting the momenta of the incoming and outgoing photons and the virtual photon respectively, and $A^{(e)\nu}(\boldsymbol{q})$ is the external Coulomb field of a nucleus in momentum space, which is given by

$$A^{(e)\nu}(\boldsymbol{q}) = (2\pi)^{-3/2} \int A^{(e)\nu}(\boldsymbol{x}) \exp(-i\boldsymbol{q}.\boldsymbol{x})\, d^3x = \frac{n^\nu}{(2\pi)^{3/2}} \frac{Ze}{|\boldsymbol{q}|^2},$$

with $n^\nu = (1,0,0,0)$. The quantities $e^{\mu}_{\lambda_i}(k_i)$ in equation (3) are photon polarization vectors.

It is convenient, following Karplus and Neuman (1950) and de Tollis (1964), to express the transition amplitude, after integration over $\boldsymbol{q}$, in the form

$$\langle k_3, k_4 | S^{(4)} | k_1 \rangle = \frac{2\alpha^2}{i\pi^2} \frac{Ze}{(2\pi)^{3/2}} \frac{1}{(2k_{10})^{\frac{1}{2}}} \frac{1}{(2k_{30})^{\frac{1}{2}}} \frac{1}{(2k_{40})^{\frac{1}{2}}} \frac{1}{|\boldsymbol{k}_1 + \boldsymbol{k}_3 + \boldsymbol{k}_4|^2}$$

$$\times M_{\lambda_1 0 \lambda_3 \lambda_4}(k_1, q, k_3, k_4)\, \delta(k_{10} + k_{30} + k_{40}),$$

where $\boldsymbol{q} = -\boldsymbol{k}_1 - \boldsymbol{k}_3 - \boldsymbol{k}_4$ and

$$M_{\lambda_1 0 \lambda_3 \lambda_4}(k_1, q, k_3, k_4) \equiv (\pi^2 i/2\alpha^2) e^{\mu}_{\lambda_1}(k_1)\, n^\nu(q)\, e^{\lambda *}_{\lambda_3}(k_3)\, e^{\sigma *}_{\lambda_4}(k_4)\, \pi_{\mu\nu\lambda\sigma}(k_1, q, k_3, k_4),$$

the zero subscript indicating the position of the external field polarization $n^\nu(q)$. The polarization amplitude $M_{\lambda_1 0 \lambda_3 \lambda_4}$ is made up of three terms, each corresponding to the sum of two equal diagrams of the kind shown in Fig. 1:

$$M_{\lambda_1 0 \lambda_3 \lambda_4}(k_1, q, k_3, k_4) = M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}(k_1, q, k_3, k_4) + M^{(1)}_{\lambda_1 0 \lambda_4 \lambda_3}(k_1, q, k_4, k_3)$$

$$+ M^{(1)}_{\lambda_1 \lambda_4 \lambda_3 0}(k_1, k_4, k_3, q), \quad (4)$$

where

$$M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}(k_1, q, k_3, k_4) = \frac{1}{4\pi^2 i} \int d^4p\, \mathrm{Tr}\{\hat{e}_{\lambda_1}(k_1)\, S_F(p_1)\, \hat{e}_q(q)\, S_F(p_2)$$

$$\times \hat{e}_{\lambda_3}(k_3)\, S_F(p_3)\, \hat{e}_{\lambda_4}(k_4)\, S_F(p_4)\},$$

with

$$S_F(p) = (\hat{p} + im)/(p^2 + m^2)$$

and

$$p_1 = p, \qquad p_2 = p - q, \qquad p_3 = p + k_4 + k_3, \qquad p_4 = p + k_1;$$

note that here a circumflex notation $\hat{a} \equiv \gamma^{\mu} a_{\mu}$ has been used in lieu of the normal 'slashed' notation.

Since there is only one particle in the initial state in photon fission, the total cross section to lowest order is

$$\sigma = \frac{(2\pi)^2}{8k_{10}k_{30}k_{40}} \frac{(Ze)^2}{(2\pi)^3} \frac{4\alpha^4}{\pi^4} \sum_i \sum_f \frac{1}{|\boldsymbol{k}_1+\boldsymbol{k}_3+\boldsymbol{k}_4|^4} |M_{\lambda_1 0 \lambda_3 \lambda_4}|^2 \delta(k_{10}+k_{30}+k_{40}). \quad (5)$$

For photons of specific polarizations, and for final photons scattered into the momentum ranges $(k_3, k_3+\mathrm{d}k_3)$ and $(k_4, k_4+\mathrm{d}k_4)$, the summation over the initial states i is not necessary and the summation over the final states f becomes an integration over $\mathrm{d}^3k_3 = k_{30}^2\,\mathrm{d}k_{30}\,\mathrm{d}\Omega_3$ and $\mathrm{d}^3k_4 = k_{40}^2\,\mathrm{d}k_{40}\,\mathrm{d}\Omega_4$. The delta function enables the integration over $\mathrm{d}k_{40}$, say, to be performed, giving the differential scattering cross section

$$\frac{\mathrm{d}^3\sigma_{\lambda_1 0 \lambda_3 \lambda_4}}{\mathrm{d}k_{30}\,\mathrm{d}\Omega_3\,\mathrm{d}\Omega_4} = \frac{\alpha^3 Z^2 r_0^2 m^2}{\pi^4} \frac{1}{|\boldsymbol{k}_1+\boldsymbol{k}_3+\boldsymbol{k}_4|^4} \frac{k_{30}k_{40}}{k_{10}} |M_{\lambda_1 0 \lambda_3 \lambda_4}|^2, \quad (6)$$

with $k_{10}+k_{30}+k_{40} = 0$ (here $r_0 = \alpha/m$). For unpolarized photons, an average over the initial polarization directions and a sum over the final polarization directions must be taken.

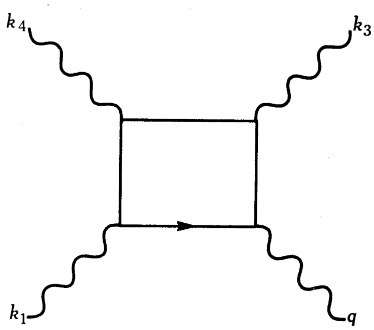

Fig. 1. Diagram for the partial amplitude $M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}$ (k_1, q, k_3, k_4). The polarization amplitude $M_{\lambda_1 0 \lambda_3 \lambda_4}$ is made up of three terms, each corresponding to the sum of two equal diagrams of this type.

Polarization Amplitudes

The calculations can be facilitated by employing the centre-of-momentum frame

$$\boldsymbol{k}_1+\boldsymbol{q} = \boldsymbol{k}_3+\boldsymbol{k}_4 = 0, \quad (7a)$$

even though this means, if the external field is to remain static and Coulomb in it, that the momentum orientations are restricted to those for which the centre-of-momentum and laboratory frames coincides. All photons are taken as outgoing:

$$k_{1\mu}+q_{\mu}+k_{3\mu}+k_{4\mu} = 0, \quad (7b)$$

with $k_i^2 = 0$, that is, $k_{i0} = |\boldsymbol{k}_i|$ $(i = 1,2,3)$ and $q_0 = 0$ because the external field is static.

The photon three-momentum directions are chosen so that the scattering is in the (x,z) plane in momentum space and $\boldsymbol{k}_1$ and $\boldsymbol{q}$ are in the z direction. Then, because of equation (7a), we have $|\boldsymbol{k}_3| = |\boldsymbol{k}_4| = k$, say, and thus equation (7b) gives

$$k_{10} = -2k, \qquad q_x = 0, \qquad q_z = -k_{10} = +2k,$$

so that

$$k_1 = (-2k, 0, 0, -2k), \qquad q = (0, 0, 0, +2k), \tag{8a}$$

$$k_3 = (+k, +k\sin\theta, 0, +k\cos\theta), \qquad k_4 = (+k, -k\sin\theta, 0, -k\cos\theta), \tag{8b}$$

with

$$\theta = \arccos(\boldsymbol{q}\,.\,\boldsymbol{k}_3/2k^2) = \arccos(\boldsymbol{k}_1\,.\,\boldsymbol{k}_4/2k^2).$$

The differential cross section in this frame reduces to

$$\frac{d^3\sigma_{\lambda_1 0\lambda_3\lambda_4}}{dk\, d\Omega_3\, d\Omega_4} = \frac{\alpha^3 r_0^2 Z^2}{32\pi^4 m}\left(\frac{m}{k}\right)^3 |M_{\lambda_1 0\lambda_3\lambda_4}(k,\theta)|^2. \tag{9}$$

With the above momentum components, the kinematic invariants r, s and t are

$$r = -(k_1+q)^2/4m^2 = -(k_3+k_4)^2/4m^2 = +k^2/m^2, \tag{10a}$$

$$s = -(k_1+k_3)^2/4m^2 = -(k_2+k_4)^2/4m^2 = -(2k^2/m^2)\sin^2\tfrac{1}{2}\theta, \tag{10b}$$

$$t = -(k_1+k_4)^2/4m^2 = -(k_2+k_3)^2/4m^2 = -(2k^2/m^2)\cos^2\tfrac{1}{2}\theta, \tag{10c}$$

with $2r+s+t = 0$.

The gauge is chosen so that the polarizations for the free photons are purely transverse and have zero time components while the polarization for the external photon has a purely time component. Thus, with $e_{\lambda_i}(i) \equiv e_{\lambda_i}(k_i)$, the linear polarizations are:

(i) normal to the scattering plane,

$$e_1(1) = e_1(3) = e_1(4) = (0, 0, 1, 0)\,; \tag{11a}$$

(ii) parallel to the scattering plane,

$$e_2(1) = (0, -1, 0, 0), \qquad e_2(3) = -e_2(4) = (0, -\cos\theta, 0, \sin\theta)\,; \tag{11b}$$

with

$$\boldsymbol{e}_1(i)\times\boldsymbol{e}_2(i) = \boldsymbol{k}_i \qquad (i = 1, 3, 4) \tag{11c}$$

and, for all possible polarizations,

$$n(q)\,.\,e_{\lambda_j}(j) = 0. \tag{11d}$$

The integrals of the partial amplitudes $M^{(1)}_{\lambda_1 0\lambda_3\lambda_4}$ over the momentum p of the intermediate state can be evaluated using Feynmann parameterization (Jauch and Rohrlich 1976). There are divergent contributions from p^4 terms in the numerator but their coefficients vanish because of the condition (11d) for all polarizations (Lindsey 1975).

The required partial amplitudes $M^{(1)}$ ultimately reduce to the expressions given in Tables 1*a* and 1*b* (Lindsey 1975), where the transcendental functions B, T and L are such that

$$\operatorname{Re}(B(r)) = -1 - \tfrac{1}{2}b(r)\log\left|\frac{b(r)-1}{b(r)+1}\right|, \qquad \text{for all } r, \tag{12a}$$

$$\operatorname{Im}(B(r)) = -\tfrac{1}{2}\pi\, b(r) \qquad \text{if } r \geqslant 1, \tag{12b}$$

$$= 0, \qquad \text{otherwise;} \tag{12c}$$

$$\operatorname{Re}(T(r)) = \tfrac{1}{4}\log^2\left|\frac{1-b(r)}{1+b(r)}\right|, \qquad \text{for all } r, \tag{13a}$$

$$\operatorname{Im}(T(r)) = \tfrac{1}{2}\pi\log\left|\frac{b(r)-1}{b(r)+1}\right| = -\pi\operatorname{arcosh} r^{\frac{1}{2}}, \qquad \text{if } r > 1, \tag{13b}$$

$$= 0, \qquad \text{otherwise;} \tag{13c}$$

$$\begin{aligned}\operatorname{Re}(L(r,s,-r)) = -\frac{1}{2ars}\operatorname{Re}\Big\{ & f\left(\frac{a+1}{a+b(r)}\right) + f\left(\frac{a+1}{a-b(r)}\right) - f\left(\frac{a-1}{a+b(r)}\right) \\ & - f\left(\frac{a-1}{a-b(r)}\right) + f\left(\frac{a+1}{a+b(s)}\right) + f\left(\frac{a+1}{a-b(s)}\right) \\ & - f\left(\frac{a-1}{a+b(s)}\right) - f\left(\frac{a-1}{a-b(s)}\right) - f\left(\frac{a+1}{a+b(-r)}\right) \\ & - f\left(\frac{a+1}{a-b(-r)}\right) + f\left(\frac{a-1}{a+b(-r)}\right) + f\left(\frac{a-1}{a-b(-r)}\right)\Big\},\end{aligned} \tag{14a}$$

for all r, s, and

$$\operatorname{Im}(L(r,s,-r)) = -\frac{\pi}{2ars}\log\left|\frac{a+b(r)}{a-b(r)}\right|, \qquad \text{if} \quad r \geqslant 1, \quad s < 1, \tag{14b}$$

$$= 0, \qquad \text{if} \quad 0 < r < 1, \quad s < 1. \tag{14c}$$

In these expressions,

$$b(r) = (1-r^{-1})^{\frac{1}{2}}, \qquad a \equiv a(r,s) = \{1-(2r+s)/rs\}^{\frac{1}{2}},$$

and the quantity $f(x)$ appearing in the real part (14a) is the Spence or dilogarithm function

$$f(x) = -\int_0^x \xi^{-1}\log|1-\xi|\,d\xi,$$

for which values have been tabulated (Mitchell 1949; Lewin 1958). The function $L(t,s,-r) = L(t,s,\tfrac{1}{2}(s+t))$ is obtained from $L(r,s,-r)$ with the substitutions $r \to t$ and $-r \to \tfrac{1}{2}(s+t)$.

Table 1. Partial amplitudes for photon fission

(*a*) $M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}$

The partial amplitudes $M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}$ in columns 2, 3 and 4 are of the form

$$\{k(t,s)\}^{-1} M^{(1)}_{\lambda_1 0 \lambda_3 \lambda_4}(r,s) = \alpha_1 + \alpha_2 B(r) + \alpha_3 B(s) + \alpha_4 B(-r) + \alpha_5 T(r) + \alpha_6 T(s) + \alpha_7 T(-r) + \alpha_8 L(r,s,-r),$$

while the partial amplitude $M^{(1)}_{2022}$ is given by

$$M^{(1)}_{2022}(r,s) = P_{2022}(r,s) + \{M^{(1)}_{1012}(r,s) - M^{(1)}_{1021}(r,s)\}(r+s)r^{-1} - M^{(1)}_{2011}(r,s),$$

where P_{2022} in column 5 is of the form

$$\{k(t,s)\}^{-1} P_{2022}(r,s) = \alpha_1 + \alpha_2 B(r) + \alpha_3 B(s) + \alpha_4 B(-r) + \alpha_5 T(r) + \alpha_6 T(s) + \alpha_7 T(-r) + \alpha_8 L(r,s,-r).$$

In these expressions, the kinematic invariants r, s and t are given by equations (10), the transcendental functions B, T and L are given by equations (12), (13) and (14) and the specific coefficients are tabulated below. Note also here that

$$2r+s+t = 0 \qquad \text{and} \qquad h = r+s = -(r+t) = \tfrac{1}{2}(s-t)$$

(1) Coefficient	(2) $M^{(1)}_{1021}(r,s)$	(3) $M^{(1)}_{1012}(r,s)$	(4) $M^{(1)}_{2011}(r,s)$	(5) $P_{2022}(r,s)$
$k(t,s)$	$2(ts)^{\frac{1}{2}}$	$-2(ts)^{\frac{1}{2}}$	$(ts)^{\frac{1}{2}}$	$-4r^{-1}ts(ts)^{\frac{1}{2}}$
α_1	0	$\frac{1}{4}h^{-1}$	$-\frac{1}{6}h^{-1}$	$-\frac{1}{12}(th)^{-1}-\frac{1}{12}(st)^{-1}$
α_2	$-\frac{1}{2}t^{-1}$	$-\frac{1}{2}t^{-1}$	t^{-1}	$\frac{1}{2}t^{-2}-\frac{1}{4}(st)^{-1}$
α_3	$\frac{5}{6}h^{-1}+t^{-1}-\frac{1}{3}(sh)^{-1}$	$-\frac{1}{2}sh^{-2}+\frac{4}{3}h^{-1}+t^{-1}+\frac{1}{2}(th)^{-1}-\frac{5}{6}(sh)^{-1}-(st)^{-1}$	$\frac{1}{3}sh^{-2}-\frac{4}{3}h^{-1}-2t^{-1}+\frac{2}{3}h^{-2}+\frac{1}{3}(sh)^{-1}$	$\frac{1}{6}h^{-2}-t^{-2}-\frac{1}{6}(th^2)^{-1}+\frac{1}{12}(sth)^{-1}+\frac{1}{2}(s^2t)^{-1}$
α_4	$-\frac{5}{6}h^{-1}-\frac{1}{2}t^{-1}-\frac{1}{3}(rh)^{-1}$	$-\frac{1}{2}rh^{-2}-\frac{5}{6}h^{-1}-\frac{1}{2}t^{-1}-\frac{1}{2}(th)^{-1}-\frac{5}{6}(rh)^{-1}-\frac{1}{2}(rt)^{-1}$	$\frac{1}{3}rh^{-2}+h^{-1}+t^{-1}-\frac{2}{3}h^{-2}$	$-\frac{1}{6}h^{-2}+\frac{1}{2}t^{-2}+\frac{1}{4}(st)^{-1}-\frac{1}{6}(rh^2)^{-1}$
α_5	$\frac{1}{2}st^{-2}+\frac{1}{4}(rt)^{-1}$	$\frac{1}{2}st^{-2}+\frac{1}{4}(rt)^{-1}$	$-st^{-2}-\frac{1}{2}(rt)^{-1}$	$-\frac{1}{2}st^{-3}-\frac{1}{8}(rt^2)^{-1}$
α_6	$\frac{1}{2}st^{-2}-\frac{1}{2}(st)^{-1}$	$\frac{1}{2}st^{-2}+\frac{1}{4}h^{-2}-\frac{1}{2}(st)^{-1}$	$-st^{-2}-\frac{1}{2}h^{-2}+\frac{1}{2}(th)^{-1}$	$-\frac{1}{2}st^{-3}+\frac{1}{4}(st^2)^{-1}-\frac{1}{4}(s^2t)^{-1}$
α_7	$-\frac{1}{2}st^{-2}-\frac{1}{4}(rt)^{-1}$	$-\frac{1}{2}st^{-2}-\frac{1}{4}h^{-2}-\frac{1}{4}(rt)^{-1}$	$st^{-2}+\frac{1}{2}h^{-2}-\frac{1}{2}(th)^{-1}$	$\frac{1}{2}st^{-3}+\frac{1}{8}(rt^2)^{-1}$
α_8	$-\frac{1}{2}st^{-1}-\frac{1}{2}rs^2t^{-2}$	$-\frac{1}{2}st^{-1}-\frac{1}{2}rs^2t^{-2}$	$-2rt^{-1}+rs^2t^{-2}$	$\frac{3}{8}st^{-2}+\frac{1}{8}s^{-1}+\frac{1}{2}rs^2t^{-3}$

Table 1 (*Continued*)

(*b*) $M^{(1)}_{\lambda_1\lambda_2\lambda_3 0}$

The partial amplitudes $M^{(1)}_{\lambda_1\lambda_2\lambda_3 0}$ in columns 2, 3 and 4 are of the form

$$\{f(t,s)\}^{-1}M^{(1)}_{\lambda_1\lambda_2\lambda_3 0}(t,s) = \beta_1+\beta_2\,B(t)+\beta_3\,B(s)+\beta_4\,B(-r)+\beta_5\,T(t)+\beta_6\,T(s)+\beta_7\,T(-r)+\beta_8\,L(t,s,-r),$$

while the partial amplitude $M^{(1)}_{2220}$ is given by

$$M^{(1)}_{2220}(t,s) = P_{2220}(t,s)+\{M^{(1)}_{1210}(t,s)-M^{(1)}_{1120}(t,s)\}(t-s)(t+s)^{-1}-M^{(1)}_{2110}(t,s),$$

where P_{2220} in column 5 is of the form

$$\{f(t,s)\}^{-1}P_{2220}(t,s) = \beta_1+\beta_2\,B(t)+\beta_3\,B(s)+\beta_4\,B(-r)+\beta_5\,T(t)+\beta_6\,T(s)+\beta_7\,T(-r)+\beta_8\,L(r,s,-r)$$

(1) Coefficient	(2) $M^{(1)}_{1120}(t,s)$	(3) $M^{(1)}_{1210}(t,s)$[A]	(4) $M^{(1)}_{2110}(t,s)$	(5) $P_{2220}(t,s)$
$f(t,s)$	$2(ts)^{\frac{1}{2}}$	$2(ts)^{\frac{1}{2}}$	$(ts)^{\frac{1}{2}}$	$-4r^{-1}ts(ts)^{\frac{1}{2}}$
β_1	$-\frac{1}{4}h^{-1}$	$-\frac{1}{4}h^{-1}$	$\frac{5}{6}h^{-1}+2hr^{-2}$	$\frac{1}{3}(st)^{-1}-\frac{1}{3}(th)^{-1}$
β_2	$-\frac{1}{2}th^{-2}+\frac{1}{12}h^{-1}+\frac{3}{4}r^{-1}-\frac{1}{3}(th)^{-1}$	$\frac{1}{2}sh^{-2}+\frac{1}{12}h^{-1}-\frac{3}{4}r^{-1}-\frac{1}{3}(sh)^{-1}$	$\frac{1}{3}th^{-2}-\frac{2}{3}h^{-1}-2r^{-1}+\frac{2}{3}h^{-2}$ $-\frac{1}{3}(th)^{-1}-\frac{1}{2}r^{-2}$	$-\frac{1}{3}h^{-2}-\frac{1}{6}(th^2)^{-1}+\frac{1}{4}(t^2h)^{-1}$
β_3	$\frac{1}{6}h^{-1}-\frac{3}{4}r^{-1}+\frac{1}{3}(sh)^{-1}$	$\frac{1}{6}h^{-1}+\frac{3}{4}r^{-1}+\frac{1}{3}(th)^{-1}$	$-\frac{1}{3}sh^{-2}-\frac{2}{3}h^{-1}+2r^{-1}+\frac{2}{3}h^{-2}$ $-\frac{1}{3}(sh)^{-1}+\frac{1}{2}r^{-2}$	$\frac{1}{3}h^{-2}+\frac{1}{6}(sh^2)^{-1}+\frac{1}{4}(s^2h)^{-1}$
β_4	$-\frac{1}{2}rh^{-2}-\frac{3}{4}h^{-1}$	$+\frac{1}{2}rh^{-2}-\frac{3}{4}h^{-1}$	$2h^{-1}$	0
β_5	$r^{-1}+\frac{3}{4}tr^{-2}+\frac{1}{4}h^{-2}-\frac{1}{2}(rt)^{-1}$	$-r^{-1}-\frac{3}{4}sr^{-2}-\frac{1}{4}h^{-2}+\frac{1}{2}(rs)^{-1}$	$2hr^{-2}-\frac{1}{2}h^{-2}-\frac{1}{2}(rh)^{-1}+\frac{1}{2}(rt)^{-1}$	$\frac{1}{4}(rt^2)^{-1}-\frac{1}{4}(rh^2)^{-1}$
β_6	$r^{-1}+\frac{3}{4}tr^{-2}+\frac{1}{2}(rh)^{-1}$	$-r^{-1}-\frac{3}{4}sr^{-2}+\frac{1}{2}(rh)^{-1}$	$2hr^{-2}+\frac{1}{2}h^{-2}-\frac{1}{2}(rh)^{-1}-\frac{1}{2}(rs)^{-1}$	$\frac{1}{4}(rh^2)^{-1}-\frac{1}{4}(rs^2)^{-1}$
β_7	$-r^{-1}-\frac{3}{4}tr^{-2}-\frac{1}{4}h^{-2}-\frac{1}{2}(rh)^{-1}$	$r^{-1}+\frac{3}{4}sr^{-2}+\frac{1}{4}h^{-2}-\frac{1}{2}(rh)^{-1}$	$-2hr^{-2}+(rh)^{-1}$	0
β_8	$-\frac{5}{8}-\frac{7}{8}tr^{-1}$	$\frac{5}{8}+\frac{7}{8}sr^{-1}$	$-2hr^{-1}-2sthr^{-2}$	0

[A] Note that $M^{(1)}_{1210}(t,s) = -M^{(1)}_{1120}(t,s)$ with $s \leftrightarrow t$.

Cross Sections for Photon Fission

The photon fission cross section can be evaluated from Table 1 and equations (12), (13) and (14) using equation (9) and

$$M_{2011}(r,s,t) = M^{(1)}_{2011}(r,s) + M^{(1)}_{2011}(r,t) + M^{(1)}_{2110}(t,s), \tag{15a}$$

$$M_{1021}(r,s,t) = M^{(1)}_{1021}(r,s) + M^{(1)}_{1012}(r,t) + M^{(1)}_{1210}(t,s), \tag{15b}$$

$$M_{1012}(r,s,t) = M^{(1)}_{1012}(r,s) + M^{(1)}_{1021}(r,t) + M^{(1)}_{1120}(t,s), \tag{15c}$$

$$M_{2022}(r,s,t) = M^{(1)}_{2022}(r,s) + M^{(1)}_{2022}(r,t) + M^{(1)}_{2220}(t,s). \tag{15d}$$

Considerable simplification occurs in the special case in which the final two photons are oppositely directed and perpendicular to the incident photon beam, that is, when $\theta = \frac{1}{2}\pi$ or $t = s = -r$. Because of the symmetry properties of the partial amplitudes, one then gets

$$M_{2011} = M_{2022} = 0 \qquad \text{and} \qquad M_{1012} = -M_{1021}\,.$$

Moreover, in this special case, the expression for M_{1021} reduces, in the low and high energy limits, to the simple forms (Lindsey 1975)

$$M_{1021} \simeq -\tfrac{22}{45}r^2, \qquad r \ll 1, \tag{16}$$

and

$$\operatorname{Re} M_{1021} \simeq -\tfrac{3}{2} + \tfrac{1}{24}\pi^2(2-r^{-1}) - \tfrac{3}{4}r^{-1}\log 4r + \tfrac{1}{4}r^{-1}(\log 4r)^2, \qquad r \gg 1, \tag{17a}$$

$$\operatorname{Im} M_{1021} \simeq \pi\{1 + \tfrac{1}{2}(2+r^{-1})\log 4r + (1+r^{-1})\log 2r\}, \qquad r \gg 1, \tag{17b}$$

with $r \equiv (k/m)^2$. The low energy limits are in agreement with the work of McKenna and Platzman (1963). The results of the exact cross section calculation when $\theta = \frac{1}{2}\pi$ are shown in Figs 2*a* and 2*b* as a function of incident energy. Extrapolations from the low and high energy approximations (16) and (17) are included there for comparison.

Discussion

The results in Fig. 2*a* show that the real part of the transition amplitude contributes more than 70% of the fission cross section in the region of maximum cross section, and it is only where the cross section falls to below 20% of its maximum value that the imaginary part of the transition amplitude takes over as the predominant contributor. It follows that the observation of photon fission would be a genuine confirmation of the nonlinear interaction of electromagnetic fields.

The total differential fission cross section is a maximum at an incident photon energy of $2k/m = 2\cdot 6$ ($\equiv 1\cdot 33$ MeV) and, for lead, it has the value

$$\mathrm{d}^3\sigma/\mathrm{d}k\,\mathrm{d}\Omega_3\,\mathrm{d}\Omega_4 = 1\cdot 90\times 10^{-32} \quad \mathrm{cm^2\,sr^{-2}\,MeV^{-1}}\,.$$

This is nearly the same order of magnitude as the maximum basic (i.e. photon–photon) nonlinear interaction cross section (also at incident energies of about 1 MeV). Thus the presence of the external field enhances that interaction much less than in the case of Delbruck scattering, where the maximum cross section is of the order of 10^3 that for photon–photon scattering. However, because there are two photons in the final state in photon fission, coincidence counting and appropriate biasing of counters enables the contributions of competing processes of comparable size (mainly double Compton scattering and random coincidence single Compton scattering) to be greatly reduced (Bolsterli 1954; Talman 1965).

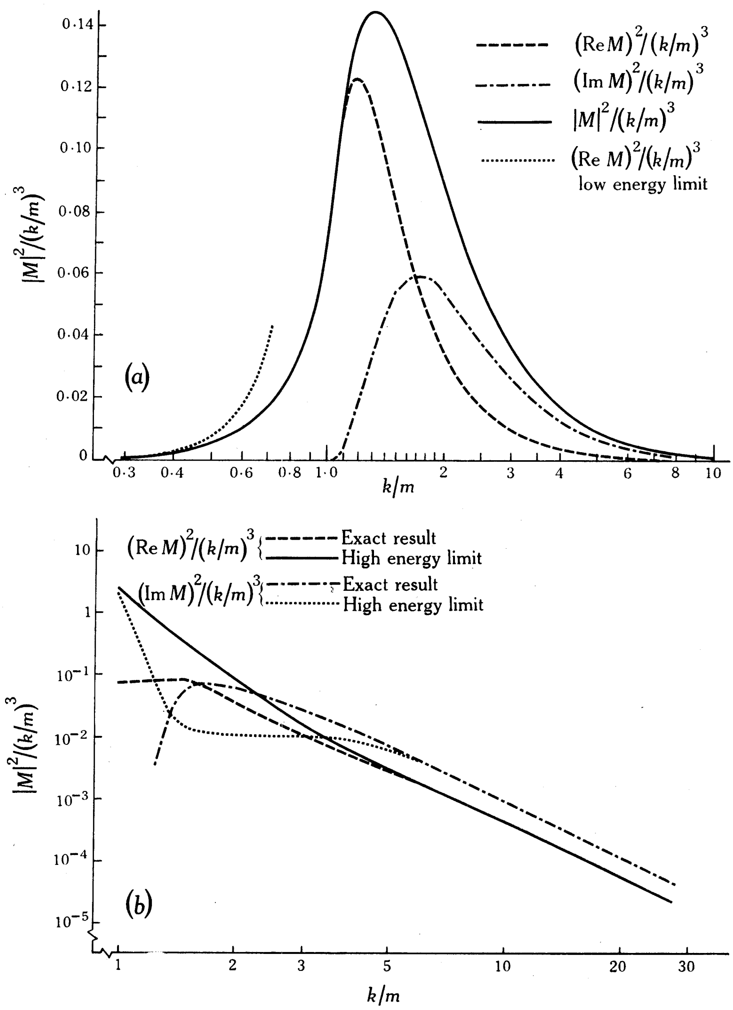

Fig. 2. Variation with energy of the calculated cross section for photon fission (*a*) in the range $0\cdot3 \leqslant k/m \leqslant 10$ and (*b*) in the high energy limits. The results shown are for the linear polarization amplitude M_{1021} in the case when the two emitted photons are oppositely directed and $\theta = \frac{1}{2}\pi$. Extrapolations from the low and high energy approximations (16) and (17) are compared with the exact results in (*a*) and (*b*) respectively.

Hughes (1978*a*, 1978*b*) has suggested a technique whereby electron–positron pairs, or muon pairs, can be 'leaked' into the focal area of a suitable cavity that is designed to receive high power laser pulses. If these pulses are of sufficiently high power they can strip nuclei of their electrons, which can then be removed by a suitable

DC electric field. Maximum laser powers continue to increase, and laser and optical technology continues to improve. In view of the 'collective photon effect' theories of Panarella (1977), with some support for his findings by Allen (1977), and of Hughes (1978*a*, 1978*b*) we suggest that photon-fission experiments using lasers should be undertaken.

References

Adler, A. W., and Cohen, S. G. (1966). *Phys. Rev.* **146**, 1001.

Adler, S. L., Bahcall, J. N., Callan, G. C., and Rosenbluth, M. N. (1970). *Phys. Rev. Lett.* **25**, 1061.

Allen, A. D. (1977). *Foundations Phys.* **7**, 609.

Bialynicka-Birula, Z., and Bialynicki-Birula, I. (1970). *Phys. Rev.* D **2**, 10, 2341.

Boivin, A., and Wolf, E. (1965). *Phys. Rev.* **138**, B1561.

Bolsterli, M. (1954). *Phys. Rev.* **94**, 367.

Bunkin, F. V., and Tugov, I. I. (1970). *Sov. Phys. Dokl.* **14**, 678.

Delbruck, M. (1933). *Z. Phys.* **84**, 144.

Ehlotsky, F., and Sheppey, G. C. (1964). *Nuovo Cimento* **33**, 1185.

Euler, H. (1936). *Ann. Phys. (Leipzig)* **26**, 398.

Glauber, R. J. (1963). *Phys. Rev.* **131**, 2766.

Grove, R. E., Wu, F. Y., and Ezekiel, S. (1977). *Phys. Rev.* A **15**, 227.

Hora, H., Kane, E. L., and Hughes, J. L. (1978*a*). *J. Appl. Phys.* **49**, 923.

Hora, H., Kane, E. L., and Hughes, J. L. (1978*b*). *Nucl. Instrum. Methods* **150**, 589.

Hughes, J. L. (1978*a*). Paper presented at 1st Australian National Laser Conf., Canberra, 20–23 March 1978.

Hughes, J. L. (1978*b*). Paper presented at 9th U.S.S.R. National Conf. on Coherence and Non-linear Optics, Leningrad, June 13–16, 1978.

Jackson, H. E., and Wetzel, K. J. (1969). *Phys. Rev. Lett.* **22**, 1008.

Jarlskog, G., Jonsson, L., Prunster, S., Schulz, H. D., Willutzki, H. J., and Winger, G. G. (1973). *Phys. Rev.* D **9**, 3813.

Jauch, J. M., and Rohrlich, F. (1976). 'The Theory of Photons and Electrons' (Springer: New York).

Karplus, R., and Neuman, M. (1950). *Phys. Rev.* **80**, 380.

Layzer, A. J. (1960). *Phys. Rev. Lett.* **4**, 580.

Lewin, L. (1958). 'Dilogarithms and Associated Functions' (McDonald: London).

Lindsey, J. F. (1975). Ph.D. Thesis, Monash University.

Lorrain, P., and Corson, D. (1970). 'Electromagnetic Fields and Waves,' 2nd edn, p. 471 (Freeman: San Francisco).

McKenna, J., and Platzman, P. M. (1963). *Phys. Rev.* **129**, 2354.

Mitchell, K. (1949). *Philos. Mag.* **40**, 351.

Panarella, E. (1977). *Phys. Rev.* A **16**, 672.

Papanyan, V. O., and Ritus, V. I. (1972). *Sov. Phys. JETP* **34**, 1195.

Papatzacos, P., and Mork, K. (1975). *Phys. Rep.* C **21**, No. 2.

Richards, B., and Wolf, E. (1959). *Proc. R. Soc. London* A **253**, 358.

Shima, Y. (1966). *Phys. Rev.* **142**, 944.

Synge, T. M. (1958). 'Relativity—the Special Theory', Ch. IX (North-Holland: Amsterdam).

Talman, J. D. (1965). *Phys. Rev.* **139**, 1644.

Toll, J. S. (1952). Ph.D. Thesis, Princeton University.

de Tollis, B. (1964). *Nuovo Cimento* **32**, 757.

Triebwasser, S., Dayhoff, E. S., and Lamb, W. E. (1953). *Phys. Rev.* **89**, 98.

Varfolomeev, A. A. (1966). *Sov. Phys. JETP* **23**, 681.

Manuscript received 12 December 1978