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The Nucleon Ground State

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Abstract

From the parameters obtained in a previous paper for the P11 pion–nucleon state, the ground state potential is determined and the meson probability density is calculated. The results hold for the simplest form of the overlap matrix and are phase-equivalent to an infinite set of nonlocal potentials.

In a previous paper (Clayton *et al.* 1977; hereafter referred to as CCR) we presented results for reaction matrix fits to low energy pion–nucleon scattering phase shifts. In particular, we obtained reasonable results for both local and nonlocal potentials which reproduced the P11 phase shifts of Roper *et al.* (1965). These data manifested the bound state pole of the nucleon ground state, but the local potential gave no such bound state. This is taken as direct evidence that the pion–nucleon interaction must be nonlocal, which raises the possibility that one can use reaction matrix theory to evaluate the ground state wavefunction and meson probability density, by extrapolating the wavefunction to the pole energy. In this note, an evaluation is given of the meson probability density together with the effective local potential at the pole.

Using the functions and constants defined in CCR, and letting the pole in the P11 state occur at an imaginary momentum $q = \mathrm{i}k_1$, where k_1 is a real constant, we have for the source density

$$\rho(q^2,r) = \int_0^a \mathrm{d}r'\, V(r,r')\,\Psi(q^2,r') \tag{1}$$

and for the nonlocal potential

$$V(r,r') = \sum_{\lambda,\mu} V_{\lambda\mu}\, U_\lambda(r')\, W_\mu(r)\,. \tag{2}$$

The quantities calculated from the CCR data in Table 1*c* are

$$\Psi(-k_1^2,r) = \sum_{\lambda} A_\lambda(-k_1^2) \sum_{\mu} B_{\lambda\mu}\, W_\mu(r) \tag{3}$$

and the local potential at the pole is

$$V(-k_1^2,r) = \rho(-k_1^2,r)/\Psi(-k_1^2,r)\,. \tag{4}$$

From equation (3) we calculated $|\Psi|^2$ and normalized it to unity. We chose the bilinear overlap matrix from CCR since it is the simplest form one can select.

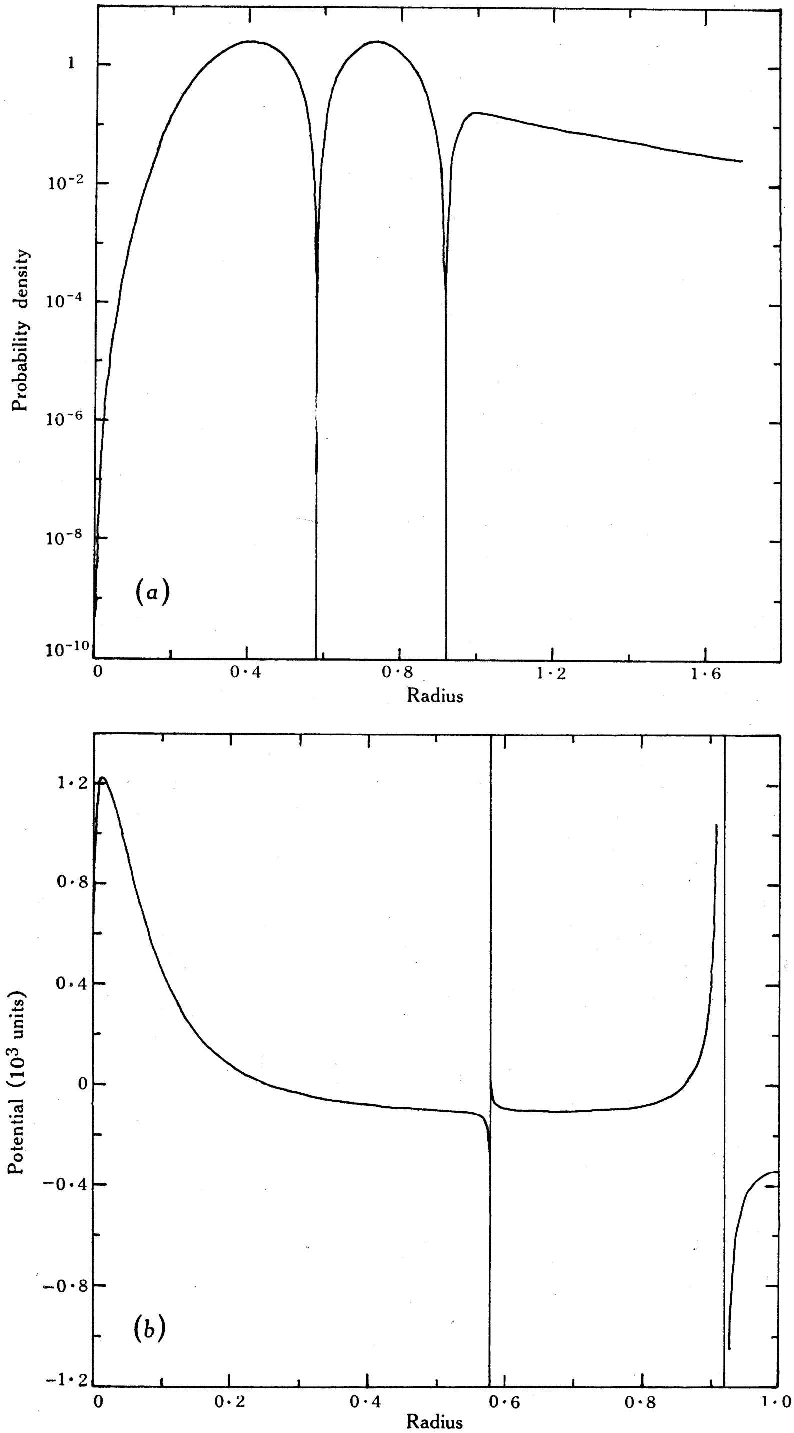

Fig. 1. Calculated results for (*a*) the meson ground state probability density and (*b*) the corresponding meson–nucleon potential. The radius is given in units of pion Compton wavelengths and the potential is in units of the pion rest mass.

A feature of nonlocal potentials is that the derived effective energy-dependent local potentials can exhibit singularities in their radial dependence, owing to zeros in the wavefunction. This leads to the unusual feature that the meson probability density possesses nodes; in the case of the present calculation there are two.

Lang and Cook (1978) have examined the general class of phase-equivalent nonlocal potentials, and their work shows that, unless the orthogonal overlap matrix can be determined in full, an infinite member set of phase-equivalent nonlocal potentials can be defined, leading to infinite sets of ground state probability densities and effective local potentials. This is not unusual in potential theory as, for example, the electromagnetic potential is indeterminate up to a particular gauge, whose selection is arbitrary. As long as the force is invariant under gauge transformations, any gauge-equivalent potential is acceptable. If we regard the scattering phase shift as the quantum mechanical manifestation of a force, then it is not unreasonable to regard the Lang–Cook transformation of the potential as an 'equiphase gauge transformation', and we are at liberty to select any overlap matrix that reproduces the phase shifts.

The results of our extrapolations are shown in Fig. 1. The meson ground state probability density (Fig. 1*a*) is seen to be divided into three regions, with zeros coinciding with the radii where singularities occur in the potential. One can approximate the potential (Fig. 1*b*) as the sum of a Yukawa-type potential and two pole potentials.

We felt it important to report these results since, to our knowledge, no previous configuration space treatment has successfully fitted the scattering data and the bound state properties of the nucleon. Dispersion relations have nothing to say about configuration space behaviour.

References

Clayton, E., Cook, J. L., and Rose, E. K. (1977). *Aust. J. Phys.* **30**, 369.
Lang, D. W., and Cook, J. L. (1978). *Aust. J. Phys.* **31**, 215.
Roper, L. D., Wright, R. M., and Feld, B. T. (1965). *Phys. Rev.* **129**, 2300.

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