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Mathematical Modelling of the Modulation Transfer Function of X-ray Film/Screen Systems

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Abstract

An exponential equation can be used to characterize the line spread function (LSF) of the X-ray film/screen combination. The analytic Fourier transform of the LSF yields the modulation transfer function (MTF). Predictions using the theoretical model show good agreement with results obtained by numerical Fourier transformation. A single figure of merit parameter may be assigned to the film/screen system. This parameter determines the system spatial frequency response.

Introduction

The modulation transfer function (MTF) is a measure of the performance of an image recording system. It describes the system in terms of the spatial frequency response and is an indicator of the quality of detail reproduction, or 'sharpness', in the photographic context.

Mathematical modelling of the MTF, using analytic transformations, provides greater insight into the relationship between certain system properties; for example, how does a change in the line spread function (LSF) affect the MTF? Another advantage is the simplification of data analysis. Mathematical models greatly reduce the computational burden. Application to the X-ray film/screen combination shows good agreement between analytic and numerical results for the data tested. However, it must be emphasized that numerical methods are preferable if maximum precision is required.

Theory

The LSF of a radiographic film/screen combination is determined by measuring the lateral spread in exposure resulting from the irradiation of a narrow platinum slit with X-ray photons. The film/screen system, located directly behind the slit, records this exposure in terms of optical density. The exposure is defined (James 1977) as the product of the irradiance I and the time t,

$$E = It. \tag{1}$$

The values for exposure are calculated from the image data by means of the sensitometric curve of the film/screen system. On the linear part of this characteristic curve, the density has the following relationship with exposure:

$$D = D_{\min} + \gamma \log_{10}(E/E_0), \tag{2}$$

where $D_{\min}$ is the minimum density and E_0 is the corresponding exposure. The slope of the linear part of the curve is given by γ. The main disadvantage of this representation is the nonlinearity in the toe and shoulder regions of the characteristic curve. The full sensitometric curve can often be represented by the following expression, which is somewhat similar in form to the Fermi function used in the study of beta decay in nuclear physics (Bayer *et al.* 1961; Enge 1966):

$$D = \frac{A}{1+\exp\{B\log_{10}(E/E_0)\}}, \tag{3}$$

where D is the density above $D_{\min}$, A and B are constants and E_0 is the exposure at the inflexion point of the curve. For the purposes of this study, however, the linear depiction of equation (2) will be sufficient—the reason for this will become apparent later.

Examination of published measurements of LSFs obtained from slit exposures (Rossmann 1964) suggests that the light intensity distribution in the film can be characterized by an exponential equation of the form

$$E(x) = a\exp(bx), \tag{4}$$

where $E(x)$ is the exposure as a function of distance, a is a constant, b takes on negative values and x is the absolute value in a centred coordinate system. Because of symmetry, only one side of the LSF need be considered for analysis.

The LSF is a measure of the spread of exposure caused by radiation scattering due to phosphors in the screen and silver halide crystals in the emulsion. Additional contributions to spread caused by geometrical and other factors can be minimized by careful adjustment of the experimental apparatus. For example, both focal spot size and distance between the slit and film cassette should be kept to a minimum. Also, the distance between the focal spot and the slit should be made as large as possible. The sides of the slit must be thick and parallel to avoid a gradual variation in X-ray absorption across an edge, which leads to a broader image. The slit itself is now made of platinum rather than uranium to avoid emission of energetic β rays which cause problems with high-speed film/screen combinations.

The fitting of various forms of exponential expressions to the LSF has been done in the past, but mainly for conventional emulsions (Frieser 1960; Paris 1961; Arnold *et al.* 1976). Very good results have been reported for films processed without appreciable adjacency effects (Paris 1961).

For equation (4) we need a method for calculating the coefficients a and b. First, we note that the equation may be linearized by means of a log transformation:

$$\ln E(x) = \ln a + bx. \tag{5}$$

The values for a and b are now calculated for n data points by a linear least-squares regression:

$$b = \frac{\sum x \sum \ln E - n^{-1}\sum x \sum \ln E}{\sum x^2 - n^{-1}(\sum x)^2}, \tag{6}$$

$$a = \exp(n^{-1}\sum \ln E - bn^{-1}\sum x). \tag{7}$$

The correlation coefficient r is given by the following expression:

$$r^2 = \frac{(\sum x \sum \ln E - n^{-1} \sum x \sum \ln E)^2}{\{\sum x^2 - n^{-1}(\sum x)^2\}\{\sum (\ln E)^2 - n^{-1}(\sum \ln E)^2\}}. \tag{8}$$

A value for r which is close to unity indicates a good fit of the proposed model to the experimental data.

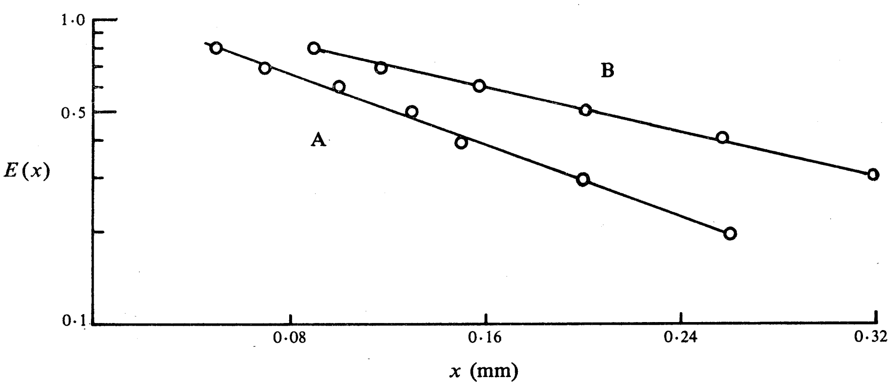

Fig. 1. Normalized LSF $E(x)$: A, Kodak Ortho G/Lanex ($b = -6{\cdot}65$ and $r^2 = 1{\cdot}00$); B, Dupont Cronex-2/Radelin STF-2 ($b = -4{\cdot}18$ and $r^2 = 1{\cdot}00$).

The MTF is calculated by finding the Fourier transform of the LSF. The normalized MTF has the following general form for a symmetric function:

$$\mathrm{MTF}(f) = \int_0^\infty E(x)\cos(2\pi f x)\,\mathrm{d}x \bigg/ \int_0^\infty E(x)\,\mathrm{d}x, \tag{9}$$

where f is the spatial frequency. Substituting equation (4) into (9) yields the normalized Fourier transform of the exponential spread function:

$$\mathrm{MTF}(f) = \frac{1}{1+(2\pi f/b)^2}. \tag{10}$$

Equation (10) is independent of the coefficient a, and so the LSF need not be normalized. Only the coefficient b is required to determine the MTF. It therefore serves as a figure of merit for comparing different emulsions. Equation (10) is essentially the MTF of the film/screen system. Correction for the MTF of the microdensitometer is unnecessary because its response is almost unity at the spatial frequencies of interest in X-ray work.

Since the regression technique is an implicit smoothing operation, it accommodates some scatter in the LSF data. Note that the parameter b can be calculated, albeit with much less precision, from equation (5) by graphical methods.

Comparison with Experimental Results

The model for the LSF was applied to the data of Arnold *et al.* (1976). In their report, they supported the applicability of an exponential expression to their LSF measurements. In Fig. 1, LSFs are compared for a rare-earth screen, Kodak's Lanex Regular with Ortho G film (case A), and a high-speed calcium tungstate screen, Radelin STF-2 with Dupont Cronex-2 film (case B). For exposure, a 12 μm slit was used with an X-ray tube voltage of 80 kVp. The slit images were scanned by a microdensitometer with a 10 μm slit width.

Both test cases described in Fig. 1 yielded correlation coefficients of unity. Data values between 10% and 90% of the peak were used. In general, results within this range will suffer very little from the toe and shoulder nonlinearities of the characteristic curve. Arnold and coworkers (1976) extrapolated the original LSF to a truncation level of less than 1% of the maximum value. They sampled at 20 μm intervals and computed the MTF digitally by numerical Fourier transformation. This MTF is plotted as a smooth curve for the two cases in Fig. 2. The circles represent the analytic transform of the exponential model using equation (10) and the value of b calculated from the data plotted in Fig. 1. It is evident that there is good agreement between the results.

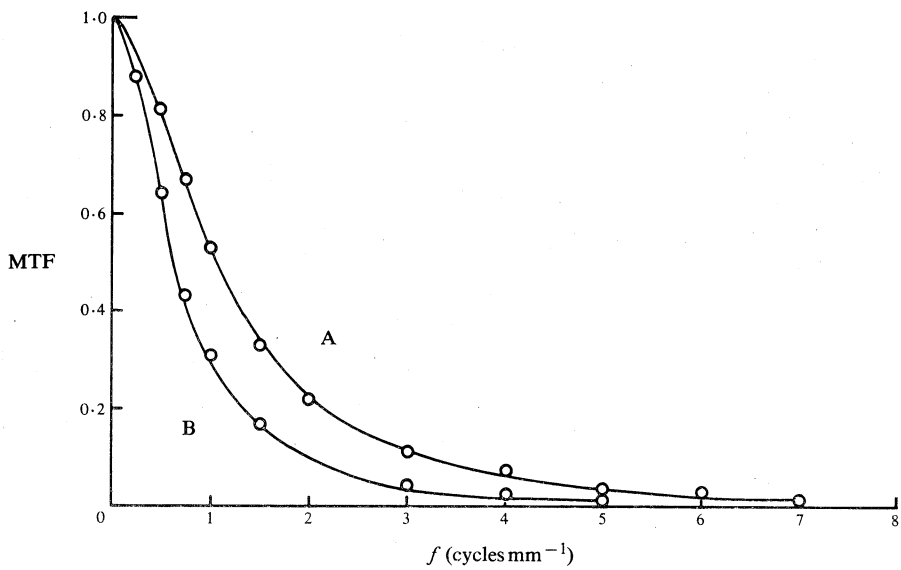

Fig. 2. Normalized MTF. Predictions from equation (10) are represented by the circles (values of b were determined from the data plotted in Fig. 1). Smooth curves represent results of numerical Fourier transformation (Arnold *et al.* 1976): A, Kodak Ortho G/Lanex; B, Dupont Cronex-2/Radelin STF-2.

Discussion

All the LSFs measured by Arnold *et al.* (1976) are adequately described by an exponential model. The analytic Fourier transform can therefore be used to calculate the MTF. The simplicity of the model is probably a consequence of the characteristically thick emulsions of X-ray films. This means that we can neglect any transmitted specular component of the light flux, which for a thin emulsion would necessarily require a compensating term in the equation. If a case should arise where the experimental data do not conform to the model, the value of the correlation coefficient would be significantly less than unity and it therefore acts as a warning signal.

One advantage of the model is that it eliminates some of the problems associated with the traditional technique of using a computer to calculate the numerical Fourier transform. These problems include a decision on the proper truncation level and sampling interval for the LSF. In order to avoid truncation errors, most of the LSF must be available for numerical analysis. For evaluation at low levels, and since the toe and shoulder regions of the characteristic curve are inadequate for precise measurements, multiple exposures are generally necessary. The final LSF is a composite of these separate exposures. This is avoided when using the model because points well away from the peak and tail regions may be chosen for calculating the parameter b.

Calculation of the numerical transform can be a lengthy exercise, even by computer, depending on the precision required and the computer memory and processing speed. In addition, a large sample size is required. The marginal loss in precision arising from the use of the model is compensated for by its simplicity and operational convenience. A program for the HP-25 calculator was used to compute b, r^2 and the MTF. Only six or seven points from the LSF were necessary for determining the MTF.

Conclusions

The LSF of the X-ray film/screen system can be characterized by a simple exponential expression. The analytic Fourier transform of the LSF yields the MTF. The model was applied to data for commercial X-ray films exposed in their regular cassettes. For the data tested, predictions using the theoretical model show good agreement with results obtained by numerical Fourier transformation. The model has been used for rapid assessments of the MTF, and in estimating the MTF variability due to random noise sources such as quantum mottle and granularity.

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