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Mode Coupling in the Solar Corona. VI* Direct Conversion of Langmuir Waves into o-mode Waves

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Abstract

Fundamental plasma radiation can result from direct coupling of Langmuir waves into o-mode waves in an inhomogeneous magnetized plasma. The coupling occurs for Langmuir waves propagating nearly along the direction of increasing plasma density, and the resulting o-mode waves propagate towards lower plasma densities. An analytic theory for the coupling (which is of z-mode waves into o-mode waves) is developed using approximate solutions of the Booker quartic equation. The results are shown to agree with the results of numerical calculations for the 'Ellis window' in the generation of the z trace in the terrestrial ionosphere. This 'direct conversion' of Langmuir waves into o-mode waves is an alternative mechanism for fundamental plasma emission, possibly replacing or supplementing the more familiar scattering from the charge clouds around thermal ions. Possible applications of 'direct conversion' to various solar radio bursts are explored, but for none is it found to be more favourable than alternative mechanisms.

1. Introduction

Coupling between z-mode waves and o-mode waves in an inhomogeneous plasma is of interest in three connections: (i) It is the accepted mechanism for the production of the z trace in ionospheric sounding (Rydbeck 1950; Ellis 1956; Budden 1961, p. 424). (ii) It is a possible mechanism for fundamental plasma emission (i.e. emission at the fundamental of the local plasma frequency) from the solar corona (Field 1956; Ginzburg and Zheleznyakov 1958, 1959; Denisse 1960; Wild *et al.* 1963; Kundu 1965, p. 58; Ginzburg 1970, p. 357; Zheleznyakov 1970, p. 385). (iii) It has been suggested as the mechanism for Jupiter's decametric radio emission and for certain emissions in the terrestrial magnetosphere (Oya 1974; Benson 1975; Jones 1976, 1977). Existing quantitative treatments of the coupling for ionospheric applications involve cumbersome numerical calculations (Budden and Terry 1971; Smith 1973; Budden and Smith 1974). In the application to fundamental plasma emission, simplifying assumptions have been made, e.g. Field (1956) assumed a sharp density gradient and Ginzburg and Zheleznyakov (1959) assumed 'vertical incidence' onto a smoothly varying stratified medium. The general case (within the framework of geometric optics or mode-coupling theory) of 'oblique incidence' onto a stratified medium is discussed in the present paper. My purpose is to find an approximate analytic expression for the coupling coefficient from the z mode to the o mode, and to apply it to fundamental plasma emission.

The coupling of z-mode waves into o-mode waves in an inhomogeneous plasma occurs due to tunnelling across a stop band. The stop band is a spatial region, in

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which the waves are evanescent, and which separates a region where the z mode can propagate from a region where the o mode can propagate. Viewing the wave properties as a function of ω/ω_p, one sees that the stop band corresponds to a range of ω/ω_p separating the z- and o-mode refractive index curves at fixed refractive index (i.e. fixed wavenumber). From this viewpoint, the coupling modes are separated in frequency at fixed wavenumber. (Of course it is ω_p and not ω which changes across the stop band.) The present case of mode coupling differs from that considered in earlier papers in this series (cf. Parts I and II, Melrose 1974*a*, 1974*b*) where the coupling waves are separated in wavenumber at fixed frequency. In the present case, coupling occurs due to a gradient in the plasma frequency (i.e. owing to a density gradient), and the stop band is across the plasma level (where the frequency equals the plasma frequency). The approach adopted in the present paper is analogous to that adopted in Part II, in that the coupling is treated by using approximate solutions of the Booker quartic equation in the general case of oblique incidence. On the one hand, this approach is qualitatively different from the phase-integral method used by most authors (e.g. Budden 1961, p. 239; Zheleznyakov 1970, pp. 353 and 385; Golant and Piliya 1972) in which vertical incidence is usually assumed. (The results of the present method are compared in Section 4 with those obtained by the phase-integral method.) On the other hand, the nature of the solutions of the Booker quartic equation (Section 2) and the manner in which they are used to treat the coupling (Section 3) are quite different from those of Part II. In the application to fundamental plasma emission, the Langmuir waves must propagate along ray paths which cause them to encounter the coupling region. This question of 'accessibility' to the coupling point is discussed in Section 5.

The terms 'Langmuir' waves and 'z-mode' waves are used here in the following sense: 'Langmuir' waves have a phase speed significantly less than the speed of light, and in their dispersion relation the effect of the finite thermal speed of electrons is more important than the effect of the ambient magnetic field. For 'z-mode' waves the refractive index is of order unity, and thermal corrections to the magnetoionic dispersion relation are negligible. It is assumed that these two modes are limiting cases of a single mode (e.g. Melrose 1976) and that they can transform into each other through refraction changing the magnitude of the wavenumber.

Fundamental Plasma Emission

Before proceeding it is appropriate to discuss the reasons for reconsidering 'direct conversion' of Langmuir waves into o-mode waves. There are two basic reasons:

The first reason is that the efficiency of the widely accepted mechanism for fundamental plasma emission (scattering by thermal ions) was overestimated by Ginzburg and Zheleznyakov (1959) and has been overestimated by many subsequent authors. It has been pointed out by Smith and Riddle (1975), Smith (1976) and Melrose (1977) that scattering by thermal ions is an inefficient process, and that one does not seem able to account for fundamental plasma emission in terms of it. A semiquantitative argument leading to this conclusion is as follows. For scattering by thermal particles (ions or electrons, depending on the circumstances) the cross section is roughly the Thomson cross section σ_T. The efficiency of conversion Q for this scattering process is given roughly by the product of the number density of scatterers ($n_i \approx n_e$), the cross section σ_T and the distance L over which escaping radiation at a given frequency can be produced. The distance L must be less than $(\delta\omega/\omega_p)L_N$, where

$\delta\omega$ is the intrinsic bandwidth of the emission, ω_p is the plasma frequency and L_N is the characteristic length for changes in the plasma density. The mechanism has an intrinsically narrow bandwidth,

$$\delta\omega/\omega_p \lesssim (m_e/m_i)^{\frac{1}{2}} \approx 1/43 .$$

For $n_e = 10^8\ \mathrm{cm}^{-3}$ one finds $Q \approx 10^{-8}$ for $L_N = 10^5$ km. As pointed out by Melrose (1977), the difficulty implied by the low efficiency of conversion due to scattering by thermal ions is so formidable that alternative mechanisms for fundamental plasma emission should be reconsidered. These arguments apply only to 'spontaneous' scattering by thermal ions, and it could be argued that induced scattering (e.g. Kaplan and Tsytovich 1967), parametric versions of this scattering (e.g. Papadopoulos *et al.* 1974; Smith and de la Noë 1976) and collapse into solitons (e.g. Bardwell and Goldman 1976) are much more efficient than 'spontaneous' scattering. This is true, but each of these processes becomes important only when a threshold energy density in Langmuir waves is exceeded. For type III bursts, Melrose (1977) argued: (i) that reabsorption of the energy in Langmuir waves could not be effective and (ii) that, for the energy losses by the stream of electrons to be consistent with the observed lack of slowing down, the energy deposited in Langmuir waves would have to be less than the threshold value for induced scattering. Similar arguments apply to the parametric conversion mechanisms and to soliton collapse. Thus an energy density in Langmuir waves which is sufficiently low to avoid unacceptable energy losses by the stream leads to a power radiated at the fundamental that is much less than is observed, while an energy density in Langmuir waves which is high enough to give sufficient power at the fundamental implies excessive energy losses. To avoid this dilemma one requires a relatively high efficiency of conversion of Langmuir waves into transverse waves at relatively low energy density in the Langmuir waves. A mechanism other than scattering by thermal ions is a possible way out of the dilemma. 'Direct conversion' is such a conceivable alternative.

The second reason for reconsidering direct conversion is that the estimate by Ginzburg and Zheleznyakov (1959) for the average efficiency of direct conversion, specifically $Q_{av} \approx 3\times10^{-9}$, holds for a smoothly varying corona with $L_N \approx 10^5$ km. There is now much evidence that the solar corona might be inhomogeneous on a fine scale. The evidence is indirect and includes the following: (i) Coronal scattering requires local density inhomogeneities with scale sizes of a few hundred kilometres (Steinberg *et al.* 1971; Riddle 1972, 1974). (ii) The depolarization of some solar radio bursts seems to require small-scale ($\lesssim 100$ km) irregularities (Melrose 1975). (iii) Various irregular structures have been invoked in connection with the split-band structure in some type II bursts (McLean 1967; Smerd *et al.* 1974), with type IIIb bursts (Takakura and Yousef 1975), with the directivity of type I bursts (Bougeret and Steinberg 1977) and with the relative positions of fundamental and second-harmonic type II and type III bursts (Duncan 1979). Suppose, e.g. that one has $L_N \lesssim 100$ km rather than $L_N \approx 10^5$ km. The estimate by Ginzburg and Zheleznyakov (1959) gives $Q_{av} \propto L_N^{-1}$, which would then imply $Q_{av} \gtrsim 3\times10^{-6}$. This value of Q_{av} is larger than their overestimated value of Q for scattering by thermal ions.

Despite the above-given rather strong reasons for reconsidering direct conversion as an alternative for fundamental plasma emission, the application to various solar radio bursts (Section 6) remains unclear. There is no clear example where the mechanism is obviously the most favourable.

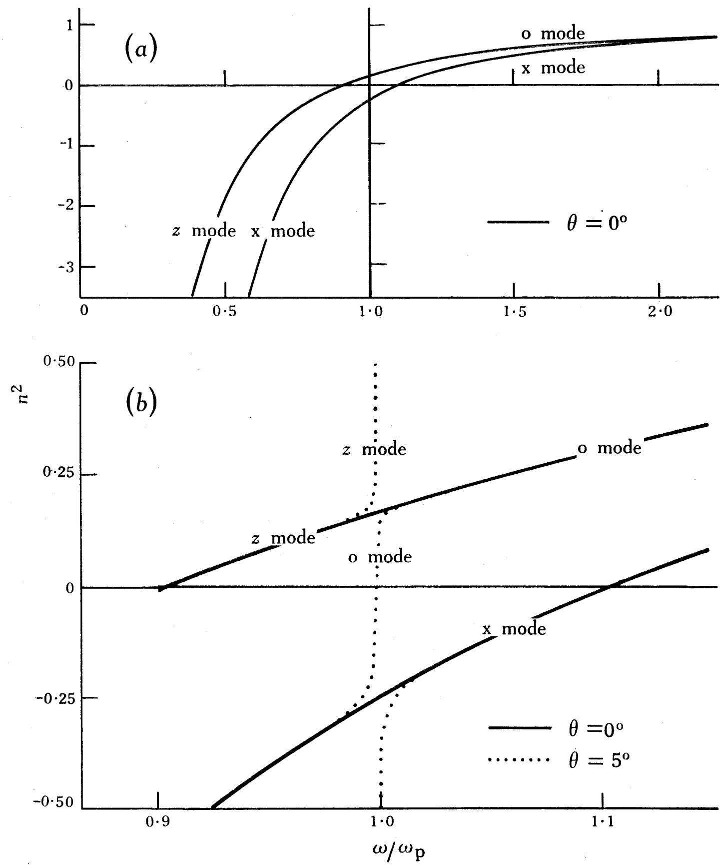


Fig. 1. Variation of the squared refractive index n^2 for the magnetoionic modes as a function of frequency ω near the plasma frequency ω_p, plotted for $\Omega_e/\omega_p = 0{\cdot}2$ and the indicated values of θ. The region near $\omega = \omega_p$ in (*a*) is shown on an expanded scale in (*b*). The upper curve is the z mode for $\omega < \omega_p$ and the o mode for $\omega > \omega_p$.

2. Wave Properties in Coupling Region

(*a*) *Coupling Point*

The refractive index curves for the z mode and the o mode are illustrated in Figs 1*a* and 1*b* for $\theta = 0$ and for small but nonzero θ respectively, where θ is the angle between the wave vector $\boldsymbol{k}$ and the ambient magnetic induction $\boldsymbol{B}$. The coupling point is that point where the z-mode and o-mode curves touch, namely, $\omega = \omega_p$ and $\sin\theta = 0$. At the coupling point, one has

$$n^2 = n_0^2 := Y_0/(1+Y_0), \qquad Y_0 := \Omega_e/\omega_p. \tag{1}$$

(The symbol $:=$ denotes a definition of the quantity on the left.) For $\sin\theta$ slightly different from zero, the refractive index curves do not join, as shown in Fig. 1*b*. There is a gap in frequency across $\omega = \omega_p$ with n^2 nearly equal to n_0^2 on either side. Coupling across this gap allows conversion of energy in z-mode waves into energy in escaping o-mode waves.

The point $\omega = \omega_p$ and $\sin\theta = 0$ is a singular point in the magnetoionic theory. This is evident from the magnetoionic dispersion equation in the form

$$(1-X)\left(n^2 - \frac{1-X+Y}{1+Y}\right)\left(n^2 - \frac{1-X-Y}{1-Y}\right) + \left(\frac{XY^2}{1-Y^2}\right)(1-n^2)n^2 \sin^2\theta = 0\,, \qquad (2)$$

where

$$X := \omega_p^2/\omega^2 \qquad \text{and} \qquad Y := \Omega_e/\omega \qquad (3)$$

are the magnetoionic parameters. In the limit $\sin\theta = 0$ and $\omega = \omega_p$ (that is, $X = 1$) one has $n^2 = n_0^2$ (see equation 1) only if the limit $\sin\theta \to 0$ is taken first. If the limit $X \to 1$ is taken first, equation (2) implies $n^2 = 1$ for the z mode and $n^2 = 0$ for the o mode. This singular nature of the point $\omega = \omega_p$ and $\sin\theta = 0$ is associated with the polarization. One can show that all waves at $\sin\theta = 0$ have circular polarization. On the other hand, one can show that all waves at $\omega = \omega_p$ have linear polarization. If one approaches $\omega = \omega_p$ and $\sin\theta = 0$ along a path where $(\sin^2\theta)/(\omega-\omega_p)$ remains small then the o mode and z mode tend to the same circular polarization and to the same value $n^2 = n_0^2$. However, if $(\sin^2\theta)/(\omega-\omega_p)$ is much greater in magnitude than unity along the path then the z mode tends to $n^2 = 1$ and the o mode tends to $n^2 = 0$, and their polarizations become orthogonal and linear at $\omega = \omega_p$.

In the discussion above, the effect of thermal motions on the wave properties is ignored. It is shown in the Appendix that thermal motions are not important.

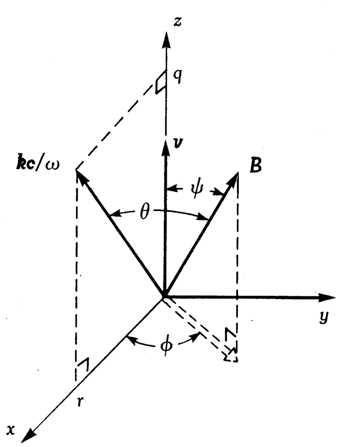


Fig. 2. Illustration of the directions and angles involved in the Booker quartic equation.

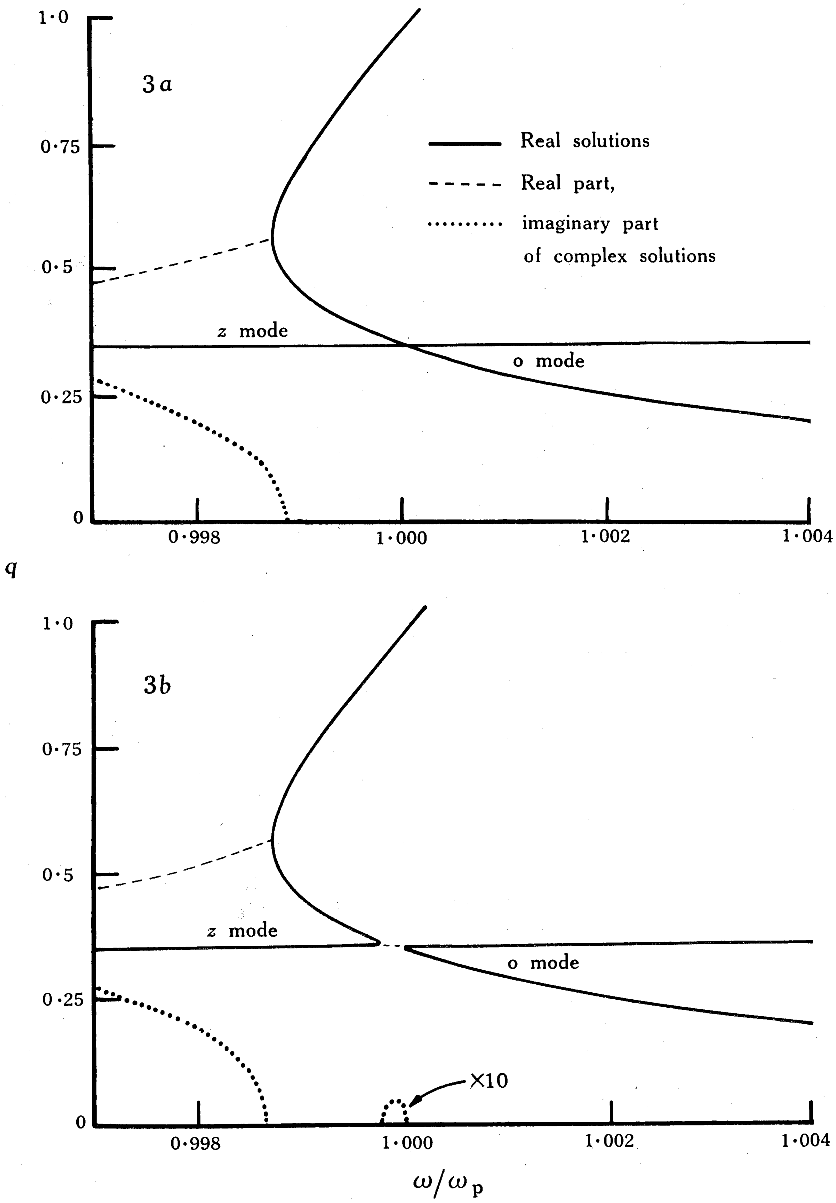
(*b*) *Booker Quartic Equation*

In order to consider mode coupling between two of the magnetoionic modes, it is convenient to rewrite equation (2) as the Booker quartic equation (Booker 1936; Budden 1961, p. 122; Parts I and II). In the present case the only relevant gradient is in the plasma frequency ω_p, and the unit vector $\mathbf{v}$ is parallel to grad ω_p. The angles used here are defined in Fig. 2. The other variables in the Booker quartic equation are the independent variable

$$r := c/\omega\,|\boldsymbol{k} \times \mathbf{v}|\,, \qquad (4)$$

whose constancy is implied by Snell's law, and the dependent variable

$$q := c/\omega\,\boldsymbol{k}\,.\,\mathbf{v}\,. \qquad (5)$$



Figs 3*a* and 3*b*. Real and complex solutions q as a function of ω/ω_p for the Booker quartic equation, plotted for $\Omega_e/\omega_p = 0{\cdot}2$, $\psi = 30^\circ$ and $\phi = 0$: (*a*) For $r = r_0$ the coupling point is where the two real solutions cross. (*b*) For $r = 0{\cdot}99\,r_0$ the solutions do not overlap, but two oppositely directed tongues (in the z mode and the o mode) are separated by only a small frequency range.

The Booker quartic equation then follows from equation (2) by writing

$$n^2 = q^2 + r^2 \tag{6a}$$

and

$$n\cos\theta = q\cos\psi + r\sin\psi\cos\phi\,. \tag{6b}$$

Solutions of the Booker quartic equation are presented in Fig. 3. In Fig. 3*a* the parameters are chosen such that the point $\sin\theta = 0$ and $\omega = \omega_p$ lies on the curves. This requires $\phi = 0$ and

$$r = r_0 := \{Y_0/(1+Y_0)\}^{\frac{1}{2}} \sin\psi\,. \tag{7a}$$

Then at $\omega = \omega_p$ the four solutions of the quartic equation include a double solution at

$$q = q_0 := \{Y_0/(1+Y_0)\}^{\frac{1}{2}} \cos\psi\,. \tag{7b}$$

In Fig. 3*b*, the value of r has been chosen slightly different from r_0 (still with $\phi = 0$). The point $q = q_0$ is approached from either side of $\omega = \omega_p$ but there is now a region across $\omega = \omega_p$ where the two relevant solutions are complex. As $|r - r_0|$ (or ϕ) is further increased the two 'tongues' recede further from each other leaving a wider gap across $\omega = \omega_p$ where the solutions are complex. In Fig. 3*c*, which is on a larger scale, the value of $|r - r_0|$ has been chosen such that the tongue in the z mode has disappeared. The solid curves in Figs 3*a* and 3*b* are indistinguishable from those in Fig. 3*c* except near the point $\omega = \omega_p$, $q = q_0$.

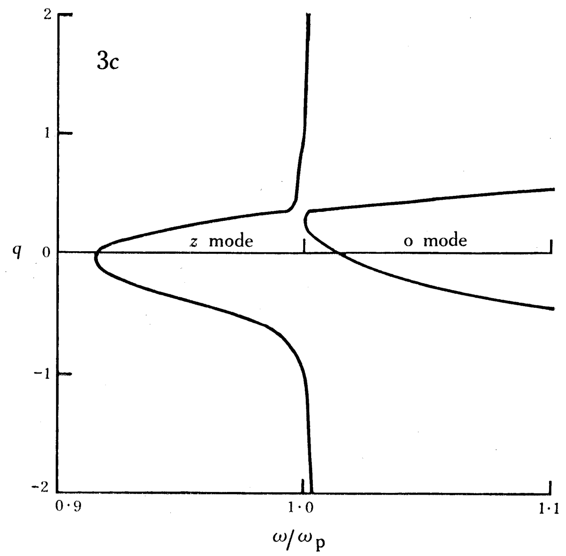


Fig. 3*c*. Solutions q as a function of ω/ω_p for the Booker quartic equation, plotted for $\Omega_e/\omega_p = 0{\cdot}2$, $\psi = 30°$ and $\phi = 0$. For $r = 0{\cdot}9\,r_0$ here the right-pointing tongue in the z mode (evident in Fig. 3*b*) has disappeared. On the same scale as Fig. 3*c*, the solutions shown in Figs 3*a* and 3*b* appear similar to that shown here.

(*c*) *Quadratic Approximation*

In order to treat the coupling between the z and o mode semiquantitatively, one requires approximate solutions for q for the two modes in the neighbourhood of the coupling point. It is clear from Figs 3*a* and 3*b* that the two solutions may be approximated by the solutions of a quadratic equation for sufficiently small values of $r - r_0$, $\omega - \omega_p$ and ϕ. Consequently, it is appropriate to find a suitable quadratic approximation.

Let us write

$$\varepsilon := (\omega - \omega_p)/\omega_p \tag{8}$$

and

$$x := (q - q_0)\sin\psi - (r - r_0)\cos\psi . \tag{9}$$

One has $\varepsilon = 0$, $\phi = 0$, $r - r_0 = 0$ and $x = 0$ at the coupling point. For small values of these parameters the quadratic approximation to equation (2) is

$$x^2 + \frac{8\varepsilon}{r_0} x\cos\psi + r_0^2\phi^2 + 8\varepsilon\frac{r - r_0}{r_0} - \frac{4\varepsilon^2(2 + Y_0)}{Y_0(1 + Y_0)} = 0 . \tag{10}$$

The solutions of equation (10) are

$$x = -\frac{4\varepsilon\cos\psi}{r_0} \pm \left(\frac{16\varepsilon^2\cos^2\psi}{r_0^2} + \frac{4\varepsilon^2(2 + Y_0)}{Y_0(1 + Y_0)} - r_0^2\phi^2 - 8\varepsilon\frac{r - r_0}{r_0}\right)^{\frac{1}{2}} . \tag{11}$$

It follows that there are complex solutions for

$$r^2\phi^2 + 8\varepsilon\frac{r - r_0}{r_0} > \frac{16\varepsilon^2\cos^2\psi}{r_0^2} + \frac{4\varepsilon^2(2 + Y_0)}{Y_0(1 + Y_0)} . \tag{12}$$

Dr K. G. Budden (personal communication) has derived a quadratic approximation similar to equation (10). He chose to use the variable Y itself, rather than Y_0 as used in equation (10). This leads to simplifications in subsequent formulae; however, the differences between the approaches are small. Dr Budden has also compared the results obtained using the quadratic approximation with results for the transmission of z-mode into o-mode waves obtained using a version of the phase-integral method. He obtained excellent agreement.

(*d*) '*Upper*' *Reflection Point*

For our purposes, it is desirable to have an approximate equation of wider validity than equation (10). In particular, it is desirable to have a semiquantitative treatment of the region above and to the left of the coupling point in Fig. 3*b*. In this region the curve for the z mode has an extremum as a function of $\omega - \omega_p$. This is a reflection point, which will be referred to as the 'upper reflection point' in the z mode to distinguish it from the reflection point at the tip of the 'tongue' in the z mode (Fig. 3*b*). It is relevant to know under what condition both reflection points occur. One could determine this condition by looking for the double solutions of the quartic equation. However, this leads to cumbersome results which are not useful for semiquantitative purposes.

A major simplification follows by noting that the upper reflection point is insensitive to small changes in ϕ and $r - r_0$. As ϕ or $|r - r_0|$ are increased, the lower reflection point in the z mode (the tip of the tongue) recedes and the upper reflection point hardly moves at all. Consequently, one may estimate the condition under which the reflection points merge as follows: (i) find the position of the upper reflection point for $\phi = 0$ and $r = r_0$ and, in particular, find the value of $\varepsilon := (\omega - \omega_p)/\omega_p$ at which it occurs; (ii) use the solutions of the quadratic inequality (12) to find ε as a function of ϕ and $r - r_0$; (iii) identify the merging of the two solutions with the equality of the two resulting expressions for ε. The validity of the procedure may be justified by comparison of the results obtained from it with the results of numerical solutions of the full quartic equation.

For $\phi = 0$ and $r = r_0$ the quartic equation reduces (to first order in ε) to

$$2\varepsilon(q^2-q_0^2)\left(q^2-q_0^2+\frac{2Y_0}{1-Y_0^2}\right)+\frac{Y_0^2}{1-Y_0^2}\left(\frac{1}{1+Y_0}-(q^2-q_0^2)\right)(q-q_0)^2\sin^2\psi = 0\,. \tag{13}$$

One of the solutions of equation (13) is $q = q_0$; this corresponds to the nearly horizontal line in Fig. 3*a*. After factoring out this solution, a cubic equation remains. One of the solutions of the cubic equation for $\omega \approx \omega_p$ is at negative q (see Fig. 3*c*) and is of no interest. The remaining two solutions for $\varepsilon = 0$ are $q = q_0$ and $q = (1-r_0^2)^{\frac{1}{2}}$. Approximating the relevant curve in Fig. 3*a* by the solution of a quadratic equation which passes through these two points, one finds that the reflection point would occur midway between them, i.e. at

$$q = q_1 := \tfrac{1}{2}\{q_0+(1-r_0^2)^{\frac{1}{2}}\}\,. \tag{14}$$

The value of ε at the reflection point may then be estimated by inserting equation (14) in (13). One finds

$$\varepsilon = -\frac{Y_0^2\sin^2\psi}{2(1-Y_0^2)}\,\frac{(q-q_0)(1-q^2-r_0^2)}{(q+q_0)\{q^2-q_0^2+2Y_0/(1-Y_0^2)\}}\,. \tag{15}$$

The parameters in Fig. 3*a* correspond to $q_0 = 0\cdot35$ and $(1-r_0^2)^{\frac{1}{2}} = 0\cdot98$, and then the definition (14) implies $q_1 = 0\cdot67$ and (13) implies $\varepsilon = 1\cdot1\times10^{-3}$. This value of ε is in satisfactory agreement with the exact result from Fig. 3*a*. In the following section, equation (15) is further approximated by

$$\varepsilon = -Y_0^2\sin^2\psi\,, \tag{16}$$

which overestimates ε by a factor of about 2 for the parameters chosen in Fig. 3*a*, and underestimates ε by a factor of 2 in the limit of small Y_0.

3. Coupling

(*a*) *Attenuation Factor*

The coupling may be treated semiquantitatively by noting that the waves decay spatially through the region where q is complex. This spatial decay may be described in terms of an attenuation factor A for the wave energy, that is,

$$A = \exp\left(-2(\omega/c)\int_{z_1}^{z_2}\mathrm{d}z\,|\operatorname{Im}q|\right), \tag{17}$$

where z denotes distance normal to the strata. Let $|\operatorname{Im}q|_{\max}$ be the maximum value of $\operatorname{Im}q$, and let $\Delta\omega$ be the separation in frequency of the tips of the two tongues. Let the characteristic length L_N for changes in the plasma density be defined by

$$|\operatorname{grad}\omega_p| := \omega_p/2L_N\,. \tag{18}$$

The factor A may be approximated by

$$A = \exp\{-(2\omega/c)\zeta\,|z_2-z_1|\,|\mathrm{Im}\,q|_{\max}\} \tag{19a}$$

$$\approx \exp\{-(4\zeta L_N\,\Delta\omega/c)\,|\mathrm{Im}\,q|_{\max}\}, \tag{19b}$$

where ζ is a number, less than but of order unity, which depends on the density profile.

(b) Estimates of $\Delta\omega$ *and* $|\mathrm{Im}\,q|_{\max}$

We may use the quadratic equation (10) and its solutions (11) to estimate the values of $\Delta\omega$ and $|\mathrm{Im}\,q|_{\max}$. Let us consider $\Delta\omega$ first. For $\phi \neq 0$ and $r-r_0 = 0$, the complex solutions are centred on $\omega = \omega_p$ and extend over a total range

$$\Delta\omega = (\tfrac{1}{2}r_0^2\,|\phi|/\cos\psi)\,\omega_p \qquad \text{for} \qquad \phi \neq 0, \qquad r-r_0 = 0. \tag{20a}$$

For $\phi = 0$ and $r-r_0 \neq 0$, the complex solutions are at $\omega < \omega_p$ for $r-r_0 < 0$, as in Fig. 3*b*, and at $\omega > \omega_p$ for $r-r_0 > 0$. The complex solutions extend over a range

$$\Delta\omega = (\tfrac{1}{2}\,|r-r_0|r_0/\cos^2\psi)\,\omega_p \quad \text{for} \quad \phi = 0, \qquad r-r_0 \neq 0. \tag{20b}$$

The maximum value of $|\mathrm{Im}\,q|$ also follows from equation (11). For $\phi \neq 0$ and $r-r_0 = 0$, one has

$$|\mathrm{Im}\,q|_{\max} = (r_0\,|\phi|/\sin\psi) \qquad \text{for} \qquad \phi \neq 0, \qquad r-r_0 = 0. \tag{21a}$$

The maximum value for $\phi = 0$ and $r-r_0 \neq 0$ depends on ε. If one sets ε equal to $\Delta\omega/\omega_p$, with $\Delta\omega$ given by half its value in equation (20b), i.e. the value midway between the two tongues, one finds

$$|\mathrm{Im}\,q|_{\max} = |r-r_0|/\sin\psi\cos\psi \qquad \text{for} \qquad \phi = 0, \qquad r-r_0 \neq 0. \tag{21b}$$

(c) Comparison with Numerical Results

We can now write down a semiquantitative condition for most of the energy in z-mode waves to tunnel through into o-mode waves. There remains an undetermined factor ζ in the approximation (19b) which depends on the details of the density profile. Rather than carry out a calculation for a specific profile to estimate ζ, it is convenient to compare the results of the present approximate analytic calculations with the numerical calculations of Smith (1973).

Smith (1973) considered coupling of the o mode into the z mode in connection with the generation of the z trace in the ionosphere. The parameters chosen were $Y_0 = 0{\cdot}5$, $\sin\psi = 0{\cdot}5$, $L_N = 5$ km, $\phi = 0$ and, by implication, $\omega_p = 1{\cdot}6\times10^7\ \mathrm{s}^{-1}$. The results imply a full width of the 'Ellis window' of $5{\cdot}8°$ for attenuation by a factor $A = \frac{1}{2}$. (The Ellis window is defined somewhat loosely as the angular width of the cone for which the z trace is observed in ionospheric sounding; the term is used loosely here to refer to the angular region in which z-mode–o-mode coupling is effective.) In the ionospheric case, the angle of incidence θ_I is related to r_0 by $r_0 = \sin\theta_I$, and a range of halfwidth of $2{\cdot}9°$ in θ_I corresponds to $|r-r_0|/r_0 = 0{\cdot}168$ for the adopted parameters.

Setting $A = \frac{1}{2}$ and inserting equations (20b) and (21b) into the approximation (19b) gives

$$\frac{|r-r_0|}{r_0} = \left(\frac{c\cos^3\psi\ln 2}{2\zeta\omega_p L_N \sin^2\psi}\right)^{\frac{1}{2}} \left(\frac{1+Y_0}{Y_0}\right)^{\frac{3}{4}}. \tag{22}$$

Inserting Smith's (1973) parameters, one identifies

$$\zeta = 0{\cdot}44 \tag{23}$$

in this case. As expected, ζ is less than but of order unity. Furthermore, the result (23) implies a satisfactory agreement between the present analytic approach and Smith's (1973) numerical approach. The result corresponding to equation (22) for $\phi = 0$ and $r-r_0 \neq 0$ is

$$\phi = \left(\frac{c\cos\psi\ln 2}{2\zeta\omega_p L_N \sin^2\psi}\right)^{\frac{1}{2}} \left(\frac{1+Y_0}{Y_0}\right)^{\frac{3}{4}}, \tag{24}$$

where equations (20a) and (21a) have been inserted into the approximation (19b) with $A = \frac{1}{2}$.

It is convenient to combine equations (22) and (24) in the form (cf. Part II)

$$\begin{aligned} \theta &= [(\phi\sin\psi)^2 + \{(r-r_0)/r_0\}^2\tan^2\psi]^{\frac{1}{2}} \\ &= \left(\frac{c\cos\psi\ln 2}{\zeta\omega_p L_N}\right)^{\frac{1}{2}} \left(\frac{1+Y_0}{Y_0}\right)^{\frac{3}{4}}. \end{aligned} \tag{25}$$

The angle θ determined by equation (25) is the effective width of the Ellis window about the direction of the magnetic field. Thus z-mode waves which approach the layer $\omega = \omega_p$ with θ less than this value couple into o-mode waves effectively, while those with larger θ are reflected and propagate towards increasing plasma densities as z-mode waves.

4. Nearly Vertical Incidence

The present method breaks down for vertical incidence ($r = 0$). In this case the Booker quartic equation reduces to equation (2) with n and θ replaced by q and ψ respectively. This equation has two real solutions for q^2, and these are just the solutions illustrated in Figs 1*a* and 1*b* for small $\theta = \psi$. Thus in this case there is no tongue in the z mode. The complex solutions which bridge the frequency gap between the z and the o modes cannot then be described using equation (10).

The case of vertical incidence was discussed by Ginzburg and Zheleznyakov (1959) based on a formula derived using the phase-integral method (cf. Zheleznyakov 1970, p. 385). The essential point in the method is to regard the coupling point as lying at a complex point in coordinate space. Thus the distinction between the method used in the present paper and the phase-integral method is that in the present method the attenuation across the stop band is attributed to the spatial decay of a mode with a complex $\boldsymbol{k}$, while in the phase-integral method it is the coordinate which is complex for real $\boldsymbol{k}$. The two methods must be related by analytic continuation.

The coupling is effective only for small θ (cf. equation 25), i.e. only when $\boldsymbol{k}$ and $\boldsymbol{B}$ are nearly parallel near the coupling point. ('Parallel' includes antiparallel in the present discussion.) Thus for nearly vertical incidence the three vectors $\boldsymbol{k}$, $\boldsymbol{B}$ and $\mathrm{grad}\, n_e$ must be nearly parallel for the coupling to be effective. (Budden and Smith (1973) have discussed the case where the three directions are nearly parallel in connection with ionospheric sounding near the magnetic poles.) Although the case of $\boldsymbol{k}$ parallel to $\mathrm{grad}\, n_e$ cannot be treated using the present method, the case of $\boldsymbol{B}$ parallel to $\mathrm{grad}\, n_e$ can be. This special case is of interest for two reasons: First, one expects it to lead to essentially the same result as one obtains for vertical incidence (treated using the phase-integral method). A detailed calculation not given here shows this to be the case. Thus, this special case may be used to establish the equivalence of the two methods, at least to a high degree of plausibility. The second reason for the special case to be of interest is that the tongue in the z mode disappears in this limit, i.e. for $\sin\psi = 0$, as implied by equations (15) or (16); any coupling for $\varepsilon \gtrsim Y_0^2 \sin^2\psi$ can be regarded as of the same form as the coupling for $\sin\psi = 0$. Thus this special case also provides an extension of the results obtained in Section 3 to larger angles where the tongue in the z mode is absent.

The result quoted by Zheleznyakov (1970, equation 25.34) for the coupling in the case of vertical incidence is equivalent to the result (25), with $\cos\psi = 1$ in this case, and with the numerical factor ζ replaced by $\frac{1}{4}\pi$. It may be concluded that equation (25) includes the special case where the three vectors $\boldsymbol{k}$, $\boldsymbol{B}$ and $\mathrm{grad}\, n_e$ are nearly parallel. Conversely, the generalization from vertical to oblique incidence turns out to be almost trivial: to within a factor of order unity the result for the size of the Ellis window is unchanged on generalizing from vertical to oblique incidence.

It seems surprising that the results obtained here for oblique incidence are virtually identical with those obtained for vertical incidence using the phase-integral method. The present method is valid only when there is a tongue in the z mode. Only then are the evanescent modes in the gap between the z mode and the o mode an obvious extension of one mode to the other. The tongue is absent for vertical incidence, and one might expect the coupling found for vertical incidence to be qualitatively different from that found here. Clearly this is not the case. It must be that the two methods are more closely related than it seems at first sight. Presumably the link is analytic continuation.

The present method breaks down when the value of $\varepsilon = \Delta\omega/\omega_p$ implied by equation (20a) or (20b) exceeds the value given in equation (16) for which the tongue in the z mode ceases to exist. This restriction is equivalent to the requirement that θ^2/ε be less than unity, and only then do the z and o modes have similar properties. For $Y_0 \ll 1$ the parameters at which one has $\theta^2 \approx \varepsilon$ correspond to

$$L_N^{-1} \approx \{(\omega_p/c)\sin^2\psi\cos\psi\}Y_0^{7/2}, \tag{26}$$

which is referred to here as the 'maximum effective gradient'. For larger gradients, waves over a wider range of θ can tunnel through the stop band. However, this is offset by the fact that the polarizations of the modes for $\theta^2 \gg \varepsilon$ are nearly orthogonal, and hence the coupling between them must be weak. Once the value defined by the approximation (26) is reached, one does not expect the efficiency of conversion to increase significantly with increasing density gradient.

5. Accessibility to Coupling Point

(*a*) *Efficiency of Conversion*

Let us assume that the initial Langmuir waves have phase speeds v and that their wave vectors are confined to a range of solid angles $\Delta\Omega$. The only waves converted completely into o-mode waves have $r = r_0$ and $\phi = 0$. For wave vectors in a small range about the direction defined by $r = r_0$ and $\phi = 0$, a partial conversion occurs. Let $\geqslant 50\%$ conversion correspond to a range $\Delta\Omega_c$. Conversion occurs only if the range $\Delta\Omega_c$ is contained in the range $\Delta\Omega$. To within a factor of order unity, the average efficiency of conversion may be identified as the ratio of the range of solid angles for which effective coupling occurs to the total range of solid angles filled by the waves. Let us define a generalization of the Ginzburg–Zheleznyakov Q_{av} by

$$Q_{av} = \Delta\Omega_c/\Delta\Omega. \tag{27}$$

Langmuir waves with $r = r_0$ are propagating initially at an angle $\rho = \rho_0$, with

$$\rho_0 = r_0\, v/c, \tag{28}$$

to the direction grad ω_p, and a given range of $|r - r_0|$ corresponds to a range

$$\Delta\rho = |r - r_0|\, v/c \tag{29}$$

about ρ_0. The waves in a range $\Delta\rho$ about ρ_0 and $\pm\phi$ about $\phi = 0$ fill a solid angle

$$\Delta\Omega_c = 2\rho_0\,\Delta\rho\,\phi = 2r_0\,|r - r_0|\,(v/c)^2\phi. \tag{30}$$

Inserting equation (30) in (29) and equating $|r - r_0|$ and ϕ to the values given by equations (22) and (24) gives

$$Q_{av} = \frac{1\cdot 1\, v^2\cos^2\psi}{c\omega_p L_N\,\Delta\Omega}\left(\frac{1+Y_0}{Y_0}\right)^{\frac{1}{2}}, \tag{31}$$

where the value (23) for ζ has been inserted. The result (31) remains valid for vertical incidence ($\sin\psi = 0$). Note from equation (28) that Langmuir waves with small phase speeds ($v \ll c$) must be propagating very nearly along the direction grad ω_p for the coupling to be effective, i.e. they must have $\rho \approx \rho_0 \ll 1$.

(*b*) *Numerical Values*

Let us now insert numbers in equation (31) to estimate the efficiency of conversion. As illustrative values, let us take $v/c = 3$, $\cos^2\psi = \frac{1}{2}$, $\omega_p = 2\pi\times 10^8\ \mathrm{s}^{-1}$, $\Delta\Omega = 4\pi$ and $Y_0 := \Omega_e/\omega_p = 0\cdot 1$. Then equation (31) gives $Q_{av} \approx 10^2/L_N$, with L_N in centimetres. For the average density gradient in the solar corona one has $L_N \approx 10^{10}$ cm, and hence $Q_{av} \approx 10^{-8}$, in reasonable agreement with the estimate made by Ginzburg and Zheleznyakov (1959). For small-scale inhomogeneities with $L_N \approx 10$ km (Melrose 1975) one would have $Q_{av} \approx 10^{-4}$. The maximum effective gradient (cf. approximation 26) corresponds to $L_N \approx 3$ km with these parameters. The efficiency of conversion could be higher by an order of magnitude or two either for higher initial phase speeds ($v \approx c$) or for highly collimated Langmuir waves ($\Delta\Omega \approx 0\cdot 1$). There is observational evidence in support of highly collimated Langmuir waves in some type III bursts (Melrose *et al.* 1978).

6. Discussion and Conclusions

The results of the foregoing discussion are summarized as follows:

(i) Direct coupling of Langmuir waves into escaping o-mode waves occurs for Langmuir waves propagating initially in a small cone of angles about the direction of increasing plasma density.

(ii) The efficiency of conversion Q may be identified as the ratio of the range of solid angles $\Delta\Omega_c$ for which the coupling occurs to the total solid angle $\Delta\Omega$ filled by the Langmuir waves. An explicit expression for Q_{av} is given by equation (31).

(iii) The coupling discussed here is not effective for $\text{grad}\, n_e$ nearly normal to $\boldsymbol{B}$ (that is, for $\cos\psi \approx 0$).

(iv) The efficiency of conversion increases with increased gradient in the plasma density ($\propto L_N^{-1}$) up to a maximum effective gradient determined by the approximation (26).

(v) The analytic results obtained here are consistent with the result of numerical calculations for parameters appropriate to the generation of the z trace in the terrestrial ionosphere (Smith 1973).

(vi) Viewed as a plasma emission mechanism, direct coupling produces (*a*) only fundamental emission which is (*b*) polarized 100% in the sense of the o mode and (*c*) has an intrinsic bandwidth which is effectively zero. The bandwidth in practice would be determined by the range of plasma frequencies from which emission arises. Direct conversion into the x mode involves tunnelling across a stop band of width $\Delta\omega \approx \frac{1}{2}\Omega_e$ and hence requires $L_N \lesssim c/\Omega_e \approx 10^{-2}$ km for the above-used parameters.

(vii) The efficiency of conversion for $\Omega_e/\omega_p = 0 \cdot 1$, $\omega_p/2\pi = 100$ MHz and $\sin^2\psi = \frac{1}{2}$, for isotropic Langmuir waves with phase speed $v = \frac{1}{3}c$ is $Q_{av} \approx (10^3\, L_N)^{-1}$, with L_N in kilometres. The maximum effective gradient corresponds to L_N slightly less than 10 km. Arguments given by Melrose (1977) suggest that $Q_{av} \gtrsim 10^{-4}$ is required to account quantitatively for type III emission. For highly collimated emission with $\Delta\Omega \approx 0 \cdot 1$ (Melrose *et al.* 1978) and $L_N = 100$ km (Melrose 1975) one has $Q_{av} \approx 10^{-3}$. Thus the required efficiency of conversion is possible in principle but requires small-scale density inhomogeneities.

Application to Solar Radio Bursts

Applying the direct conversion mechanism to the interpretation of solar radio bursts leads to equivocal results. We compare the direct conversion mechanism with scattering by thermal ions for bursts of types I, II and III as follows:

Efficiency of Conversion. The strongest argument in favour of direct conversion is that it can be efficient enough to account for the inferred efficient conversion of Langmuir waves into transverse waves. Scattering by thermal ions is not efficient enough except when the Langmuir waves are generated in intense clumps (Melrose 1977).

Harmonic Structure. Types II and III bursts show harmonic structure, and the fact that direct conversion leads only to fundamental emission could be used to argue against it. On the other hand, all proposed fundamental emission mechanisms are qualitatively different from the accepted second-harmonic generation mechanism (coalescence of two Langmuir waves) and the same argument would apply to all fundamental emission mechanisms. Type I emission shows no harmonic structure and is probably at the fundamental plasma frequency (see e.g. the review of Elgarøy 1977).

Polarization. Types II and III bursts are unpolarized or weakly polarized in the sense of the o mode. Direct conversion leads to 100% polarization in the sense of the o mode and this could be used to argue against it. On the other hand, scattering by thermal ions should also lead to 100% polarization in the sense of the o mode (Melrose 1975). For all the mechanisms, one needs to invoke a depolarizing agent to account for the observed relatively low polarizations (Melrose 1975). Type I emission is 100% polarized in the sense of the o mode.

Bandwidth. For all forms of plasma emission the observed bandwidth is thought to be due to the spread of plasma frequencies in the emitting region.

Fine Structure. Many solar radio bursts show fine structure, and it is tempting to attribute this to local plasma inhomogeneities which are just what is required for direct coupling of Langmuir waves into transverse waves. However, effective direct coupling requires such large gradients that each inhomogeneity would be quite small, for example, $\lesssim 100$ km. More generally, one requires Langmuir waves propagating nearly along local density gradients which must not be strictly perpendicular to the magnetic field. Fine structure could be due to these conditions being satisfied for local inhomogeneities in one part of a source but not in others. Under special circumstances, e.g. parallel sheets of enhanced density, it is conceivable that the emission from specific inhomogeneous layers could be observed.

From the above discussion it may be concluded that little would be gained by invoking the direct conversion mechanism for types II and III bursts. It is more plausible for type I bursts. However, an alternative mechanism involving coalescence of Langmuir waves with ion sound waves or other low frequency waves (Melrose 1980) seems at least as favourable. Although the direct conversion mechanism is not obviously the most favourable for the more familiar types of solar radio bursts, it should be kept in mind as a possible alternative to scattering by ions.

In the Introduction, a third application of the coupling of z-mode waves to o-mode waves across $\omega = \omega_p$ was mentioned; this is connected with some theories for the Jovian decametric emission and for certain terrestrial emissions (Oya 1974; Benson 1975; Jones 1976, 1977). The theory presented here could be applied to the (terrestrial) nonthermal continuum along the lines suggested by Jones (1976). In the case of the Jovian decametric emission and the terrestrial kilometric emission, it is thought that one has $\Omega_e > \omega_p$ in the source region. Although the present theory can be extended to treat the z-mode to o-mode coupling across $\omega = \omega_p$ for $\Omega_e > \omega_p$, there is a further relevant coupling from the z mode to the x mode across $\omega = \Omega_e$. The case $\Omega_e > \omega_p$ should be explored further in connection with these applications.

In conclusion, direct conversion of Langmuir waves into transverse waves due to density inhomogeneities should not be excluded as a possible fundamental plasma emission mechanism, but there is no clear example where it is obviously more favourable than alternative mechanisms.

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Appendix

Thermal Corrections near Coupling Point

Thermal corrections to the magnetoionic dispersion relations near the coupling point could conceivably alter the results obtained above. Near the coupling point at $n^2 = Y_0/(1+Y_0)$ between the z mode and the o mode, the waves are nearly circularly polarized. (All waves are circularly polarized for $\sin\theta = 0$ and $\omega = \omega_p$ in a cold plasma, and the coupling point is approached by taking $\sin\theta \to 0$ before $\omega \to \omega_p$.) The dispersion relation for a hot electronic plasma for $\sin\theta = 0$ for the relevant mode is

$$n^2 = 1 - \{\omega_p^2/\omega(\omega+\Omega_e)\}\,\phi(y), \tag{A1}$$

where

$$\phi(y) := 2y\exp(-y^2)\int_0^y \mathrm{d}t\,\exp(t^2) = 1+\tfrac{1}{2}y^{-2}+\dots \qquad \text{for} \qquad y^2 \gg 1 \tag{A2}$$

is a form of the plasma dispersion function with

$$y = (\omega+\Omega_e)/\sqrt{2}\,kV_e \tag{A3}$$

here. Near the coupling point one has

$$y \approx y_0 = \frac{c}{\sqrt{2}\,V_e}\frac{(1+Y_0)^{3/2}}{Y_0^{1/2}} \gg 1 \tag{A4}$$

and thermal corrections are of order $1/y_0^2$, which is small and of no significance here.

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