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Radiation Properties of Arc Antennas in a Warm Plasma

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Abstract

General expressions for the radiation properties of centre-fed standing wave circular arc antennas in a compressible electron plasma are derived in terms of the plasma and antenna parameters. The effects of these parameters on the antenna radiation characteristics are evaluated numerically and shown graphically for quarter-, semi- and three-quarter-circular arc and loop antennas. It is observed that the radiation properties depend significantly on the plasma frequency. For a given antenna size and plasma frequency, the power radiated in the electromagnetic mode decreases with increasing antenna curvature, while that radiated in the plasma mode remains almost independent of the antenna's shape. The radiation efficiency of an arc antenna thus decreases with increasing curvature.

Introduction

In the past, numerous theoretical investigations have studied the radiation characteristics of various types of antennas immersed in a homogeneous warm electron plasma of infinite extent. The presence of the plasma excites an electron plasma (EP) wave in addition to the usual electromagnetic (EM) wave. Using different approaches, Wait (1964), Cook and Edgar (1966), Talekar and Gupta (1966, 1967), Wunsch (1968) and many others have expressed the radiation properties of dipole and linear antennas in terms of the basic parameters of the plasma and the antenna. These methods have been extended to loop and helical (nonlinear) antennas by Adachi *et al.* (1969), Freeston and Gupta (1971), Talekar and Soni (1973, 1974*a*, 1974*b*) and Soni (1978). It is the purpose of the present study to evaluate the radiation fields and resistances of centre-fed circular arc antennas in an electron plasma, to derive their dependence on the plasma and antenna parameters, and to compare the obtained results with existing calculations for linear and loop antennas.

Far-zone Fields

The orientation of the thin circular arc antenna under consideration is shown in Fig. 1. It has a radius a and a circular-arc angle ϕ_0, and it is embedded in a warm neutral homogeneous isotropic plasma of infinite extent. Perturbation due to the source is taken to be small and to have an oscillatory time dependence of the form $\exp(j\omega t)$. The electrons are assumed to be the only effective components of the plasma, the ions being a stationary neutralizing background. Collisions between electrons and neutral particles are neglected.

The antenna is symmetrically fed at $\phi' = 0$ and the current distribution along it is assumed to be sinusoidal, being given by

$$I(\phi') = I_{\mathrm{m}} \sin\big(\beta a(\tfrac{1}{2}\phi_0 - |\phi'|)\big), \tag{1}$$

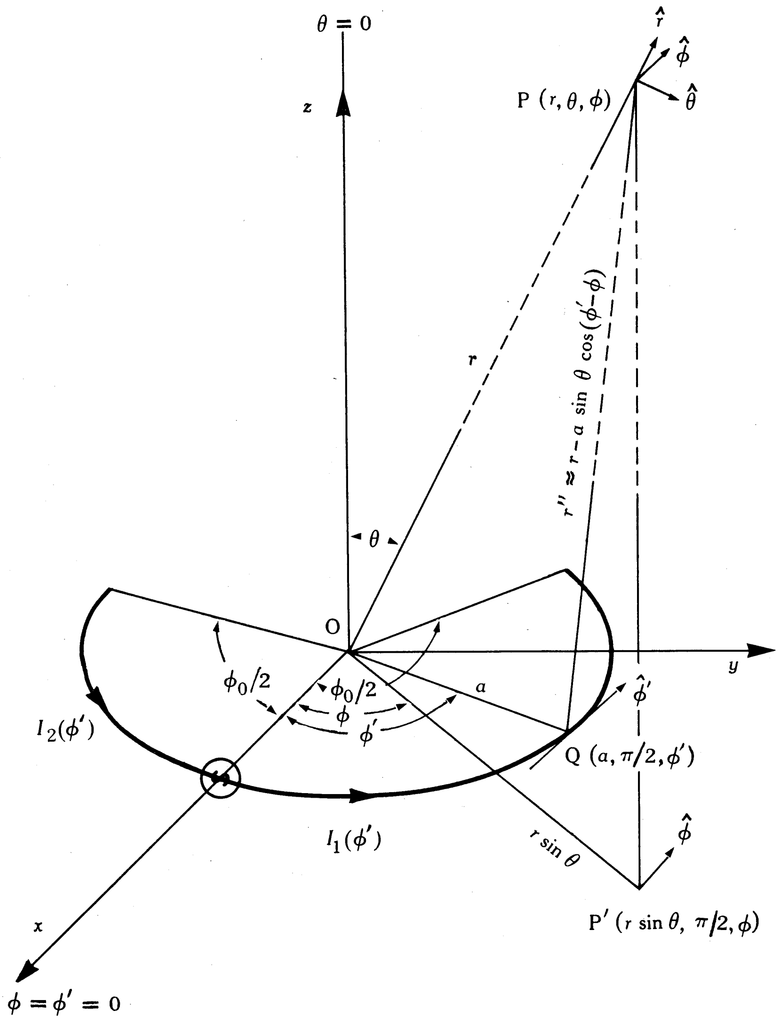


Fig. 1. Geometry of a circular arc antenna in a spherical coordinate system.

with I_{m} the maximum current on the antenna and β the general phase propagation constant of the current wave. The vector potential $\boldsymbol{A}$ of the EM-mode component $\boldsymbol{E}_{\mathrm{e}}$ of the electric intensity due to the antenna at a space point P (r, θ, ϕ) can be expressed as (Wait 1965)

$$\boldsymbol{A} = (\mu_0/4\pi r) \int_{-\frac{1}{2}\phi_0}^{\frac{1}{2}\phi_0} I_{\mathrm{m}} \sin\big(\beta a(\tfrac{1}{2}\phi_0 - \phi' \operatorname{sgn}\phi')\big)\, \hat{\phi}'\, a \exp\big(-\mathrm{j}\beta_{\mathrm{e}}\, r''\big)\, \mathrm{d}\phi'. \tag{2}$$

Here

$$\hat{\phi}' = -\hat{r}\sin\theta\sin(\phi'-\phi) - \hat{\theta}\cos\theta\sin(\phi'-\phi) + \hat{\phi}\cos(\phi'-\phi) \tag{3}$$

is the unit vector tangential to the current element $a\,d\phi'$ at the point Q $(a, \frac{1}{2}\pi, \phi')$; $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are the unit vectors of the field point P in spherical coordinates;

$$r'' \approx r - a\sin\theta\cos(\phi'-\phi) \tag{4}$$

is the distance PQ of the field point from the current element; β_e is the phase propagation constant of an EM wave in the plasma; μ_0 is the permeability of free space; $\operatorname{sgn}\phi'$ is the signum function which is 1, 0 or -1 as ϕ' is positive, zero or negative.

The plasma pressure p of the EP-mode component $\boldsymbol{E}_p$ of the electric intensity can be expressed as (Wait 1965)

$$p = \frac{j}{4\pi r}\frac{\omega_p^2}{\omega}\frac{m}{e}\int_{-\frac{1}{2}\phi_0}^{\frac{1}{2}\phi_0} \nabla\big(I_m \sin\big(\beta a(\tfrac{1}{2}\phi_0 - \phi'\operatorname{sgn}\phi')\big)\, a\exp(-j\beta_p r'')\big)\, d\phi', \tag{5}$$

where β_p is the phase propagation constant of the EP mode, e is the magnitude of the electronic charge, m is the mass of the electron, ω_p is the plasma angular frequency of the electrons and ω is the source angular frequency.

In the far zone, $\boldsymbol{E}_e$ and $\boldsymbol{E}_p$ are related to the components of $\boldsymbol{A}$ and to p respectively by

$$E_{e(\theta)} = -j\omega A_\theta, \qquad E_{e(\phi)} = -j\omega A_\phi; \tag{6}$$

$$E_{p(r)} = -jep/m\varepsilon_0\beta_p u^2; \tag{7}$$

where ε_0 is the permittivity of free space and u is the acoustic velocity of the electrons.

Proceeding on lines similar to those followed by Talekar and Soni (1973) we obtain the following expressions for the components of $\boldsymbol{E}_e$ and $\boldsymbol{E}_p$:

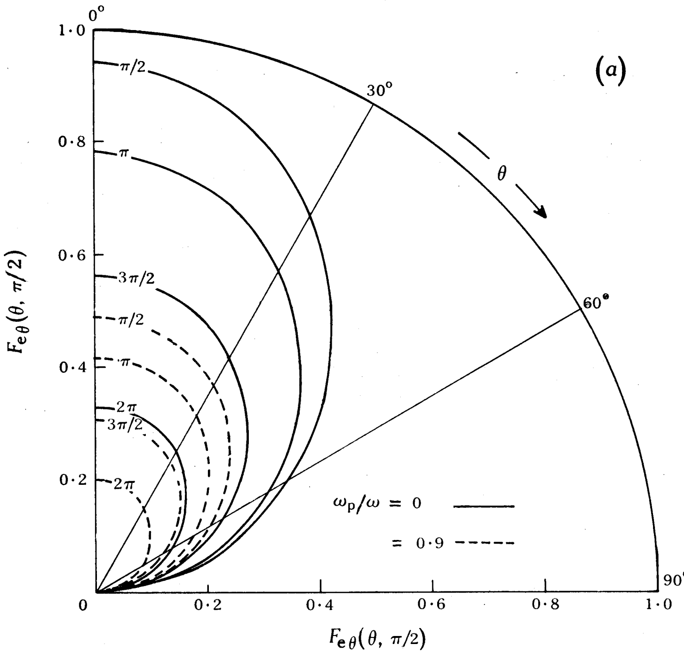
$$E_{e(r)} = 0, \tag{8a}$$

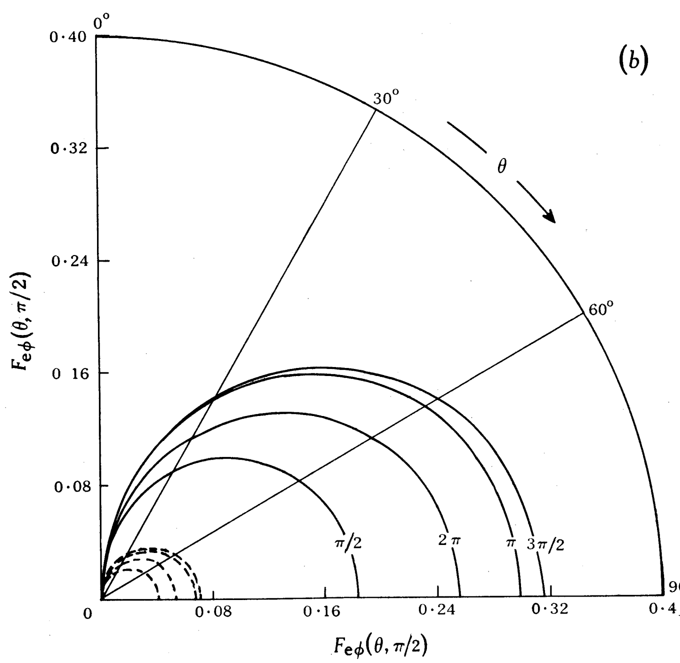
$$E_{e(\theta)} = \frac{ZI_m}{\pi r}\beta a\cot\theta\left(\sum_{q=1}^{\infty} j^q q\, J_q(\beta_e a\sin\theta)\, f(q,\beta,\phi_0)\sin(q\phi)\right)\exp(-j\beta_e r), \tag{8b}$$

$$E_{e(\phi)} = -\frac{ZI_m}{\pi r}\beta\beta_e a^2\left\{\tfrac{1}{2}J_1(\beta_e a\sin\theta)\, f(0,\beta,\phi_0) - \left(\sum_{q=1}^{\infty} j^q J_q'(\beta_e a\sin\theta)\, f(q,\beta,\phi_0) \times\cos(q\phi)\right)\right\}\exp(-j\beta_e r); \tag{8c}$$

$$E_{p(r)} = \frac{I_m}{\pi r}\frac{1}{\varepsilon_0\beta_p u^2}\frac{\omega_p^2}{\omega}\beta a\left(\sum_{q=1}^{\infty} j^q q\, J_q(\beta_p a\sin\theta)\, f(q,\beta,\phi_0)\sin(q\phi)\right)\exp(-j\beta_p r), \tag{9a}$$

$$E_{p(\theta)} = E_{p(\phi)} = 0; \tag{9b}$$





Figs 2a and 2b *[see caption below Fig. 2c opposite]*

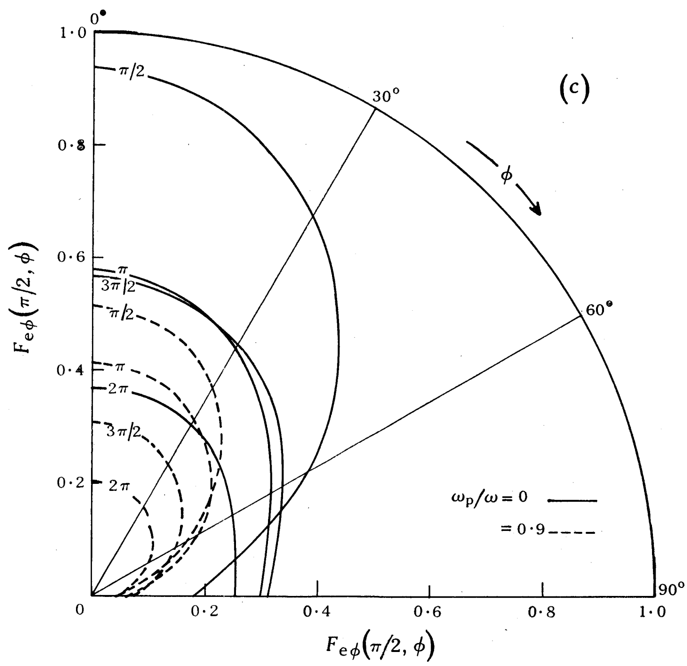


Fig. 2. Angular variation of the far-field pattern factors:

(*a*) $F_{e\theta}$ in the $\phi = \frac{1}{2}\pi$ plane,

(*b*) $F_{e\phi}$ in the $\phi = \frac{1}{2}\pi$ plane,

(*c*) $F_{e\phi}$ in the $\theta = \frac{1}{2}\pi$ plane

for half-wave ($S_{\lambda_0} = 0\cdot5$) quarter-circular arc, semi-circular arc, three-quarter-circular arc and open loop antennas, as indicated on the curves. The unlabelled dashed curves in (*b*) follow the same labelling sequence as the full curves.

where the functional form of f is

$$f(q, \beta, \phi_0) = \{\cos(\tfrac{1}{2}\beta a\phi_0) - \cos(\tfrac{1}{2}q\phi_0)\}/\{(\beta a)^2 - q^2\}, \tag{10}$$

Z is given by

$$Z = (\mu_0/\varepsilon_0)^{\frac{1}{2}}(1 - \omega_p^2/\omega^2)^{-\frac{1}{2}}, \tag{11}$$

$J_n(x)$ is a Bessel function of the first kind and $J_n'(x)$ is its derivative.

Field Patterns

For the determination of the spatial distribution of the electric intensity in the far zone, the field pattern factor F is defined by

$$F(\theta, \phi) = (2\pi r/Z_0 I_m)|E|, \tag{12}$$

where Z_0 is the intrinsic impedance of free space. This definition can also be applied to the components of the electric intensity. Thus, the field pattern factors for the components of the electric intensity can be written as follows

$$F_{e\theta}(\theta,\phi) = (2\pi r/Z_0 I_m)\,|E_{e(\theta)}|, \qquad F_{e\phi}(\theta,\phi) = (2\pi r/Z_0 I_m)\,|E_{e(\phi)}|; \tag{13}$$

$$F_{pr}(\theta,\phi) = (2\pi r/Z_0 I_m)\,|E_{p(r)}|. \tag{14}$$

For numerical calculations of the pattern factors we took $\beta = \beta_e$, which is suggested by the experimental results of Judson *et al.* (1968). The distribution was studied in the $\theta = \frac{1}{2}\pi$ (the plane of the arc) and $\phi = \frac{1}{2}\pi$ planes. For the half-wave arc antenna we took the normalized length $S_{\lambda_0} = a\phi_0/\lambda_0$ to be 0·5. To observe the effects of plasma, we computed the field pattern factors for $\omega_p/\omega = 0$ (free space) and $\omega_p/\omega = 0{\cdot}90$ (plasma frequency very close to the source frequency). The results are given in Fig. 2 for four different values of ϕ_0: $\phi_0 = \frac{1}{2}\pi$, quarter-circular arc; $\phi_0 = \pi$, semi-circular arc; $\phi_0 = \frac{3}{2}\pi$, three-quarter-circular arc; $\phi_0 = 2\pi$, open loop. Since $F_{e\theta}(\frac{1}{2}\pi,\phi)$ vanishes in the $\theta = \frac{1}{2}\pi$ plane, only $F_{e\theta}(\theta,\frac{1}{2}\pi)$, $F_{e\phi}(\theta,\frac{1}{2}\pi)$ and $F_{e\phi}(\frac{1}{2}\pi,\phi)$ are presented. Further, because of their symmetry about the $\theta = 0$ axis (the axis of the arc) and the $\theta = \frac{1}{2}\pi$, $\phi = 0$ and $\phi = \frac{1}{2}\pi$ planes, the field pattern factors are shown in one quadrant only. (The EP-mode field pattern factor $F_p(r)$ has numerous lobes, and is impossible to represent in a similar diagram.)

It is evident from Fig. 2 that the presence of a plasma greatly decreases the magnitude of the electric intensity radiated by the arc. For any given plasma frequency and antenna length, an increase in the curvature of the antenna decreases the electric intensity along the antenna axis. In the plane of the antenna, the intensity also changes with a change in curvature; the beam becomes more circular for an increased antenna curvature and is nonzero in all directions.

Power Considerations

The total power radiated in the far zone by a source immersed in a plasma is the sum of the powers radiated in the transverse EM wave and the longitudinal EP wave. This can be obtained by integrating the Poynting vector $\mathscr{S}_r$, where

$$\mathscr{S}_r = \tfrac{1}{2}Z^{-1}(|E_{e(\theta)}|^2 + |E_{e(\phi)}|^2 + u\omega^2/c\omega_p^2\,|E_{p(r)}|^2), \tag{15}$$

over an infinite spherical surface. Here c is the velocity of light in free space.

The total radiation resistance R_T (the sum of the EM- and EP-mode radiation resistances R_e and R_p respectively) of the antenna referred to the current maximum I_m is defined by

$$\tfrac{1}{2}R_T I_m^2 \equiv \lim_{r\to\infty} \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \mathscr{S}_r\, r^2 \sin\theta\, d\theta\, d\phi. \tag{16}$$

Consequently we have

$$R_e = \frac{1}{ZI_m^2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} (|E_{e(\theta)}|^2 + |E_{e(\phi)}|^2)\, r^2 \sin\theta\, d\theta\, d\phi \tag{17}$$

and

$$R_p = \frac{1}{ZI_m^2}\frac{u}{c}\frac{\omega^2}{\omega_p^2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} |E_{p(r)}|^2\, r^2 \sin\theta\, d\theta\, d\phi. \tag{18}$$

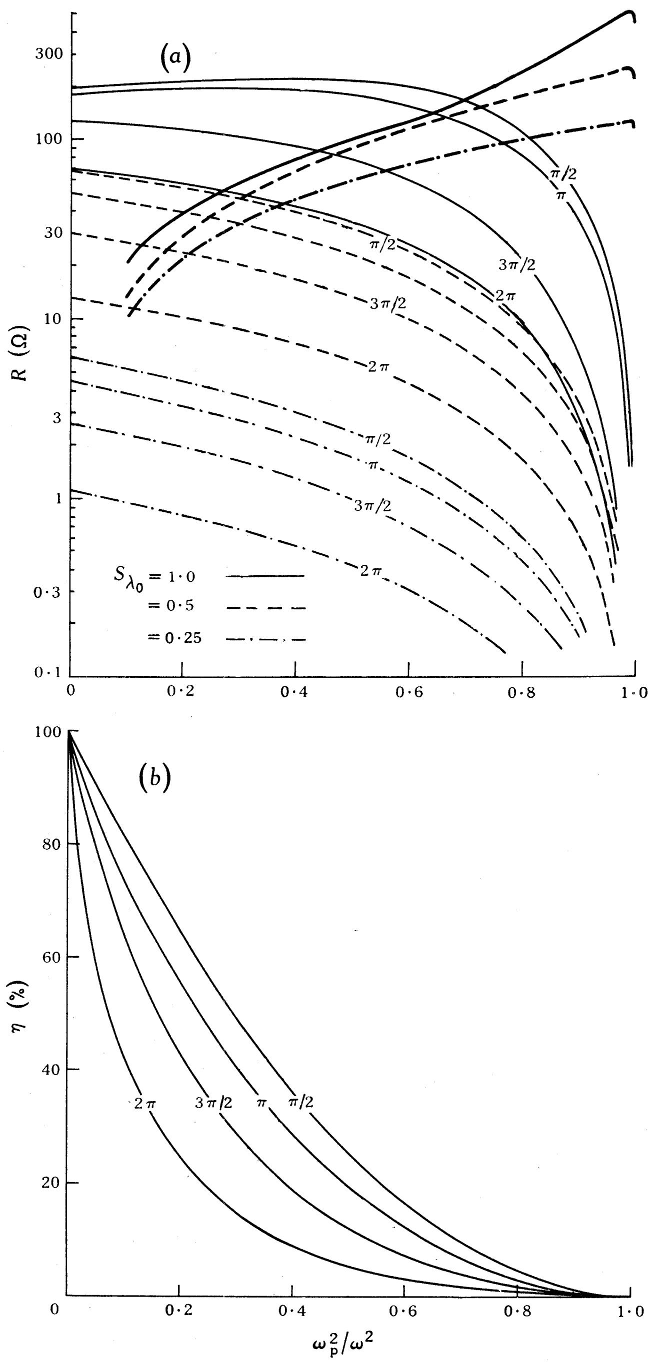


Fig. 3. Dependence on the square of the normalized frequency ω_p^2/ω^2 of:

(*a*) the radiation resistance R_e (light curves) and R_p (bold curves) in the EM and EP modes for antennas with the indicated normalized antenna lengths S_{λ_0}, and with $c/u \geqslant 10^3$;

(*b*) the radiation efficiency η for half-wave ($S_{\lambda_0} = 0 \cdot 5$) antennas.

In (*a*) the curves for R_e are labelled by the ϕ_0 values of the antennas. Those for R_p apply to all four ϕ_0 values.

Using equations (8) and (9) for the EM- and the EP-mode fields respectively we can obtain the following expressions for R_e and R_p:

$$R_e = (Z/\pi)\beta\beta_e a^3 \Bigg\{ \sum_{q=0}^{\infty} J_{2q+1}(2\beta_e a) \times \Bigg(f^2(q+1,\beta,\phi_0) - f^2(q,\beta,\phi_0) - f^2(0,\beta,\phi_0) + 2(\beta_e a)^{-2} \sum_{p=0}^{q} (\beta_e^2 a^2 - p^2) f^2(p,\beta,\phi_0) \Bigg) \Bigg\} \tag{19}$$

and

$$R_p = \frac{2Z}{\pi} \frac{\beta^2 a^2}{\beta_e a} \frac{\omega_p^2}{\omega^2} \sum_{q=1}^{\infty} J_{2q+1}(2\beta_p a) \sum_{p=0}^{q} p^2 f^2(p,\beta,\phi_0). \tag{20}$$

Numerical evaluations of R_e and R_p were made for three different antenna lengths: $S_{\lambda_0} = 1 \cdot 0$, $0 \cdot 5$ and $0 \cdot 25$ (full-wave, half-wave and quarter-wave antennas respectively). For each antenna length, four different shapes were considered: $\phi_0 = \frac{1}{2}\pi$, π, $\frac{3}{2}\pi$ and 2π. Fig. 3*a* shows the resulting dependence of R_e and R_p on ω_p^2/ω^2 in the range 0–1·0.

The radiation efficiency η of a matched antenna in an isotropic plasma medium can be expressed as the ratio of the power radiated in the EM mode to the total input power. Hence we have

$$\eta = R_e/(R_e + R_p + R_L), \tag{21}$$

where R_L is the ohmic resistance, which depends on the length of the antenna and is assumed to be negligible in equation (21). The dependence of η on ω_p^2/ω^2 is shown in Fig. 3*b* for half-wave arcs.

From Fig. 3*a* we see that, for a given angle subtended by an arc antenna and a given plasma frequency, both R_e and R_p increase with the normalized length and the radius of curvature of the antenna, with R_e increasing more rapidly than R_p. However, for a given antenna length and plasma frequency, R_e decreases with increasing curvature, being greatest for a linear antenna (Talekar and Gupta 1967) and least for a loop, while R_p is almost independent of the antenna shape. Hence, R_p depends primarily on the length of the antenna and only secondarily on its shape. This implies that small-angled arcs are more efficient than large-angled arcs, which is evident from Fig. 3*b*.

Conclusions

As was to be expected from the earlier studies, we find that the presence of a plasma significantly affects the free-space radiation characteristics of an arc antenna. The electric intensity radiated in the EM mode decreases with the plasma frequency, which in fact decreases the useful power. For any given plasma frequency and antenna size, the field distribution of an arc antenna covers more space than that of a linear antenna, with the power radiated in the EM mode decreasing with increasing curvature of the arc. However, the power radiated in the EP mode depends primarily on the size of the antenna, and hence the radiation efficiency of an arc antenna decreases with the curvature of the antenna, reaching a maximum for a linear antenna and a minimum for a loop. Thus our results suggest that small-angled arcs are more useful radiators in a plasma medium than large-angled arcs or loop antennas.

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