

Analysis of the (p, 2p) Reaction in an Asymmetric Energy-sharing Mode*

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Abstract

A half distorted-wave formalism, applicable specifically to the analysis of the (p, 2p) reaction in an asymmetric energy-sharing mode, is developed. The formalism explicitly includes *t*-matrix operators of central and tensor forms, and the expansion of the technique to include an intermediate giant resonance excitation mechanism is discussed. Application is made to analyses of the ¹²C(p, 2p) reaction in both symmetric and asymmetric energy-sharing geometries and, when semi-phenomenological effective interaction forms of the type in current use in the analysis of (p, p') data are used, it is shown that the spectroscopic information extracted from the analysis of the asymmetric mode (p, 2p) data is in good agreement with the predictions of 0p shell model calculations.

1. Introduction

The discrepancy between calculated and experimental (p, 2p) cross sections at low energy has remained unresolved for some time, with each factor appearing in the theoretical amplitude being examined in its turn as a source of the discrepancy (Riou and Ruhla 1970). In particular, nonlocal and off-shell effects not included in standard two-nucleon *t* matrices, the bound state wavefunctions and the optical model distorted waves have each been thought possible sources of the discrepancy (Ritchie and Wright 1967; Redish *et al.* 1970; Welch *et al.* 1970; Grimm *et al.* 1971; Birrell *et al.* 1976; Wright *et al.* 1977). However, it has been argued that the comparatively good agreement between data and calculations of (p, 2p) reactions initiated by quite high incident energy protons (Shanta and Jain 1971) demonstrates that the wavefunctions used are at least reasonable. As there are a number of equivalent local model two-nucleon *t* matrices (i.e. ones whose use enables predictions that reproduce both the shape and absolute magnitude of the experimental inelastic cross sections), the remaining discrepancies may well be a consequence of an overlooked reaction mechanism. Indeed, this argument was adopted by Geramb and Strobel (1972), who produced an excellent fit to some ¹⁶O(p, 2p)¹⁵N data by including in the reaction mechanism a two-step process in which they allowed for virtual excitation of giant resonance states of the target nucleus. The subsequent work of Geramb and Eppel (1973) and of Wright *et al.* (1977), in which a distorted wave *t*-matrix approximation was used and special attention was given to off-shell behaviour, confirmed that at low energies the inclusion of such a 'resonance' mechanism was

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essential for the prediction of the observed structure in the double differential cross sections.

It is expected that the importance of such 'two-step' processes should decrease with increasing projectile energy, at least in analyses of data taken in the symmetric energy mode. Thus, at high projectile energy (>100 MeV), analyses should differentiate between 'equivalent' t matrices, simplifications to a complete distorted-wave approach (Birrell *et al.* 1976) and nuclear structure models (Deutchman and Old 1977).

In all of the studies discussed above, without exception, data from symmetric coplanar geometry experiments were analysed. In this paper we will also examine the alternative, asymmetric energy-sharing geometry experiment; an experimental arrangement that we believe has some distinct advantages. Such an experiment requires that the incident proton energy be of the order of, or greater than, 100 MeV and that there be a large difference between the exit channel proton energies (typically with the 'knocked-out' proton having an energy which is less than 30% of that of the emergent 'incident' proton). Under such circumstances the state vectors of the incident and the high energy exit protons can be approximated by attenuated plane wavefunctions in theoretical analyses while, nevertheless, retaining the usual distorted wavefunction to describe the relative motion of the low energy exit proton. The consequent saving in computational time for such a theoretical model over that required to make a full distorted-wave calculation is significant. Further, one can argue that in such a geometry the effects of antisymmetrization between the two exit nucleons will be small, as the low energy and momentum transfer probability amplitudes involved in the 'direct' knockout terms dominate those of the exchange type, in which quite large transfer energies and momenta are involved. Thus we develop a formalism for the analysis of (p, 2p) reactions in an asymmetric energy-sharing mode, within the above unantisymmetrized framework.

We note also that the energy-asymmetric mode conditions involve t -matrix elements that are much closer to being on-shell than those involved in the energy-symmetric mode analyses (Redish *et al.* 1970). Thus, the simple model t matrices to be used herein, based as they are upon their successful use in (p, p') analyses, should be adequate for analyses of the asymmetric mode data.

Of further interest is the role of the intermediate giant resonance mechanism in (p, 2p) reactions in an asymmetric mode. In Section 2 we show that under very specific kinematic conditions the two-step resonance mechanism, seen strongly in the analyses of low energy coplanar *symmetric* geometry experiments, may also strongly affect the analyses of *asymmetric* mode data. Unfortunately, the asymmetric mode data currently available, namely those of Devins *et al.* (1979) for the reaction $^{12}\text{C}(p, 2p)$, would not, from kinematic considerations, be expected to reflect the strong intermediate E1 and E2 strength of ^{12}C (although it may be affected in part by E3 excitation strength). In the subsequent sections of this paper, therefore, we concentrate on developing a formalism for the knockout reaction mechanism and on the spectroscopic information that can be drawn from analyses based upon that formalism.

In Section 3 we justify the use of attenuated plane wavefunctions in our analyses and use a well-understood reaction mechanism ((p, p') to collective states of nuclei) to establish the relevant parameters for subsequent use in (p, 2p) analyses. The (p, 2p) formalism is then presented in Section 4 and the results of application to ^{12}C are presented in the subsequent Sections 5, 6 and 7.

2. Kinematics for the Asymmetric Mode

We suppose that the (p, 2p) reaction is initiated by a proton having mass M , energy E_0 and momentum \mathbf{K}_0 which subsequently is detected to have an energy E_L and momentum \mathbf{K}_L . Likewise we suppose that the other proton, originally bound in the target of mass M_t , is detected to have an energy E_R and momentum \mathbf{K}_R . The appropriate energy-asymmetric mode has $E_R \ll E_L$. Conservation of energy and momentum readily determine the recoil energy and momentum of the residual nucleus (mass M_b) and any intrinsic excitation it may have.

The (p, 2p) reaction may proceed via the direct interaction of the incident proton with that detected in coincidence or via some higher order process. As there exists a particular interest in the kinematics of an intermediate giant resonance mechanism (Geramb and Strobel 1972; Geramb and Eppel 1973), we consider that, in the intermediate stage, the target has an energy of E_t^* and a momentum of \mathbf{K}_t . These kinematic properties are displayed in Fig. 1. The scattering amplitudes associated

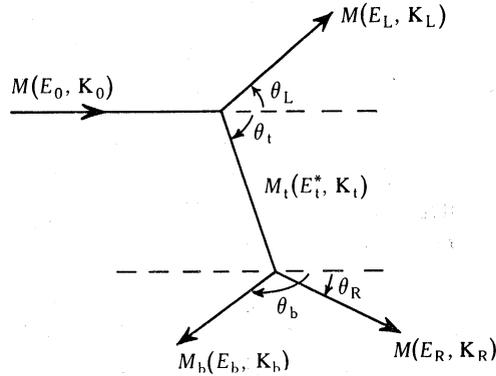


Fig. 1. Kinematics assumed for a (p, 2p) intermediate state excitation mechanism.

with this resonance mechanism should be significant only if the 'inelastic scattering' and 'particle emission' coupling constants (widths) are large. Direct excitation of giant resonance states by inelastic hadron scattering (Satchler 1974, and references therein) and the importance of intermediate giant resonance excitation as a doorway state in the analysis of inelastic proton scattering to discrete low lying states of nuclei (Geramb *et al.* 1975) would indicate that these widths are large if the intermediate (particle-hole) states form a group confined in a narrow energy band that exhausts a significant fraction of an appropriate energy-weighted sum rule. Only the giant resonances meet this criterion and hence such amplitudes will be significant only if the energy available to the intermediate excitation spans the giant resonance region.

If we suppose the intermediate target state to have energy

$$E_t^* = (\hbar^2/2M_t)K_t^2 + \langle E_x \rangle \quad (1)$$

and we force the reaction channel to be 'on the energy and momentum shells', then the excitation energy of the target in the intermediate state is given by

$$\langle E_x \rangle = (M_t - M)E_0/M_t - (M_t + M)E_L/M_t + (2M/M_t)(E_0 E_L)^{\frac{1}{2}} \cos \theta_L. \quad (2)$$

For the asymmetric energy-sharing mode data from ^{12}C (Devins *et al.* 1979), namely

$$E_0 = 98.7 \text{ MeV}, \quad E_L = 59.5 \text{ MeV}, \quad \theta_L = 25^\circ,$$

the excitation energy (2) determines an on-shell intrinsic excitation energy of

$$\langle E_x \rangle = 37.6 \text{ MeV}. \quad (3)$$

The value (3) lies above the well-established giant E1 strength in ^{12}C which, as revealed by analyses of electron scattering from ^{12}C (Yamaguchi *et al.* 1971) and of proton capture by ^{11}B (Brassard *et al.* 1972), has its onset at 20 MeV, a major strength centred around 23 MeV and a secondary peak around 26 MeV in excitation. There is some evidence of E2 strength with centroids at 28, 32 and 42 MeV (Geramb *et al.* 1975) but in each case the distribution has a width of 2 MeV or less. Evidence for a weak E3 coupling strength, from the same analysis, in the 30–40 MeV excitation region is ambiguous at best. Thus, as the (p, 2p) amplitudes in which virtual excitation of intermediate states occurs should involve those collective states whose excitation centres upon the on-shell value of $\langle E_x \rangle$, we expect that in the ^{12}C data of Devins *et al.* (1979) only the effects of the weak and poorly known E3 resonance in ^{12}C should appear. Therefore, in our first analyses of (p, 2p) data for the asymmetric energy-sharing mode, we make use of a half distorted-wave formalism to examine only the operator forms (both central and tensor) for the t matrix, and to note what spectroscopic information can be drawn from analyses based upon a standard knockout reaction mechanism.

In adopting our method of analysis, we *do not* discount the evidence (Geramb and Strobel 1972; Geramb and Eppel 1973; Wright *et al.* 1977) for intermediate resonance amplitudes seen in analyses of coplanar symmetric (p, 2p) data. Rather we suggest that the asymmetric energy-sharing mode, under very specific kinematic conditions, may be a most useful means of probing the giant resonance excitation structure of nuclei. Furthermore, one might also expect that multistep processes, in which, for example, the knockout follows or precedes an inelastic scattering event, may have significant influences, at least upon select transitions. Indeed, Pugh *et al.* (1967) postulated that the excitation of the $5/2^-$ (4.44 MeV) and $7/2^-$ (6.74 MeV) states in ^{11}B , specifically, may be promoted in part by the excitation of the 4.43 MeV (2^+) state in ^{12}C , followed by knockout from that state. Such possible reaction mechanisms will not be discussed in depth in this paper.

3. Attenuated Plane Wave Approximation and Inelastic Scattering

The attenuated plane wave (APW) model was first suggested by a study of the probability flux in the α -particle optical model (McCarthy 1959) and was subsequently confirmed as realistic for α -particle scattering by Austern (1961) and for nucleon scattering by Amos (1966). In this section we shall use a well-understood reaction mechanism, the extended optical model version of the DWBA for collective (p, p') reactions (Love and Satchler 1967), to establish a realistic parameterization of APW functions for subsequent use in (p, 2p) analyses.

The differential cross section for inelastic proton scattering in which the target nucleus is excited from a state J_i to a state J_f , is given by

$$\frac{d\sigma}{d\Omega} = \frac{5k_f}{k_i} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 J_i^{-1} \sum_{v_1 v_2 M_1 M_f} |T_{if}|^2, \quad (4)$$

in which k_i (k_f) is the incident (exit) nucleon wave number, μ is the reduced mass of the system, J_i denotes $2J_i + 1$ and the summation extends over all spin projections of the projectile and target states that enter the scattering amplitude T_{if} . This scattering amplitude can be expressed as

$$T_{if} = \langle \chi_f^{(-)}(0) | \langle \Phi_{J_f} | V | \Phi_{J_i} \rangle | \chi_i^{(+)}(0) \rangle. \quad (5)$$

By using the collective model representation of the reaction mechanism (Love and Satchler 1967)

$$V = R_0 \sum_{\lambda\mu} \alpha_{\lambda\mu}(\xi) Y_{\lambda\mu}^*(\Omega_0) \partial U(r_0) / \partial r_0, \quad (6)$$

in which U is the usual optical model potential for the projectile energy that best fits elastic scattering data, the scattering amplitude (5) reduces to

$$T_{if} = \sum_{\lambda\mu} \langle \Phi_{J_f} | \alpha_{\lambda\mu} | \Phi_{J_i} \rangle R_0 \langle \chi_f^{(-)}(0) | (\partial U / \partial r_0) Y_{\lambda\mu}^*(\Omega_0) | \chi_i^{(+)}(0) \rangle. \quad (7)$$

As our interest is with even mass targets having zero ground state spin, the target state integrations give

$$\langle \Phi_{J_f} | \alpha_{\lambda\mu} | \Phi_{J_i} \rangle = \beta_{J_f} \delta_{\lambda J_f} \delta_{\mu M_f}, \quad (8)$$

and hence

$$T_{if} = \beta_{J_f} R_0 \langle \chi_f^{(-)}(0) | (\partial U / \partial r_0) Y_{J_f M_f}^*(\Omega_0) | \chi_i^{(+)}(0) \rangle, \quad (9)$$

in which the $\chi^{(\pm)}$ are continuum (projectile) wavefunctions. Usually these continuum wavefunctions are chosen to be eigenfunctions of a Schrödinger equation which includes an appropriate optical model potential (the distorted wave approximation). Herein we will use the simpler APW representation (McCarthy and Pursey 1961; Amos 1966; Janus and McCarthy 1974; Birrell *et al.* 1976) which has the form

$$\chi^{(\pm)} = N^{(\pm)} \exp(\pm i \mathbf{K}^{(\pm)} \cdot \mathbf{r}), \quad (10a)$$

involving the complex wave numbers

$$\mathbf{K}^{(+)} = (a + ib) \mathbf{k}_i, \quad \mathbf{K}^{(-)} = (c + id) \mathbf{k}_f \quad (10b)$$

and a normalization factor $N^{(\pm)}$ which we will choose to yield unit magnitudes for these functions at the nuclear surface R_0 . Thus

$$N^{(+)} = \exp(-bk_i R_0), \quad N^{(-)} = \exp(-dk_f R_0). \quad (10c)$$

The preceding wavefunctions image appropriate distorted wavefunctions in the interaction region, with the wave-number modification parameters a and c changing

the wave number in the interaction region from that for the incident beam. The local WKB approximation may be used to establish their appropriate values (Janus and McCarthy 1974). Likewise, the absorption parameters b and d allow for the fact that the distorted wave is attenuated (reflecting particle absorption) as it enters the nucleus. We note that this simple model, whilst suitable for nucleon scattering in the 100 MeV region, is not appropriate at energies below 50 MeV where the effects of the focus in the usual distorted wavefunctions will be severe (Kromminga and McCarthy 1961). It is for this reason that in the following (p, 2p) asymmetric energy-sharing mode formalism we use an APW approximation for the incident and exit high energy nucleons only, and retain full distortion for the exit low energy nucleon.

The use of the simple functions described above facilitates evaluation of the nucleon inelastic scattering transition amplitude, which can now be written as

$$T_{if} = \beta_{J_f} R_0 N^{(+)} N^{(-)} \int dr \exp(i \mathbf{Q} \cdot \mathbf{r}) (\partial U / \partial r) Y_{J_f M_f}^*(\Omega), \quad (11)$$

in which \mathbf{Q} is the complex momentum transfer vector

$$\mathbf{Q} = (a + ib) \mathbf{k}_i - (c + id) \mathbf{k}_f. \quad (12)$$

From equation (11) it is a simple matter to deduce that

$$T_{if} = \beta_{J_f} R_0 N^{(+)} N^{(-)} \{4\pi(2J_f + 1)\}^{\frac{1}{2}} (i)^{J_f} \int r^2 dr j_{J_f}(Qr) (\partial U / \partial r). \quad (13)$$

It then follows that the normalized differential cross section can be evaluated from

$$\frac{d\sigma}{d\Omega} = \frac{5k_f}{k_i} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 4\pi (N^{(+)} N^{(-)})^2 (\beta_{J_f} R_0)^2 \left(\int r^2 dr j_{J_f}(Qr) (\partial U / \partial r) \right)^2. \quad (14)$$

At energies for which the APW approximation should be reasonable, the (inelastic scattering) reaction Q values for the excitation of low lying collective states are small by comparison. Thus we equate the parameters of the attenuated wavefunctions for the incident and emergent projectiles, with the consequence that

$$\mathbf{Q} \approx (a + ib)(\mathbf{k}_i - \mathbf{k}_f) \quad \text{and} \quad N^{(+)} = N^{(-)} = N \quad (15)$$

can be used to further simplify equation (14).

Thus with electromagnetic transition probabilities (or, better, DWBA collective model analyses of the same scattering data) defining appropriate values for β_{J_f} , R_0 and $U(r)$, inelastic scattering data can then be used to select appropriate values for N , a and b ; the last two parameters primarily influence the structure predictions whilst the normalization factor N scales the overall results.

As trial cases of the credibility of the APW approximation for proton scattering, and in particular with a view to its use in analyses of (p, 2p) reaction data, we have used equation (14) to analyse the inelastic proton scattering from ^{12}C as initiated by 100 and 65 MeV protons, and in which the 2^+ (4.43 MeV) and 3^- (9.6 MeV) states were excited. These data have been analysed previously by using the distorted wave

Table 1. Optical potential and deformation parameter values for ^{12}C form factor

J_f^π	Energy	Optical potential							Deform.
	E_{lab} (MeV)	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W_0 (MeV)	$4W_d$ (MeV)	r_d (fm)	a_d (fm)	β_{J_f}
2^+	65.0	20.7	1.44	0.60	0	13.80	1.03	0.48	0.625
2^+	100.0	25.6	1.02	0.65	6.66	0	1.70	0.216	0.625
3^-	100.0	25.6	1.02	0.65	6.66	0	1.70	0.216	0.45

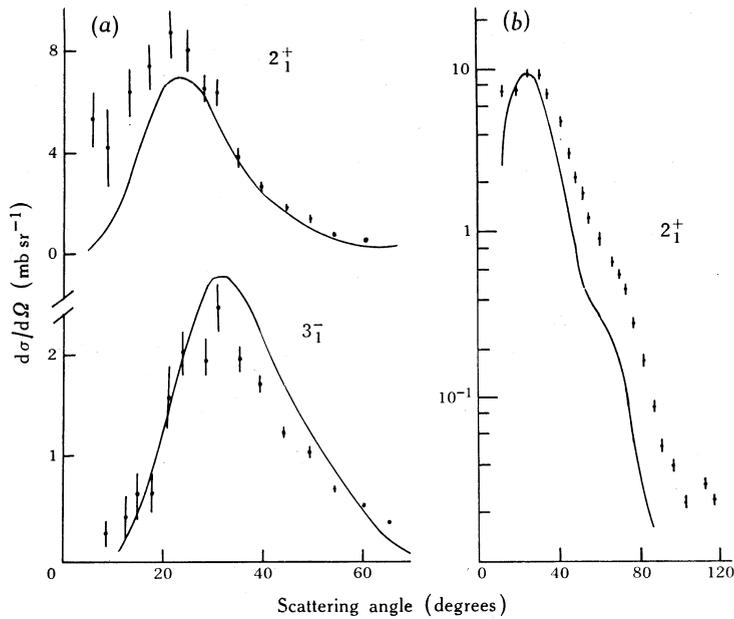


Fig. 2. Predictions from attenuated plane wavefunction, collective model form factor, analyses of the $^{12}\text{C}(p, p')$ reaction compared with experimental data for the inelastic scattering of (a) 100 MeV protons to the 2^+ (4.43 MeV) and 3^- (9.6 MeV) states of ^{12}C and (b) 65 MeV protons to the 2^+ (4.43 MeV) state of ^{12}C .

approximation and/or WKB methods (Haybron 1969; Hosono *et al.* 1978), and pertinent parameter values for the optical potentials ($R_0 = r_0 A^{1/3}$) and deformation parameters (β_{J_f}) are given in Table 1. The APW parameters can be constrained slightly if one uses the WKB approximation at the nuclear radius to estimate the parameter a , giving values of 1.15 and 1.17 for the 100 and 65 MeV proton wavefunctions respectively. Then, with respective values of b of 0.13 and 0.2 and the normalization N set to yield unit magnitudes at the nuclear surface, the predictions for the 2^+ and 3^- cross sections are compared with the data in Fig. 2.

If we decrease the value of b in the 100 MeV analyses (at least as far as the value -0.2), the predicted cross sections become more sharply peaked with little change in the values forward of 20° scattering angle. Likewise, if a is decreased from 1.15, the predicted differential cross section shifts almost laterally to larger scattering angles. Thus the results are sufficiently sensitive that optimal values of the parameters

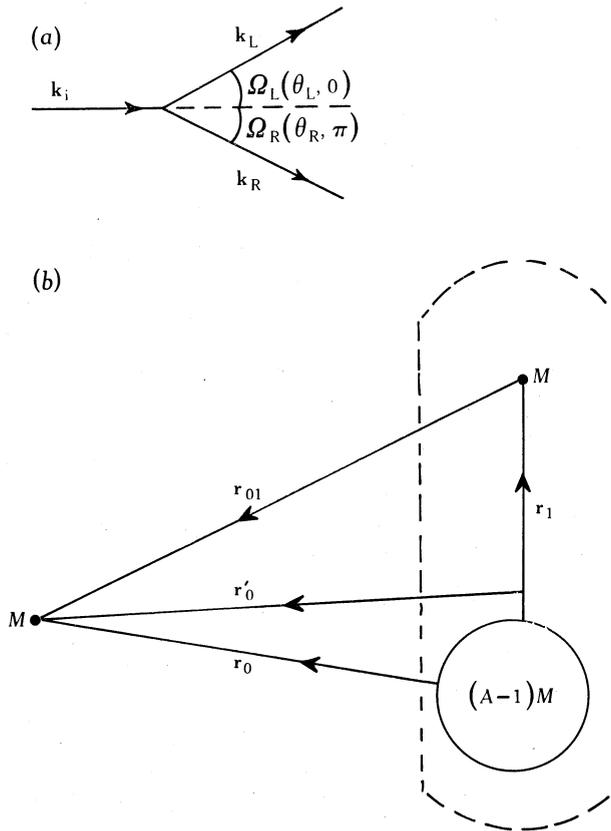


Fig. 3. Showing for the (p, 2p) reaction: (a) the coplanar geometry assumed and (b) the coordinate system for the present formalism.

a , b and thence N can be obtained. The fits shown in Fig. 2 required values of 1.15 (0.13) and 1.17 (0.20) for the parameters a (b) in the 100 MeV (Fig. 2a) and 65 MeV (Fig. 2b) cases respectively; these values were then used in the (p, 2p) analyses reported in Section 6 below.

4. Theory of (nucleon, 2 nucleon) Reactions in an Asymmetric Energy-sharing Mode

We suppose that the (nucleon, 2 nucleon) reaction from a target of mass number A is initiated by nucleons having energy E_i and, with the incident beam defining the z axis, we suppose also that nucleons emerge in a coplanar fashion with one either side (left (L) and right (R)) of the z axis, as depicted in Fig. 3a. For these circumstances, the transition amplitudes, as mediated by a two-nucleon t matrix, can be approximated by

$$T_{fi} = N' \langle \chi_L^{(-)}(0) \chi_R^{(-)}(1) \psi_{J_i T_i}^{(A-1)}(2 \dots A) | t(01) | \psi_{J_i T_i}^{(A)}(1 \dots A) \chi_i^{(+)}(0) \rangle, \quad (16)$$

with the coordinates as defined in Fig. 3b, the $\psi_{J_i T_i}^{(x)}$ being states of the x nucleon nucleus, N' being the proton (neutron) number of the target for the (p, 2p) ((n, 2n))

reaction and $\chi^{(\pm)}$ being the appropriate continuum wavefunctions. From the coordinate definition in Fig. 3b we have

$$\mathbf{r}'_0 = \mathbf{r}_0 - A^{-1} \mathbf{r}_1, \quad (17)$$

with the origin of the \mathbf{r}'_0 coordinate being at the centre of mass of the A nucleon target system.

It is convenient, at this point to make a cofactor expansion of the initial state,* namely

$$|\psi_{JT}^{(A)}(1 \dots A)\rangle = \sum_{j m \alpha} A^{-\frac{1}{2}} |\phi_{j m \alpha}(1)\rangle a_{j m \alpha} |\psi_{JT}^{(A)}(1 \dots A)\rangle, \quad (18)$$

as this enables the transition amplitude of equation (16) to be expanded to

$$\begin{aligned} T_{fi} &= N' A^{-\frac{1}{2}} \sum_{j m \alpha} \langle \psi_{J_f T_f}^{(A-1)}(2 \dots A) | a_{j m \alpha} |\psi_{J_i T_i}^{(A)}(1 \dots A)\rangle \mathcal{M}_j \\ &= \sum_{j m \alpha} N' A^{-\frac{1}{2}} F(j; J_i J_f T_i T_f) (-)^{j+m+\frac{1}{2}+\alpha} (\hat{J}_f \hat{T}_f)^{-\frac{1}{2}} \\ &\quad \times \langle J_i j M_i - m | J_f M_f \rangle \langle T_i \frac{1}{2} P_i - \alpha | T_f P_f \rangle \mathcal{M}_j. \end{aligned} \quad (19)$$

Here the spectroscopic amplitudes

$$F(j; J_i J_f T_i T_f) = \langle \psi_{J_f T_f}^{(A-1)} ||| b_{j \frac{1}{2}}^+ ||| \psi_{J_i T_i}^{(A)} \rangle \quad (21)$$

are the reduced (angular momentum and isospin) matrix elements of the hole creation operator

$$b_{j \frac{1}{2} m \alpha}^+ = (-)^{j-m+\frac{1}{2}-\alpha} a_{j-m(\frac{1}{2})-\alpha},$$

and they weight the two-nucleon matrix elements

$$\mathcal{M}_j = \langle \chi_L^{(-)}(0) \chi_R^{(-)}(1) | t(01) | \chi_i^{(+)}(0') \phi_{j m \alpha}(1) \rangle. \quad (22)$$

In the foregoing, antisymmetrization has been ignored since, as discussed in the Introduction, for a large asymmetric energy-sharing situation such an approximation is both reasonable and greatly reduces computational effort.

For projectile energies of the order of 100 MeV, the asymmetric-energy condition makes the low energy nucleon emerge with energies of the order of tens of MeV and we must make a half distorted-wave approximation to facilitate further development of (and eventually evaluation of) the transition amplitudes. This approximation consists of retaining a distorted wavefunction (from an appropriate optical model potential) for the low energy exit nucleon ($\chi_R^{(-)}(1)$) but using APW functions for the incident and left emergent nucleons. Thus, in developing the two-nucleon transition amplitudes

$$\mathcal{M}_j = \langle P_L^{(-)}(0) \chi_R^{(-)}(1) | t(01) | P_i^{(+)}(0') \phi_{j m \alpha}(1) \rangle, \quad (23)$$

the following expansions of the wavefunctions will be used:

$$|P_i^{(+)}(0')\rangle = N_i \exp(i \mathbf{K}_i \cdot \mathbf{r}'_0) |\frac{1}{2} v\rangle_S |\frac{1}{2} \tau\rangle_T, \quad (24a)$$

* While spherical spectroscopy has been assumed here, it is for simplicity and not of necessity (Amos *et al.* 1976).

$$\langle P_L^-(0) | = N_f^* \exp(-i \mathbf{K}_L \cdot \mathbf{r}_0) \langle \frac{1}{2} v' |_S \langle \frac{1}{2} \tau' |_T, \quad (24b)$$

$$\begin{aligned} \langle \chi_R^-(1) | = & \sum_{L'' J'' M'' \xi''} 4\pi i^{-L''} \exp(i\alpha) f_{L'' J''}^*(k_R r_1) \\ & \times Y_{L'', M'' + v'' - \xi''}^*(\Omega_1) \langle L'' \frac{1}{2} M'' + v'' - \xi'' \xi'' | J'' M'' + v'' \rangle \\ & \times \langle L'' \frac{1}{2} M'' v'' | J'' M'' + v'' \rangle Y_{L'' M''}(\theta_R, \pi) \langle \frac{1}{2} \xi'' |_S \langle \frac{1}{2} \tau'' |_T, \end{aligned} \quad (24c)$$

$$| \phi_{jm}(1) \rangle = R_{lj} \sum_{\mu} \langle l \frac{1}{2} m_l \mu | j m \rangle Y_{lm_l}(\Omega_1) | \frac{1}{2} \mu \rangle_S | \frac{1}{2} \nu \rangle_T. \quad (24d)$$

The coordinates have been decomposed into an intrinsic spin (S) part and an isotopic spin (T) part and the radial functions involve the Coulomb phase shifts α and the normalized partial wavefunction solutions f_{LJ} of the radial optical-model equation.

(a) *Central Force Component of t Matrix*

For a central force component of the t matrix

$$t(01) = \sum_{ST} v_{ST}(01) P^S P^T, \quad (25)$$

the two-nucleon amplitudes expand to

$$\begin{aligned} \mathcal{M}_j = & \sum 4\pi i^{-L''} \exp(i\alpha) \langle \frac{1}{2} \frac{1}{2} \tau y | T M_T \rangle \langle \frac{1}{2} \frac{1}{2} \tau' \tau'' | T M_T \rangle \langle \frac{1}{2} \frac{1}{2} \nu \mu | S M_S \rangle \langle \frac{1}{2} \frac{1}{2} \nu' \xi'' | S M_S \rangle \\ & \times \langle l \frac{1}{2} m_l \mu | j m \rangle \langle L'' \frac{1}{2} M'' + v'' - \xi'' \xi'' | J'' M'' + v'' \rangle \langle L'' \frac{1}{2} M'' v'' | J'' M'' + v'' \rangle \\ & \times Y_{L'' M''}(\theta_R, \pi) N_i N_f^* \\ & \times \int d\mathbf{r}_0 \int d\mathbf{r}_1 \exp\{i(\mathbf{K}_i - \mathbf{K}_L) \cdot \mathbf{r}_0\} \exp\{-iA^{-1} \mathbf{K}_i \cdot \mathbf{r}_1\} \\ & \times v_{ST}^-(01) f_{L'' J''}^*(k_R r_1) R_{lj}(r_1) Y_{lm_l}(\Omega_1) Y_{L'', M'' + v'' - \xi''}^*(\Omega_1). \end{aligned} \quad (26)$$

By defining the (complex) transfer momenta

$$\mathbf{Q} = \mathbf{K}_i - \mathbf{K}_L, \quad \mathbf{X} = \mathbf{Q} - A^{-1} \mathbf{K}_i \quad (27)$$

and by using a simple transformation of coordinates, we obtain the double integral involved in equation (26) as

$$\begin{aligned} I_{r_0} I_{r_1} = & (-) \int d\mathbf{r} \exp\{i \mathbf{Q} \cdot \mathbf{r}\} v_{ST}(\mathbf{r}) \\ & \times \int d\mathbf{r}_1 f_{L'' J''}^*(k_R r_1) Y_{L'', M'' + v'' - \xi''}^*(\Omega_1) R_{lj}(r_1) Y_{lm_l}(\Omega_1) \exp\{i \mathbf{X} \cdot \mathbf{r}_1\}. \end{aligned} \quad (28)$$

Further reduction requires partial wave expansion of the exponential in the r_1 integral (choosing \hat{X} as the integration z axis), to permit use of the Wigner-Eckart theorem. Thereafter, standard angular momentum algebra yields for the (p, 2p) transition amplitude

$$\begin{aligned}
 T_{fi} = & \sum Z A^{-\frac{1}{2}} F(j; J_i J_f T_i T_f) (4\pi)^2 i^{L-L'} \hat{L} \hat{S} [\hat{j} \hat{G} \hat{B} / \hat{J}_i \hat{T}_i]^{\frac{1}{2}} (-)^{S+v'-\frac{1}{2}} \\
 & \times \langle \frac{1}{2} \frac{1}{2} \tau y | T M_T \rangle \langle \frac{1}{2} \frac{1}{2} \tau' \tau'' | T M_T \rangle \langle l L 0 0 | L' 0 \rangle \begin{Bmatrix} B & \frac{1}{2} & \frac{1}{2} \\ S & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \langle \frac{1}{2} \frac{1}{2} v' - v | B \beta \rangle \\
 & \times \langle L B 0 - \beta | G - \beta \rangle \langle j G m - \beta | J'' M'' + v'' \rangle \langle L' \frac{1}{2} M'' v'' | J'' M'' + v'' \rangle \\
 & \times \langle j J_f m M_f | J_i M_i \rangle \langle \frac{1}{2} T_f y P_f | T_i P_i \rangle Y_{L'' M''}(\theta_R + \Delta, \pi) \begin{Bmatrix} B L & G \\ \frac{1}{2} l & j \\ \frac{1}{2} L'' & J'' \end{Bmatrix} N_i N_f^* \\
 & \times \int r^2 dr j_0(Qr) v_{ST}(r) \\
 & \times \int r_1^2 dr_1 j_L(Xr_1) \exp(i\alpha) f_{L'' J''}(k_R r_1) R_{ij}(r_1), \tag{29}
 \end{aligned}$$

in which \hat{a} denotes $2a + 1$, it is to be understood that all relevant isospin projections are those for protons and the summation extends over quantum numbers $l, j, L, L'', S, B, G, J''$ and T , with all angular momentum projection quantum numbers being determined from the set $\{v, v', v'', M_i, M_f\}$ over which incoherent sums must be performed to determine cross sections. The shift in the polar angle ($\theta_R \rightarrow \theta_R + \Delta$) in the spherical harmonic $Y_{L'' M''}$ in equation (29) is that required by the shift of the z axis from the beam direction to the \hat{X} direction, in the evaluation of the r_1 integral.

(b) *Tensor Force Component of t Matrix*

If the t matrix contains a tensor force component (superscript abbreviation 'ten') of the form (Brink and Satchler 1968)

$$t(01) = \sum_T v_T^{\text{ten}}(r_{01}) S_{01}, \tag{30}$$

in which we define

$$\begin{aligned}
 S_{01} = & \{(\boldsymbol{\sigma}_0 \cdot \mathbf{r}_{01})(\boldsymbol{\sigma}_1 \cdot \mathbf{r}_{01})/r_{01}^2 - \frac{1}{3}(\boldsymbol{\sigma}_0 \cdot \boldsymbol{\sigma}_1)\} \\
 = & (\frac{2}{3})^{\frac{1}{2}} T_2(\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_1) \cdot C_2(\Omega_{01}), \tag{31}
 \end{aligned}$$

then we may use the identity (Brink and Satchler 1968)

$$\begin{aligned} \langle \frac{1}{2} \frac{1}{2} S' M_S' | \sum_T v_T^{\text{ten}} S_{01} | \frac{1}{2} \frac{1}{2} S M_S \rangle \\ = \sum_{Tq} v_T^{\text{ten}}(r_{01}) \frac{2}{3} \sqrt{10} (-)^q \langle 12 M_S q | 1 M_S' \rangle C_{2-q}(\Omega_{01}) \delta_{SS'} \delta_{S1} \end{aligned} \quad (32)$$

to expand the two-nucleon (proton) amplitude to

$$\begin{aligned} \mathcal{M}_j^{\text{ten}} = & \sum 4\pi i^{-L''} \exp(i\alpha) \langle \frac{1}{2} \frac{1}{2} \tau y | T M_T \rangle \\ & \times \langle \frac{1}{2} \frac{1}{2} \tau' \tau'' | T M_T \rangle \langle \frac{1}{2} \frac{1}{2} \nu \mu | 1 M_S \rangle \langle \frac{1}{2} \frac{1}{2} \nu' \xi'' | 1 M_S' \rangle \\ & \times \langle l \frac{1}{2} m_l \mu | j m \rangle \langle L' \frac{1}{2} M'' + \nu'' - \xi'' \xi'' | J'' M'' + \nu'' \rangle \langle L'' \frac{1}{2} M'' \nu'' | J'' M'' + \nu'' \rangle \\ & \times Y_{L'' M''}(\theta_R, \pi) N_i N_f^* \\ & \times \frac{2}{3} \sqrt{10} (-)^q \langle 12 M_S q | 1 M_S' \rangle \\ & \times \int dr_0 \int dr_1 \exp\{i(\mathbf{K}_i - \mathbf{K}_L) \cdot \mathbf{r}_0\} \exp\{-iA^{-1} \mathbf{K}_i \cdot \mathbf{r}_1\} \\ & \times v_T^{\text{ten}}(r_{01}) C_{2-q}(\Omega_{01}) f_{L'' J''}^*(k_R r_1) R_{l_j}(r_1) Y_{lm_i}(\Omega_1) Y_{L'' M'' + \nu'' - \xi''}^*(\Omega_1). \end{aligned} \quad (33)$$

If we again introduce the (complex) transfer momenta \mathbf{Q} and \mathbf{X} (equations 27) then a simple transformation of coordinates and a partial wave expansion of the exponentials in the double integrals above reduces this transition amplitude to

$$\begin{aligned} T_{fi}^{\text{ten}} = & \sum Z A^{-\frac{1}{2}} F(j; J_i J_f T_i T_f) \left(\frac{3}{2} \pi^2\right)^{3/2} i^{L-L''} [15/\hat{L}'' \hat{J}_i \hat{T}_i]^{\frac{1}{2}} (\hat{L}/4\pi) \\ & \times \langle \frac{1}{2} \frac{1}{2} \tau y | T M_T \rangle \langle \frac{1}{2} \frac{1}{2} \tau' \tau'' | T M_T \rangle \langle l L 0 0 | L' 0 \rangle \\ & \times \langle \frac{1}{2} \frac{1}{2} \nu' \xi'' | 1 M_S \rangle \langle \frac{1}{2} \frac{1}{2} \nu \mu | 1 M_S \rangle \langle 12 M_S 0 | 1 M_S \rangle \\ & \times \langle L'' \frac{1}{2} M'' \nu'' | J'' M'' + \nu'' \rangle \langle L' \frac{1}{2} M'' + \nu'' - \xi'' \xi'' | J'' M'' + \nu'' \rangle \langle l \frac{1}{2} m_l \mu | j m \rangle \\ & \times \langle l L m_l 0 | L' M'' + \nu'' - \xi'' \rangle \langle j J_f m M_f | J_i M_i \rangle \langle \frac{1}{2} T_f y P_f | T_i P_i \rangle \\ & \times Y_{L'' M''}(\theta_R + \Delta, \pi) N_i N_f^* \\ & \times \int r^2 dr j_2(Qr) v_T^{\text{ten}}(r) \\ & \times \int r_1^2 dr_1 j_L(Xr_1) \exp(i\alpha) f_{L'' J''}^*(k_R r_1) R_{l_j}(r_1). \end{aligned} \quad (34)$$

Again \hat{a} denotes $2a+1$ here and the summation extends over quantum numbers l, j, L, L'', M_S, J'' and T , with all projection quantum numbers determined from the set $\{\nu, \nu', \nu'', M_i, M_f\}$. Contraction of the Clebsch-Gordan products to a product of $6j$ and $9j$ symbols, whilst possible, is not considered since the extra summations that would be introduced result in little if any savings in computational effort.

(c) Total Matrix Elements

The total matrix elements for a reaction are complex numbers and they are specified by equation (23) as a linear combination of all allowed two-nucleon (proton) amplitudes for each specified transition operator, for the allowed values of ν , ν' , ν'' , M_i , M_f and θ_R . The differential cross sections then are determined from

$$\frac{d^3\sigma}{d\Omega_L d\Omega_R dE_L} = \frac{1}{(2\pi)^5} \left(\frac{k_L k_R}{k_0} \right) \left(\frac{m_0}{\hbar^2} \right)^3 \frac{10}{2(2J_i+1)} \sum_{\nu\nu'\nu'', M_i M_f} |T_{fi}|^2, \quad (35)$$

where the cross sections are normalized to units of $\text{mb sr}^{-2} \text{MeV}^{-1}$.

In the analyses reported here the transition operator will be limited to the central and tensor forms, the functional character of which is described in the following section, together with a detailed description of the optical model and bound state wavefunctions and the spectroscopic amplitudes that we can deduce from a variety of models of nuclear structure.

5. Calculation Details

(a) Spectroscopic Amplitudes

The spectroscopic amplitudes as defined by equation (21) satisfy the sum rule (Bohr and Mottelson 1975)

$$\sum_{J_f T_f} |F|^2 \equiv (2J_i+1)(2T_i+1)n(j), \quad (36)$$

where $n(j)$ is the nucleon occupancy of the j shell. In the case of the ^{12}C ground state, $J_i = T_i = 0$ and, if the knockout of a nucleon populates a unique state in the final $(A-1)$ nucleus, then the direct scattering (single-step) transition strength for that single-nucleon knockout (or pickup) is

$$|F(j; 0j 0\frac{1}{2})| = \{n(j)\}^{\frac{1}{2}}, \quad (37)$$

in which case the ground state density distributions totally determine the weighting due to structure in the reaction amplitudes.

Usually, however, more than one residual nuclear state share the population distribution of given nucleons in the target, and some spectroscopic model of the residual nucleus must be defined before one can evaluate the spectroscopic amplitudes. Furthermore, it is usual to specify the particular nucleus involved and therefore the particular type of nucleon, proton or neutron, that is removed from the target. Thus, using (p, 2p) on ^{12}C as our example, we must make a model for the spectroscopy of ^{11}B , from which the proton spectroscopic amplitudes

$$\begin{aligned} S(j; J_i J_f) &\equiv \langle \Psi_{J_f T_f P_f}^{(A-1)} \| b_{j\frac{1}{2}-\alpha}^+ \| \Psi_{J_i T_i P_i}^{(A)} \rangle \\ &\equiv (2T_f+1)^{-\frac{1}{2}} \langle T_i \frac{1}{2} P_i - \alpha | T_f P_f \rangle F(j; J_i J_f T_i T_f) \\ &\equiv 2^{-\frac{1}{2}} \delta_{P_f-\alpha} F(j; J_i J_f T_i T_f) \end{aligned} \quad (38)$$

satisfy a proton sum rule (for $N = Z$ nuclei), namely

$$\sum_{J_f} |S(j; 0J_f)|^2 \equiv \frac{1}{2} n(j). \quad (39)$$

A set of such (proton) occupancies in the ^{12}C ground state is presented in Table 2; the values are to be compared with the j - j coupling, spherical shell model limits of 2, 0 and 4 for the $0s_{1/2}$, $0p_{1/2}$ and $0p_{3/2}$ shells respectively. The results in columns 2, 3 and 4 of Table 2, labelled PHFB, PHFV' (Boekker 1968) and PHFBA (Bassichis *et al.* 1967), were obtained from large basis (three major shells) projected Hartree-Fock studies that predicted a large oblate deformation in ^{12}C . The calculations by Boekker ignored the spin-orbit splitting of the $p_{1/2}$ and $p_{3/2}$ shells, but this was included in the calculations of Bassichis *et al.* Also given in Table 2, column 5 (MCHF), are the ground state occupancies obtained if a multiconfiguration Hartree-Fock analysis (Faessler and Schmid 1971) is made. The results obtained from an $SU(3)$ representation of the ^{12}C ground state are given in column 6. Here we used the single oblate (0, 4) $SU(3)$ representation of the ^{12}C g.s.:

$$|^{12}\text{C g.s.}\rangle = |[f](\lambda\mu); LSJT\rangle = |[444](04); 0000\rangle. \quad (40)$$

This simple model (L - S limit) implies that the shell occupancies vary as the number of states available, that is,

$$n(\frac{3}{2})/n(\frac{1}{2}) = 2/1,$$

and it gives maximum deformation in ^{12}C .

Table 2. Calculated ground state (proton) occupancies in ^{12}C

(1) Shell nl_j	(2) PHFB	(3) Occupancies PHFV'	(4) $\frac{1}{2}n(j)$ from different models ^A PHFBA	(5) MCHF	(6) $SU(3)$	(7) SM
$0s_{1/2}$	1.97	1.86	1.94	1.99	2.00	2.00
$0p_{1/2}$	1.19	1.30	0.96	1.02	1.33	0.74
$0p_{3/2}$	2.38	2.60	2.99	2.98	2.67	3.27
$1s_{1/2}$	0.02	0.02	0.01	—	—	—
$0d_{3/2}$	0.01	0.07	0.02	—	—	—
$0d_{5/2}$	0.00	0.05	0.03	—	—	—
$1p_{1/2}$	0.01	0.01	0.00	—	—	—
$1p_{3/2}$	0.02	0.02	0.01	—	—	—
$0f_{5/2}$	0.17	0.03	0.01	—	—	—
$0f_{7/2}$	0.23	0.04	0.03	—	—	—

^A See the text for a description of the models used.

The last column in Table 2 (SM) gives the ^{12}C ground state occupancies that were obtained from a standard shell model calculation in which the Cohen and Kurath (1965) 8-162BME matrix elements were used.

The $SU(3)$ results stress collective deformation, as is evident by their close agreement with the large basis PHFB and PHFV' calculations. The PHFBA, MCHF and SM results, on the other hand, are much more aligned with the j - j coupling shell model limit. Hence, on this basis alone, relative strengths of the (p, 2p) (or pickup) cross sections will reflect deformation of the target and the relative importance of the spin-orbit splitting of the p shells. Specifically, the PHFB, PHFV' and $SU(3)$ models for the structure result in a 2 to 1 weighting of the (total) $3/2^-$ transitions over the

$1/2^-$ transitions (L - S limit) whilst the PHFBA, MCHF and SM descriptions yield weightings of 3.1 to 1, 3 to 1 and 4.4 to 1 respectively giving a tendency to the j - j limit.

The foregoing has been predicated upon the assumption of a single state in ^{11}B being populated by particle knockout and upon the sum rule (39). In addition to the $3/2^-$ ground state there is also a $3/2^-$ state at 5.02 MeV excitation in ^{11}B , and the use of a structure model of ^{11}B is therefore necessary to ascertain just how the $0p_{3/2}$ proton knockout strength will be divided between these two final states. In the $SU(3)$ scheme, this is a simple matter since only components in negative parity ^{11}B states, belonging to the [443] space symmetry and the (13) $SU(3)$ representation, can be reached directly by proton knockout.

Thus states in ^{11}B characterized by

$$|LJ\rangle = |[443](13); L\frac{1}{2}\frac{3}{2}\frac{1}{2}\rangle, \quad (41)$$

having values of 1 or 2 for L , are possible, with $L = 1$ components being accessible by p-shell knockout. In the simplest case therefore we have

$$|^{11}\text{B g.s.}\rangle = \alpha |1\frac{3}{2}\rangle + (1-\alpha^2)^{\frac{1}{2}} |2\frac{3}{2}\rangle, \quad |^{11}\text{B } 5.02\rangle = (1-\alpha^2)^{\frac{1}{2}} |1\frac{3}{2}\rangle - \alpha |2\frac{3}{2}\rangle. \quad (42)$$

Consequently, on the basis of this spectroscopy alone, the (p, 2p) transition data from ^{12}C should scale in ratios of $2.67\alpha^2$ and 1.33 and $2.67(1-\alpha^2)$ for the ^{11}B $3/2^-$ ground state and the $1/2^-$ (2.13 MeV) and $3/2^-$ (5.02 MeV) excitations respectively. By contrast the simple shell model (p-shell) results are given in Table 3 for three different matrix element sets as defined by Cohen and Kurath (1965) and from a calculation by Amit and Katz (1964). These exhaust the sum rule limits.

Table 3. Shell-model spectroscopic factors

Simple shell model results with different sets of matrix elements are listed for the spectroscopic factor $|S(j; 0J_f)|^2$ for the reaction $^{12}\text{C}(0^+) \rightarrow ^{11}\text{B}(J_f^\pi)$

Matrix element set	Reference ^A	J_f^π for ^{11}B state		
		$3/2^-$ (g.s.)	$1/2^-$ (2.13 MeV)	$3/2^-$ (5.02 MeV)
6-162BME	1	2.770	0.732	0.458
8-162BME	1	2.811	0.727	0.433
8-16POT	1	2.843	0.753	0.378
AK	2	3.180	0.184	0.444

^A From: 1, Cohen and Kurath (1965); 2, Amit and Katz (1964).

(b) Wavefunctions

Both the energy-symmetric and energy-asymmetric (p, 2p) results that are presented in Section 6 below were obtained by using the half DWA theory described previously, and with an incident projectile energy of 98.7 MeV for which the 100 MeV APW function, as detailed in Section 3, was used. In the energy-asymmetric mode analyses, the high energy (59.5 MeV) exit proton wavefunction was chosen to be the 65 MeV APW function that was also defined in Section 3. For simplicity, the same wavefunction was used to represent the left exit proton in our analysis of the energy-symmetric mode (p, 2p) reaction on ^{12}C ; albeit that at the emergent energy (42 MeV) distortion is probably much more severe than such a simple model suggests.

Nevertheless, this was adequate for our purpose of comparing the properties of model two-nucleon t -matrices.

In the energy-asymmetric mode, $^{12}\text{C}(p, 2p)$ data have been taken, initiated by 98.7 MeV protons and with the left emergent proton having an energy of 59.5 MeV at a scattering angle of 25° (Devins *et al.* 1979). Kinematics then determine that the other reaction proton (right and coplanar) emerges with an energy that varies from 20 to 24 MeV according to its scattering angle. For this low energy proton we have used a distorted wavefunction in our calculations, and the relevant optical model parameters (Karban *et al.* 1969) for all energies in the small energy range were assumed to be the same and are identified as the 23 MeV set in Table 4. For the energy-symmetric case, as we sought an equal angle correlation, the (41.35 MeV) right proton distorted wave was obtained using the optical model parameter values identified as the 42 MeV set in Table 4.

For most of the calculations that we have made using this half DWA theory, harmonic oscillator wavefunctions ($\hbar\omega = 17.9$ MeV) were used to represent the initial state, bound proton. In one calculation, however, a Woods–Saxon potential, the parameter values for which are given in Table 4, was used to obtain a wavefunction appropriate for a $0p_{3/2}$ proton bound in ^{12}C by 13 MeV.

Table 4. Optical model potential parameters

Parameter set	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W_d (MeV)	W_0 (MeV)	r_d (fm)	a_d (fm)	V_{so} (MeV)	r_{so} (fm)	a_{so} (fm)	r_c (fm)
23 MeV	45.18	1.09	0.59	3.38	—	1.30	1.01	7.79	0.98	0.57	1.29
42 MeV	41.50	1.15	0.66	5.01	—	1.10	0.52	7.80	1.00	0.60	1.25
Bound	58.18	1.25	0.7	—	—	—	—	30.0	1.25	0.70	1.25

(c) Model t Matrices

To complete the specifications of the calculations, only details of our model two-nucleon t matrices remain to be given. We have used three specific model t matrices, all of which are local in nature. The first is the pseudopotential of Lim and McCarthy (1964, 1966), hereafter designated as the LM force, and which consists of a linear combination of three Yukawa functions. In our analyses, however, we permit the LM force to act equally in the singlet and triplet channels and, as we have also neglected exchange effects due to antisymmetrization, an exact comparison of our symmetric-energy mode LM force results with those previously published (Lim and McCarthy 1966) is not possible. The other two model t matrices are designated as the WW and EH forces, the former being the local gaussian functions of Wong and Wong (1967) that model the even state internucleon interaction and the latter being the two-nucleon tensor interaction of Eikemeier and Hackenbroich (1971). The combination of WW and EH forces is a model t matrix that has been used with success in analyses of inelastic proton scattering data; albeit at proton energies usually below 60 MeV (see e.g. Geramb *et al.* 1975, and references therein; Amos *et al.* 1976, 1978).

The preceding model t matrices (pseudopotentials) have been chosen for simplicity and for the ensuing reduction in calculational complexity from that we would have faced with a more realistic choice, such as one of the nonlocal t matrices which pay special attention to off-shell behaviour (e.g. Redish *et al.* 1970; Birrell *et al.* 1976).

Further, as our primary interest is with the energy-asymmetric mode experiments, the off-shell effects should be less problematic than they have been assessed to be for the energy-symmetric mode studies in which the 'on-shell' and 'off-shell' momenta are large and at right angles to each other (Redish *et al.* 1970). With increasing energy discrepancy between the emergent protons and decreasing emission angle of the 'high' energy proton, the reaction becomes increasingly on-shell and hence it is of primary concern in our analyses that the t matrix have appropriate on-shell characteristics.

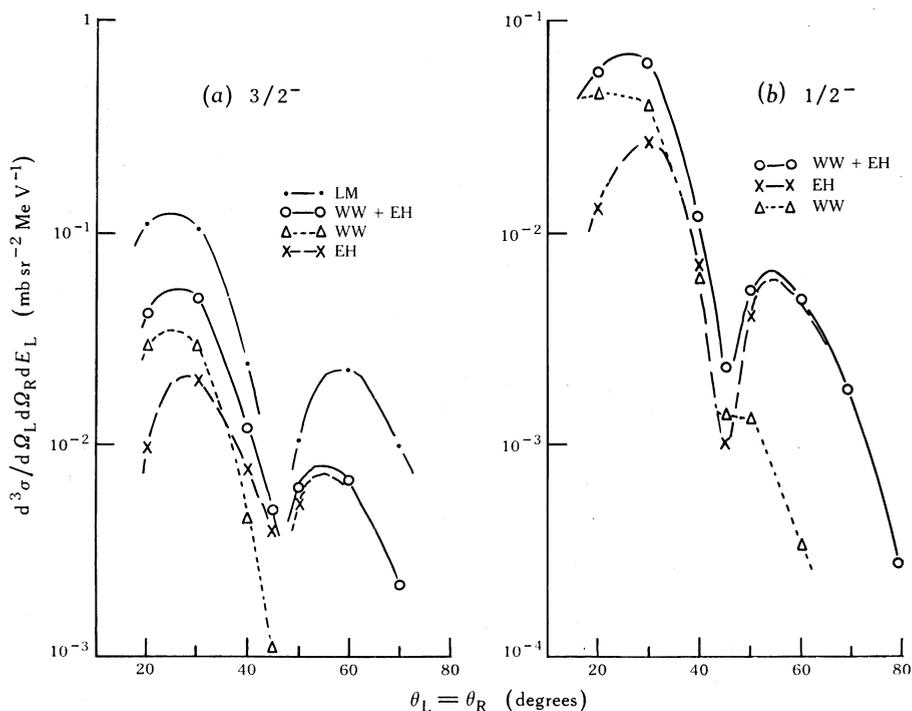


Fig. 4. Symmetric-energy mode predictions for (a) $3/2^-$ ($0p_{3/2}$) and (b) $1/2^-$ ($0p_{1/2}$) transitions in the reaction $^{12}\text{C}(p, 2p)$. The predictions are shown for the LM, WW and EH forces specified in Section 5c. The incident proton energy was 98.7 MeV and both exit proton energies were taken as 42 MeV.

6. Results

In this, our first study of asymmetric mode (p, 2p) data, we consider the reactions from ^{12}C leading to the $3/2^-$ (ground), $1/2^-$ (2.13 MeV) and $3/2^-$ (5.02 MeV) states in ^{11}B specifically, and we compare our analyses with the data initiated by 98.7 MeV protons (Devins *et al.* 1979). This projectile energy is lower than desirable for application of the half DWA formalism, as is the asymmetry in energy sharing of the emergent protons. Nevertheless, it is modern data of the appropriate type and useful as a first test of the method of analysis proposed herein.

We consider firstly the energy-symmetric mode correlations, and the predictions obtained using the half DWA method with the various model forces are presented in Fig. 4, for the $3/2^-$ (ground) and $1/2^-$ (2.13 MeV) transitions. None of the predicted

correlations adequately match the experimentally observed structure (see Devins *et al.* 1979). Specifically, there is a mismatch in the peak height ratios, a failure that has been explained as due to the incorrect off-shell characteristics of forces of the type we have used (Redish *et al.* 1970; Birrell *et al.* 1976). The overall magnitude of the WW + EH force results has been obtained using the 6-162BME spectroscopic factors (see Table 3) and it underestimates the data magnitude, particularly at back angles, but this may be due as much to the neglect of antisymmetrization as to the off-shell effects of the full t matrix, both of which effects have also been neglected in the distorted wave impulse approximation (DWIA) analyses of Devins *et al.* (1979).

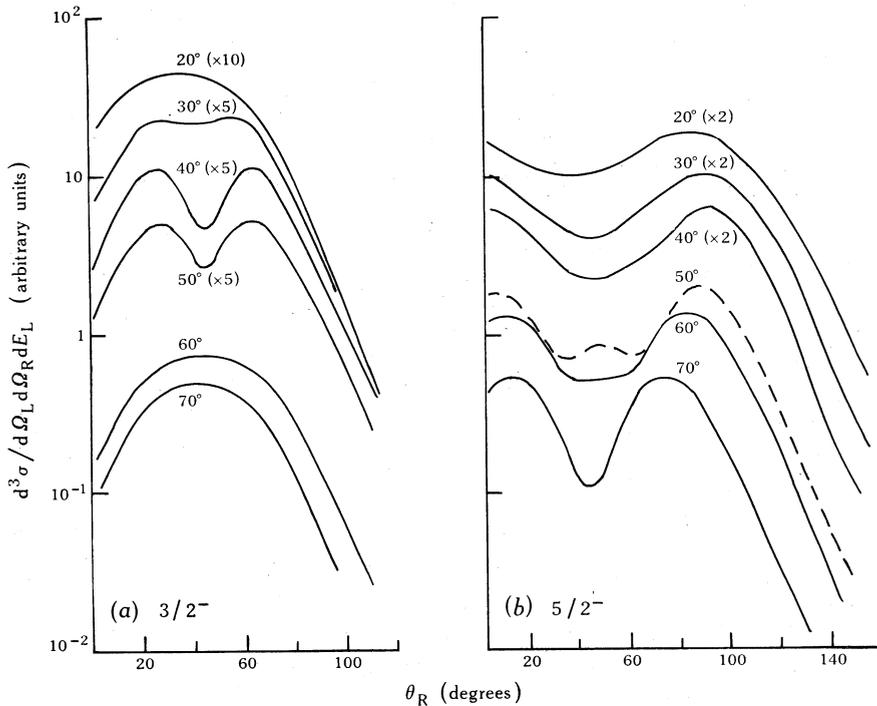


Fig. 5. Symmetric-energy mode predictions as functions of the right exit proton emission angle θ_R for the (a) $3/2^-$ ($0p_{3/2}$) and (b) $5/2^-$ ($0f_{5/2}$) transitions in $^{12}\text{C}(p, 2p)$. The values on the curves of the left exit proton emission angle and the relative scaling factor identify each result.

Thus we do not attempt to ascertain spectroscopic information from the symmetric mode analyses. Nevertheless, it is evident that the tensor force (EH results in Fig. 4), owing to its strong short-range character, greatly influences predictions, particularly at large momentum transfers (i.e. at the second peak in the p-particle knockout correlation). Such is not the case at small angles since the order-2 Bessel transform of the tensor force form is involved (see equation 34).

In the recent analysis by Devins *et al.* (1979) of $^{12}\text{C}(p, 2p)$ data initiated by 98.7 MeV protons, conclusions were drawn concerning the $l = 1$ versus $l = 3$ nature of the knockout reaction mechanism. Those conclusions were based upon calculation comparisons with four or five data points in each distribution, taken over a scattering region for which the structure of the calculated $l = 1$ and $l = 3$ distributions differed only slightly. A rigorous assignment of $l = 1$ or 3 knockout mechanisms and, in

particular, a rejection of the possibility of some $l = 3$ component based upon such structure arguments is therefore doubtful. To study this problem, we have evaluated the correlation functions for both the $0p_{3/2}$ and $0f_{5/2}$ particle knockouts from ^{12}C in the energy-symmetric mode, the results of which are shown in Fig. 5. With the fixed values for the left emergent proton given in the figure and scalings to spread the results as shown in parentheses, quite distinctive correlations for $l = 1$ and 3 knockout mechanisms result, particularly at θ_R scattering angles of greater than 80° .

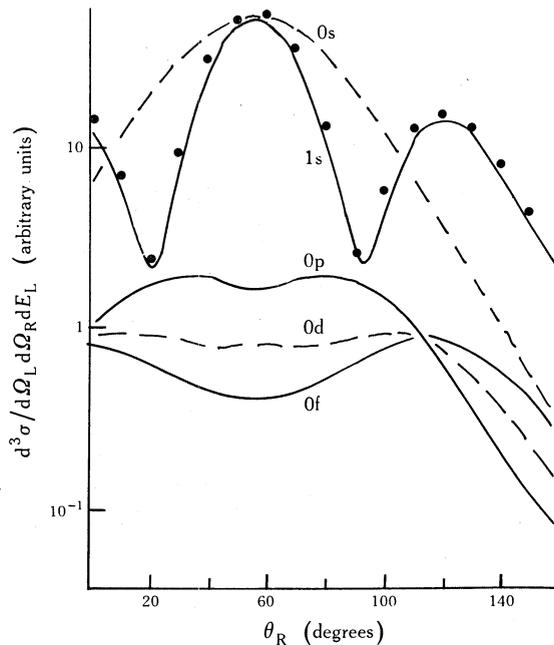


Fig. 6. Asymmetric-energy mode results (arbitrary normalizations) for proton knockout from the indicated orbitals in the reaction $^{12}\text{C}(p, 2p)$, with an incident proton energy of 98.7 MeV and a left exit proton emission angle of 25° . The curves are the results of calculations using the WW+EH forces, while the solid circles show, for comparison, the results for a $1s$ proton knockout found using the LM force.

If these calculations are at least reasonable, there should be no confusion between a p and f shell particle knockout, and it is regrettable that data taken to date have been limited to equal angle geometry or to so few scattering angle values that the distinction between the $l = 1$ and 3 knockout mechanism structures has not been clearly seen.

However, the half DWA method is designed for analyses of asymmetric data, and the results of our calculations for the kinematic conditions set by the data of Devins *et al.* (1979) are shown in Figs 6 and 7.

In Fig. 6 we present the correlations for proton knockout from various single-particle orbits (as shown in the diagram). The normalizations are arbitrary as no specific final state in ^{11}B has been specified. The WW+EH force results are shown by the curve for each transition, with the LM force results for $1s$ knockout being shown by the solid circles for comparison. In this case the WW+EH results concur

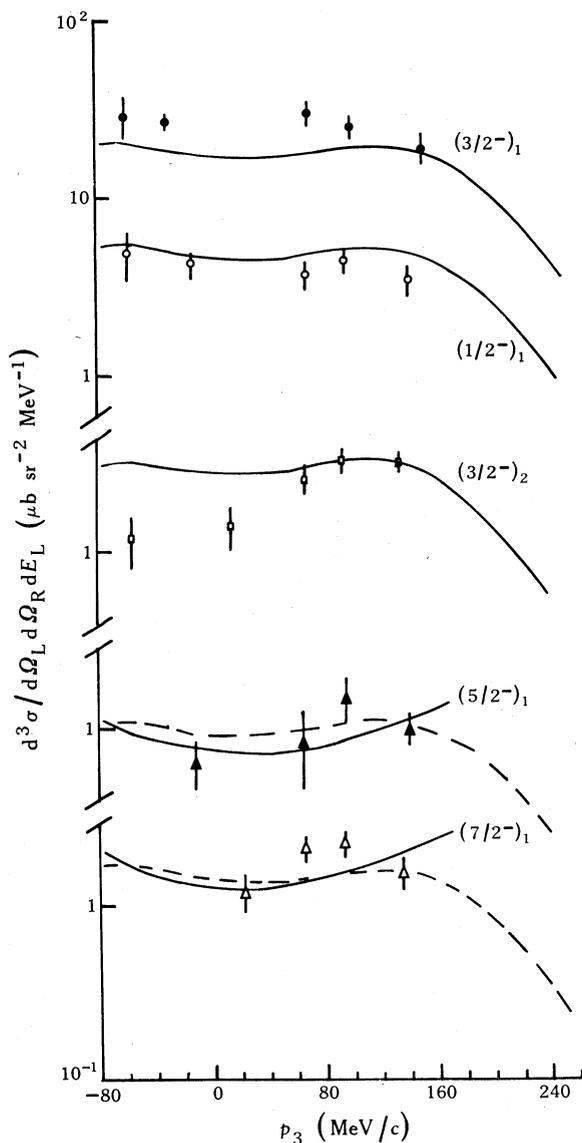


Fig. 7. Asymmetric-energy mode data (from Devins *et al.* 1979) compared with curves showing the present half distorted-wave predictions for proton knockout in the reaction $^{12}\text{C}(p, 2p)$, with an incident proton energy of 98.7 MeV and a left exit proton emission angle of 25° . The predictions for the $(3/2^-)_1$, $(1/2^-)_1$ and $(3/2^-)_2$ transitions have been normalized using $0p$ shell spectroscopy. The predictions for the $(5/2^-)_1$ and $(7/2^-)_1$ transitions are unnormalized, with $l = 1$ (dashed curves) and $l = 3$ (full curves) knockout being compared with the data.

with those found using the LM force since the roles of the tensor and off-shell attributes of the forces are less important than in the energy-symmetric mode calculations. The relatively small angle (25°) and high energy (60 MeV) of the (left) emergent proton make the t -matrix momenta tend to their on-shell values (Birrell *et al.* 1976). Thus, the distinctive structure seen in the calculated correlations of the 0s and 1s particle knockout should provide a data signature, as may the large-angle trends of the p, d and f particle knockout reactions.

In Fig. 7 the p-shell knockout data of Devins *et al.* (1979) are compared with our predictions obtained using the shell model spectroscopy of the 6–162BME type, the spectroscopic factors of which are given in Table 3. This spectroscopy is far removed from the pure spherical shell model limit in which the $1/2^-$ state in ^{11}B would not be populated, save by higher order processes. If the large basis PHFB, PHFV' or $SU(3)$ spectroscopies (see Table 2) are used, the relative population of the $1/2^-$ state as compared with the summed strengths of the two $3/2^-$ transitions is overestimated. Analyses of inelastic scattering form factors in ^{12}C (Amos and Morrison 1979) have shown that, whereas L - S coupled Hartree-Fock type states (PHFB, PHFV') can adequately reproduce the electron scattering longitudinal form factors, they fail completely in the case of the transverse form factors. The PHFBA state, which includes the spin-orbit splitting of the 0p shells, can adequately reproduce both form factors with no free parameters. Within the 0p shell, the Cohen and Kurath (1965) interaction can reproduce most features of the inelastic scattering data, but the role of the $p_{3/2}$ - $p_{1/2}$ splitting is obscured by the incorrect radial dependence of the wavefunctions, requiring the use of effective charges.

The (p, 2p) correlations, at least their magnitudes, are primarily dependent upon the particle occupancy in the ground state of the target and hence give a much clearer picture of the role of deformation and spin-orbit splitting. The excellent agreement with (p, 2p) correlation magnitudes shown in Fig. 7 using spectroscopies including spin-orbit suppression of the $0p_{1/2}$ occupancy indicates that the types of ground state correlations given by the shell model for ^{12}C are more appropriate than those of the single determinant PHFB, PHFV' or $SU(3)$ models. The tendency of the PHFBA and MCHF values in Table 2 to move from the PHFB and PHFV' values towards those of the shell model testifies also to that conclusion.

The sensitivity of the calculation to the radial form of the bound state wavefunction used was examined by comparing the $p_{3/2}$ proton knockout predictions of Fig. 7 (obtained using harmonic oscillator wavefunctions) with those obtained using the Woods-Saxon potential of Table 4. The change in the predicted correlation, from that shown in Fig. 7, was insufficient to warrant showing it separately on that graph.

Finally, in Fig. 7 the (unnormalized) f-shell knockout correlations (full curves) are compared with those for p-shell knockout (dashed curves) and with the data for the excitation of the $5/2^-$ (4.445 MeV) and $7/2^-$ (6.743 MeV) states of ^{11}B . It is evident again, that the data do not extend to the regions at which a distinction between the $l = 1$ and 3 knockout shapes can be made.

7. Conclusions

A half distorted-wave approximation formalism, appropriate for use in analyses of energy-asymmetric (p, 2p) data initiated by protons in the energy range 100–200 MeV and with a sizable disparity in the exit proton energies, has been developed and applied

to analyses of the recent Indiana data of Devins *et al.* (1979). In so doing we have used for simplicity t matrices (pseudopotentials) that facilitate computation. Their off-shell characteristics are inappropriate, as the energy-symmetric results demonstrate, but, since the half DWA was designed to analyse energy-asymmetric data for which conditions can be set that stress near to on-shell properties of the t matrices, the effective interaction prescriptions, as ascertained by (p, p') analyses, should be appropriate and $(p, 2p)$ correlation data should then reflect properties of nuclear structure.

From our analyses of the ^{12}C data leading to the 'p-shell knockout' states in ^{11}B ($3/2^-$ (ground), $1/2^-$ (2.13 MeV) and $3/2^-$ (5.02 MeV)), it can be concluded that the particle occupancies as predicted by the shell model with the 6-162BME fitted matrix elements are most appropriate. It remains to be seen whether this conclusion is borne out with refinement of the structure models for both ^{12}C and ^{11}B and with refinement of the reaction mechanisms to include off-shell effects of the t matrices and higher order processes such as post or prior inelastic scattering. Information regarding the latter may be provided by data from knockout to the $5/2^-$ and $7/2^-$ states, but first one must ascertain the upper limits to direct feeding of these states by f-particle knockout. The current data are inconclusive about this since they cannot be clearly assigned an $l = 1$ (e.g. p-particle knockout from the 2_1^+ state in ^{12}C) or an $l = 3$ (f-particle knockout from the ground state of ^{12}C) structure, and information concerning the appropriate mixture of these distributions is not obtained.

Finally, it still remains to be seen whether the virtual excitations of giant resonances will influence data in the asymmetric-mode experiment as they do in other $(p, 2p)$ geometries and in other reactions, since the current data do not span the appropriate excitation energy region. More appropriate conditions to see such effects should be the use of 150 MeV incident protons and the detection of 120 MeV protons at an angle of 30° to the beam which, for the ^{11}B ground state transition, would yield intermediate excitation energies of ~ 26 – 27 MeV, in which the effects of the E1 and possibly E2 resonance strength in ^{12}C might be evident.

References

- Amit, D., and Katz, A. (1964). *Nucl. Phys.* **58**, 388.
 Amos, K. A. (1966). *Nucl. Phys.* **77**, 225.
 Amos, K. A., Faessler, A., Morrison, I., Smith, R., and Muether, H. (1978). *Nucl. Phys. A* **304**, 191.
 Amos, K. A., and Morrison, I. (1979). *Phys. Rev. C* **19**, 2108.
 Amos, K., Nesci, P., and Smith, R. (1976). *Aust. J. Phys.* **29**, 233.
 Austern, N. (1961). *Ann. Phys. (New York)* **15**, 299.
 Bassichis, W. H., Kerman, A. L., and Svenne, J. P. (1967). *Phys. Rev.* **160**, 746.
 Birrell, N. D., McCarthy, I. E., and Noble, C. J. (1976). *Nucl. Phys. A* **271**, 469.
 Boekker, E. (1968). *Nucl. Phys. A* **119**, 435.
 Bohr, A., and Mottelson, B. R. (1975). 'Nuclear Structure' (Benjamin: New York).
 Brassard, C., Shay, H. D., Coffin, J. P., Scholz, W., and Bromley, D. A. (1972). *Phys. Rev. C* **6**, 53.
 Brink, D. M., and Satchler, G. R. (1968). 'Angular Momentum' (Clarendon: Oxford).
 Cohen, S., and Kurath, D. (1965). *Nucl. Phys.* **73**, 1.
 Deutchman, P. A., and Old, J. G. (1977). *Nucl. Phys. A* **283**, 289.
 Devins, D. W., *et al.* (1979). *Aust. J. Phys.* **32**, 323.
 Eikemeier, H., and Hackenbroich, H. H. (1971). *Nucl. Phys. A* **169**, 407.
 Faessler, A., and Schmid, K. W. (1971). *Nucl. Phys. A* **174**, 26.
 Geramb, H. V., *et al.* (1975). *Phys. Rev. C* **12**, 1697.
 Geramb, H. V., and Eppel, D. (1973). *Z. Phys.* **261**, 177.
 Geramb, H. V., and Strobel, G. L. (1972). *Phys. Lett. B* **39**, 611.
 Grimm, R. C., McCarthy, I. E., and Storer, R. G. (1971). *Nucl. Phys. A* **166**, 330.

- Haybron, R. M. (1969). *Nucl. Phys. A* **124**, 662.
- Hosono, K., *et al.* (1978). *Phys. Rev. Lett.* **41**, 621.
- Janus, R. T., and McCarthy, I. E. (1974). *Phys. Rev. C* **10**, 1041.
- Karban, O., Lowe, J., Greaves, P. D., and Hnizdo, V. (1969). *Nucl. Phys. A* **133**, 255.
- Kromminga, A. J., and McCarthy, I. E. (1961). *Nucl. Phys.* **24**, 36.
- Lim, K. L., and McCarthy, I. E. (1964). *Phys. Rev. Lett.* **13**, 446.
- Lim, K. L., and McCarthy, I. E. (1966). *Nucl. Phys.* **88**, 457.
- Love, W. G., and Satchler, G. R. (1967). *Nucl. Phys. A* **101**, 424.
- McCarthy, I. E. (1959). *Nucl. Phys.* **10**, 583.
- McCarthy, I. E., and Pursey, D. L. (1961). *Phys. Rev.* **122**, 578.
- Pugh, H. G., Hendrie, D. L., Chabre, M., Boschitz, E., and McCarthy, I. E. (1967). *Phys. Rev.* **155**, 1054.
- Redish, E. F., Stephenson, G. J., and Lerner, M. L. (1970). *Phys. Rev. C* **2**, 1665.
- Riou, M., and Ruhla, Ch. (1970). 'Progress in Nuclear Physics', Vol. 11 (Eds D. M. Brink and J. H. Mulvey), p. 195 (Pergamon: Oxford).
- Ritchie, K. E., and Wright, B. T. (1967). *Phys. Rev.* **159**, 839.
- Satchler, G. R. (1974). *Phys. Rep. C* **14**, 99.
- Shanta, R., and Jain, B. K. (1971). *Nucl. Phys. A* **175**, 417.
- Welch, L. C., Chang, C. C., Forster, H. H., Kim, C. C., Devins, D. W., and Deutchman, P. A. (1970). *Nucl. Phys. A* **158**, 644.
- Wong, C. W., and Wong, C. Y. (1967). *Nucl. Phys. A* **91**, 433.
- Wright, P. C., Storer, R. G., and McCarthy, I. E. (1977). *Phys. Rev. C* **17**, 473.
- Yamaguchi, A., Terasawa, T., Nakahara, K., and Torizuka, Y. (1971). *Phys. Rev. C* **3**, 1750.

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