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An Analysis of the γ-ray Decay of Weak s Resonances in ^{40}Ca

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Abstract

The structure of the negative-parity low-lying levels in ^{41}Ca is calculated on the basis of a core–nucleon model. Good agreement is obtained between the calculated and experimental level characteristics. The resulting p-level wavefunctions are used to analyse the mechanism of the γ-ray decay of the weak s resonances in ^{40}Ca at 20·4 and 42·1 keV.

Introduction

A clear understanding of the nucleon capture mechanism requires a satisfactory model description of the structure of the final levels populated in the product nucleus by direct γ-ray transitions from the capture state. Thus the choice of a product nucleus suitable for theoretical investigation is rather limited. One such nucleus is ^{41}Ca, whose low-lying level structure is well described by the core–nucleon model of Federman *et al.* (1969). This affords us a means of studying the neutron capture mechanism in ^{40}Ca.

The ^{40}Ca(n, γ)^{41}Ca reaction has been thoroughly investigated experimentally with both thermal neutrons (Gruppelaar and Spilling 1967) and resonance neutrons (Musgrove *et al.* 1976). In the case of thermal neutron capture, the direct reaction mechanism is supposed to be of appreciable importance, since a positive correlation exists between the reduced probabilities of direct γ-ray transitions (where $B(\mathrm{E1}) \approx I_\gamma/E_\gamma^3$) and the spectroscopic factors of the final p levels. The situation is quite different in the case of resonant neutron capture (s resonances with energies of 20·4 and 42·1 keV), where valence capture contributions to the measured radiative widths are less than a few per cent (Musgrove *et al.* 1976). It is obvious that γ-ray decays of these resonances proceed primarily through configurations other than the entrance channel.

The main aim of the present paper is to obtain quantitative information on the role of simple configurations in the primary E1 γ-ray transitions that are observed in the γ-ray decay of the weak s resonances in ^{40}Ca. We have used the calculation procedure suggested by Knat'ko and Rudak (1972), according to which the wavefunctions ψ_i and ψ_f for a capture state and a final state respectively of the product nucleus are presented in the form of a series expansion in the basis functions of the same model. Moreover, the expansion coefficients in the final-level wavefunctions are calculated, whereas the coefficients in the capture-state wavefunctions are considered as parameters to be found by fitting to the experimental partial γ-ray widths.

Model Description of ^{41}Ca

The negative-parity level spectrum of ^{41}Ca has been described theoretically in previous papers. Gerac and Green (1967) concentrated on the low-lying levels with spins $3/2^-$ and $7/2^-$ as well as on the fragmentation of the $1f_{7/2}$ and $2p_{3/2}$ single-particle neutron states. However, because they did not take the $1f_{5/2}$ and $2p_{1/2}$ neutron orbits into account, the results of their calculations are of restricted application. Federman *et al.* (1969) calculated the spectrum of the negative-parity levels in ^{41}Ca using a simple phenomenological model in which a neutron in the 2p and 1f shells is coupled to the ground state and the first rotation band in ^{40}Ca (the states at 3·35 (0^+), 3·90 (2^+) and 5·28 MeV (4^+)). Their description of the level spectrum for spins of $1/2^-$, $3/2^-$ and $7/2^-$ is quite satisfactory, but that for spins of $5/2^-$ is poor.

More complete spectroscopy information on ^{41}Ca than was available to Federman *et al.* (1969) has been acquired recently (Seth *et al.* 1974; Tabor *et al.* 1975). In particular, Seth *et al.*, who made high resolution investigations of the ^{40}Ca(d, p)^{41}Ca reaction, discovered a doublet of d and p states in the 3·73 MeV energy range, and assigned a spin of $1/2^-$ to the 3·730 MeV level. This spin assignment has been confirmed by Tabor *et al.* Utilizing the more recent data, we here present independent model calculations for ^{41}Ca. We use a phenomenological core–particle model that does not completely specify the nature of the core states (Thankappan and True 1965).

Taking into account only the low-lying levels we limited our calculations to three core states: the ground 0_1^+ state and the first excited states at 3·55 (0_2^+) and 3·90 (2_1^+) MeV. The $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$ and $1f_{7/2}$ states, with energy values taken from the experimental data of Belote *et al.* (1965), were considered to be single-particle valence neutron states. In order to characterize the quadrupole interaction of the valence neutron with the core, the following 'coupling strength' parameters were used:

$$\chi_1 = \eta\langle 0_1^+ \,\|\, Q_c \,\|\, 2_1^+\rangle, \qquad \chi_1' = \eta\langle 0_2^+ \,\|\, Q_c \,\|\, 2_1^+\rangle, \qquad \chi_2 = \eta\langle 2_1^+ \,\|\, Q_c \,\|\, 2_1^+\rangle.$$

In principle the values of χ_1, χ_1' and χ_2 can be determined from the experimental reduced B(E2) probabilities in ^{40}Ca (see e.g. Bindal *et al.* 1975). In our calculation the coupling strengths χ_1, χ_1' and χ_2 were varied within a reasonable range of the experimental values in order to obtain the best agreement with the experimental level energies and their spectroscopic factors. The dipole interaction constant ξ was varied in the range $-0\cdot2 \to 0\cdot2$, as in the work by Thankappan and True (1965). The parameter values which yielded the best description are as follows: energies E, 0·00 ($f_{7/2}$), 2·07 ($p_{3/2}$), 4·10 ($p_{1/2}$), 5·50 ($f_{5/2}$), 3·35 (0_2^+) and 3·90 (2_1^+) MeV; dipole interaction constant $\xi = -0\cdot10$; coupling strengths, $\chi_1 = 0\cdot1$, $\chi_1' = 0\cdot70$ and $\chi_2 = -0\cdot60$ MeV.

The results of the present model calculations, together with those of Gerac and Green (1967) and Federman *et al.* (1969), are compared with the experimental data of Seth *et al.* (1974) and Tabor *et al.* (1975) in Tables 1 and 2. Table 1 compares the energies and spectroscopic factors, and Table 2 compares the B(M1) and B(E2) transition rates for various p and f levels. The present model and that used by Federman *et al.* (1969) yield practically the same description of the energy level spectrum for energies below 4 MeV (see Table 1). Both models correctly predict the energies and spectroscopic factors of all levels with $1/2^-$, $3/2^-$ and $7/2^-$ spins but are unable to explain the existence of the three $5/2^-$ levels within this energy range. In our opinion the present model gives a better description of the reduced probabilities for the M1 and E2 γ-ray transitions than do the previous calculations (see Table 2). In most cases, the present calculated values agree with the experimental ones within the limits of experimental error. On the whole, the properties of the negative-parity low-lying levels in ^{41}Ca are satisfactorily described by the core–nucleon model, and so we can use the obtained wavefunctions to analyse the mechanism of the ^{40}Ca(n,γ)^{41}Ca reaction.

Table 1. Energies and spectroscopic factors of p and f levels in ^{41}Ca

Energies are expressed in keV

	Experiment		Model calculations					
	Seth *et al.*		Gerac and Green		Federman *et al.*		Present work	
J_f^π	E_f	$S(J_f)$	E_f	$S(J_f)$	E_f	$S(J_f)$	E_f	$S(J_f)$
3/2⁻	1943	0·74	1950	0·59	1910	0·79	1945	0·77
3/2⁻	2462	0·23	2470	0·19	2760	0·21	2491	0·22
3/2⁻	3730	0·017	3900	0·22	4100	0·00	3949	0·004
1/2⁻	3613	0·08			3430	0·08	3075	0·09
1/2⁻	3944	0·62			4160	0·90	4212	0·92
7/2⁻	0	0·80	0	0·85	0	0·99	0	0·99
7/2⁻	2960	0·007	3220	0·15	2520	0·008	2395	0·002
5/2⁻	2574	(0·007)						
5/2⁻	3050	0·007			3400	0·003	3082	0·001
5/2⁻	3494	0·008						

Table 2. B(M1) and B(E2) transition rates for p and f levels in ^{41}Ca

B(M1) and B(E2) values are expressed in μ_n^2 and $e^2\,\mathrm{fm}^4$ respectively

		Experiment	Model calculations		
J_i^π	J_f^π	Tabor *et al.*	Gerac and Green	Federman *et al.*	Present work[A]
			B(M1) values		
$3/2_2^-$	$3/2_1^-$	$0\cdot073\pm0\cdot018$	0·34	$6\cdot6\times10^{-4}$	0·025
$3/2_3^-$	$3/2_1^-$	$0\cdot005\pm0\cdot002$	0·37		0·007
$3/2_3^-$	$3/2_2^-$	$0\cdot026\pm0\cdot007$	0·10		0·006
$1/2_1^-$	$3/2_1^-$	$0\cdot026\pm0\cdot007$			0·085
$1/2_1^-$	$3/2_2^-$	$0\cdot052\pm0\cdot015$			0·103
$1/2_2^-$	$3/2_1^-$	0·12			0·091
$1/2_2^-$	$3/2_2^-$	0·03			0·056
$7/2_2^-$	$7/2_1^-$	$0\cdot034^{+0\cdot016}_{-0\cdot008}$	0·08		8×10^{-6}
			B(E2) values		
$3/2_1^-$	$7/2_1^-$	37 ± 4	52	77	48·65
$3/2_1^-$	$7/2_2^-$	0·025	1·2	0·49	0·42
$7/2_2^-$	$7/2_1^-$	$5\cdot3\ ^{+2\cdot7}_{-1\cdot6}$	1·8		10·74
$3/2_3^-$	$7/2_1^-$	$0\cdot74\pm0\cdot20$	2·1		5·42

[A] Our B(M1) calculations are for $g_s = -3\cdot826$ and $g_R = 0\cdot5$, while our B(E2) calculations are for $e_{\rm eff} = 1$.

Gamma-ray Decay of s Resonances at 20·4 and 42·1 keV

The neutron s resonances at 20·4 and 42·1 keV in ^{40}Ca are characterized by very small dimensionless reduced neutron widths given by $\theta_n^2 = 8\cdot2\times10^{-6}$ and $1\cdot9\times10^{-6}$ respectively (for the experimental data, see Musgrove *et al.* 1976). (By way of comparison we note that the mean value of θ_n^2 for s resonances in ^{40}Ca is $3 \cdot 3 \times 10^{-3}$.) The θ_n^2 values were calculated from the expression

$$\theta_n^2 = \Gamma_n/(2k\hbar^2/mR),$$

where Γ_n is the neutron width of the resonance (taken from Mughabghab and Garber 1973), R is the nuclear radius (which is 4·3 fm for ^{41}Ca) and m is the neutron mass. The radiative widths Γ_γ of the 20·4 and 42·1 keV resonances are 0·7 and 2·0 eV respectively, and are comparable with the mean $\langle \Gamma_\gamma \rangle = 1 \cdot 8$ eV for s resonances in ^{40}Ca (see Musgrove *et al.*). Thus, obviously the 20·4 and 42·1 keV resonances distinguish themselves sharply among the other s resonances by the small contributions of the valent component to γ-ray decay.

In our calculations, the partial radiation widths were presented as

$$\Gamma_{\gamma if}^{E1} = \left([\Gamma_{\gamma if}^{vm}]^{\frac{1}{2}} + \sum_{k=1}^{3} C_k [\Gamma_{\gamma if}(k)]^{\frac{1}{2}} \right)^2, \tag{1}$$

where $\Gamma_{\gamma if}^{vm}$ is the valence component of the partial γ-ray width, $\Gamma_{\gamma if}(k)$ is the γ-ray width for the transition from the doorway state, and C_1, C_2 and C_3 are the expansion coefficients for the resonance wavefunction configurations $|0_2^+, s_{1/2}; 1/2^+\rangle$, $|2_1^+, d_{5/2}; 1/2^+\rangle$ and $|2_1^+, d_{3/2}; 1/2^+\rangle$ respectively. The $\Gamma_{\gamma if}^{vm}$ values were calculated by the method of Cugnon (1976). We evaluated the γ-ray width $\Gamma_{\gamma if}(k)$ from the expression of Knat'ko and Rudak (1972), using the single-particle radial matrix elements obtained by Lynn (1968). The coefficients C_1, C_2 and C_3 were considered to be free parameters whose values were found through fitting the experimental partial γ-ray widths by means of the CERN program MINSQ which minimizes the function

$$F = \sum_f (\Gamma_{\gamma if}^{exp} - \Gamma_{\gamma if}^{calc})^2 \tag{2}$$

with respect to the parameters C_k. Five γ-ray transitions from the resonance states to the p levels at 1·943, 2·462, 3·613, 3·730 and 3·944 MeV were examined. The calculated partial γ-ray widths and C_k^2 values are compared with experimental data in Tables 3 and 4 respectively.

The γ-ray decay scheme for the s resonance at 42·1 keV is explained satisfactorily by the present model (as can be seen from Table 3), with one exception, namely the transition to the level at 3·944 MeV for which the calculated partial γ-ray width is approximately an order of magnitude less than the experimental value. This discrepancy arises from deficiencies in the description of the 3·944 MeV level structure by the core–nucleon model, which predicts a spectroscopic factor of 0·92, whereas its experimental value is 0·62. As a consequence, the role of the doorway states in the transition to the 3·944 MeV level is underestimated several times, even though the γ-ray decay of the 42·1 keV resonance is determined solely by a configuration different from the entrance channel. In particular, a 20% decrease in the amplitude of the $|0_1^+, p_{3/2}; 3/2^-\rangle$ configuration and a corresponding variation in that of the $|2_1^+, p_{3/2}; 3/2^-\rangle$ configuration in the wavefunction of the 3·944 MeV level result in an increase of $\Gamma_{\gamma if}^{calc}$ by more than an order of magnitude for the same values of C_k.

It should be noted, that the reduced probabilities B(M1) for γ-ray transitions from the 3·944 MeV level to the levels at 1·943 and 2·462 MeV predicted by the present core–nucleon model agree satisfactorily with the experimental ones. Table 5 shows that the basic contribution to B(M1) originates from the $|0_1^+; lj\rangle$ configurations, and so the B(M1) values are not affected by an underestimation of contributions from configurations with the excited core.

Table 3. Partial γ-ray widths for transitions from s resonance states in ^{40}Ca

The listed experimental errors allow for errors in the γ-ray transition intensities and in the total γ-ray width of the s resonance

E_f (keV)	J_f^-	42·1 keV resonance $\Gamma_{\gamma if}^{\rm exp}$ (meV)	$\Gamma_{\gamma if}^{\rm vm}$ (meV)	$\Gamma_{\gamma if}^{\rm calc}$ (meV)	20·4 keV resonance $\Gamma_{\gamma if}^{\rm exp}$ (meV)	$\Gamma_{\gamma if}^{\rm vm}$ (meV)	$\Gamma_{\gamma if}^{\rm calc}$ (meV)
1943	3/2⁻	380^{+51}_{-142}	0·14	280	165^{+40}_{-35}	0·28	63
2462	3/2⁻	508^{+64}_{-187}	0·02	535	88^{+24}_{-22}	0·22	95
3613	1/2⁻	52^{+22}_{-30}	0·005	57	12^{+6}_{-5}	0·02	16
3730	3/2⁻	<10 (1·0%)[A]	$1\cdot7\times10^4$	12	<2 (0·5%)[A]	0·02	46
3944	1/2⁻	294^{+38}_{-109}	0·023	5	8 ± 5	0·09	4·5

[A] Values in parentheses are the corresponding intensities $I_{\gamma if}$.

According to our results, the C_k^2 values for the 42·1 keV resonance agree to within an order of magnitude with the value of $\bar{\theta}_n^2$ and they are similar to the mean C_k^2 values (see Table 4) obtained from calculations based on varying $\Gamma_{\gamma if}^{\rm exp}$ in the range $\Gamma_{\gamma if}^{\rm exp} \pm \Delta\Gamma_{\gamma if}$, where $\Delta\Gamma_{\gamma if}$ is the experimental error (the fitting was carried out for 100 sets of $\Gamma_{\gamma if}^{\rm exp}$ values, generated arbitrarily within prescribed limits).

Table 4. Contributions of simple configurations to resonance wavefunctions in ^{40}Ca

Configuration	42·1 keV resonance C_k^2	20·4 keV resonance C_k^2
$\|0_2^+; s_{1/2}; 1/2^+\rangle$	0·54% (0·55%)[A]	0·01%
$\|2_1^+; d_{3/2}; 1/2^+\rangle$	0·11% (0·10%)	0·81%
$\|2_1^+; d_{5/2}; 1/2^+\rangle$	1·25% (1·23%)	2·25%

[A] Values in parentheses are mean values of C_k^2 (see text).

The agreement between the calculated and experimental partial γ-ray widths is worse in the case of the 20·4 keV resonance. For the most intense γ-ray transition to the first p level at 1·943 MeV, the discrepancy far oversteps the limits of experimental error. The most probable cause of the discrepancy is the influence of the 19·3 keV d resonance. It should be noted that the total intensities of the γ-ray transitions from the 19·3 and 20·4 keV resonances were measured by Musgrove *et al.* (1976). The present calculations are thus based on the assumption that $\Gamma_{\gamma if}^{\rm exp}(20\cdot4) \gg \Gamma_{\gamma if}^{\rm exp}(19\cdot3)$. Taking into account the experimentally determined γ-ray transition to the ground state of ^{41}Ca with spin 7/2⁻, we find that the most probable value for the 19·3 keV resonance spin is 5/2. Thus, only the intensities of γ-ray transitions to 3/2⁻ levels will be affected by the presence of the 19·3 keV resonance.

The calculations for the 42·1 keV resonance thus show that γ-ray transitions with a total intensity of about 40% can be explained satisfactorily within the framework of the core–nucleon model by means of γ-ray decay through configurations involving the excited core. Additional experimental data are required before definite conclusions can be formed regarding doorway state contributions to the primary high-energy γ-ray transitions in the case of the 20·4 keV resonance. However, the present results indicate that the dominating role of the doorway states in the population of p levels in ^{41}Ca seems to be a characteristic feature of both resonances.

It should be noted that resonant neutron capture in light nuclei seems to be of particular interest from the point of view of the highly excited state structure, since resonances with very small reduced neutron widths are discovered experimentally in these nuclei.

Table 5. Structure of matrix elements for M1 γ-ray transitions in ^{41}Ca

The matrix elements are for γ-ray transitions from the 3944 keV level to the levels at 1943 and 2462 keV, and are expressed in arbitrary units

$\mid R_i, (l_j)_i\rangle \to \mid R_f, (l_j)_f\rangle$	$1/2_2^- \to 3/2_1^-$	$1/2_2^- \to 3/2^-$
$\mid 0_1, p_{1/2}\rangle \to \mid 0_1, p_{3/2}\rangle$	0·070	−0·037
$\mid 0_2, p_{1/2}\rangle \to \mid 0_2, p_{3/2}\rangle$	−0·003	−0·006
$\mid 2_1, p_{3/2}\rangle \to \mid 2_1, p_{1/2}\rangle$	−0·006	0·007
$\mid 2_1, p_{3/2}\rangle \to \mid 2_1, p_{3/2}\rangle$	0·002	0·003
$\mid 2_1, f_{5/2}\rangle \to \mid 2_1, f_{7/2}\rangle$	−0·004	−0·009

Conclusions

The basic results of the present paper are as follows: (i) The structure of the negative-parity low-lying levels ($E_x < 4$ MeV) in ^{41}Ca can be described satisfactorily within the framework of a core–nucleon model, without detailed consideration of the core excited state nature. (ii) The use of the core–nucleon model for the analysis of partial γ-ray widths for the 20·4 and 42·1 keV resonances in ^{40}Ca reveals the dominant role of configurations involving the excited core in the direct γ-ray transitions to low-lying p levels in ^{41}Ca. This effect can be accounted for by an anomalously small reduced neutron width for these resonances and the simple structure of the low-lying p levels in ^{41}Ca.

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