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'Plasma Emission' without Langmuir Waves

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Abstract

Recent observations have confirmed that the level of Langmuir waves associated with type III streams of electrons in the interplanetary medium is usually too low to account for the observed radio emission by the accepted 'plasma emission' processes, and it has been suggested that emission mechanisms which do not require Langmuir waves should be explored. Four such mechanisms are discussed. One is a parametric instability leading directly to second-harmonic emission; it is found inapplicable under conditions of interest here. The other three processes all involve ion-sound turbulence. One which is known in a different context is turbulent bremsstrahlung. Turbulent bremsstrahlung of transverse waves is found to compare unfavourably with the other two processes, which are scattering of an ion-sound (s) wave into a transverse (t) wave and double emission of both waves simultaneously. These latter two processes are related by a crossing symmetry and are treated together with the following results: (i) The processes become greatly enhanced when the beat $(\omega \pm \omega', \boldsymbol{k} \pm \boldsymbol{k}')$ between the t wave and the s wave nearly satisfies the dispersion relation for Langmuir (l) waves. (ii) A bump-in-the-tail instability (due to electrons with $dF(v)/dv > 0$) can cause the transverse waves to grow due to double emission; this growth has been likened to a free-electron maser. (iii) The familiar bump-in-the-tail instability for resonant l waves can be suppressed by the ion-sound waves, and the double-emission instability then takes over with about the same growth rate as the original l-wave instability. (iv) The conditions for the double-emission instability to occur are probably satisfied at least some of the time for type III streams. It is concluded that although 'plasma emission' without Langmuir waves may be possible in principle, it is unlikely to play any role in type III bursts.

1. Introduction

Since the first attempt at a quantitative theory of solar radio bursts by Ginzburg and Zheleznyakov (1958), it has been widely accepted that the observed emission is due to conversion of energy in Langmuir waves into escaping radiation near the plasma frequency ω_p and/or its second harmonic. In the case of type III bursts the generation of the Langmuir waves is attributed to the so-called bump-in-the-tail instability due to 10–100 keV electrons. Type III electrons and their associated radio emission were first observed in situ in the interplanetary medium in the late 1960's and have since been studied extensively (see e.g. Lin 1974). However, two difficulties have emerged from studies of the Langmuir waves expected to accompany the electrons. The first concerns the intensity of the Langmuir waves. Early searches near the orbit of the Earth were largely negative (Gurnett and Frank 1975). Later searches closer to the Sun confirmed the presence of Langmuir waves in some type III events, but mostly the Langmuir waves were at too low a level to account for the observed radio emission (Gurnett and Anderson 1977). These observations indicate that either the conversion processes (of Langmuir into transverse waves) are much more efficient than has previously been thought, or the emission process does not involve the observed Langmuir waves. The second difficulty concerns the causal relation between the Langmuir waves and the radio emission. A preliminary point is that the radio emission is interpreted as being at the second harmonic (see e.g. Fainberg *et al.* 1972; Haddock and Alvarez 1973; Kaiser 1975), with possibly a few exceptions (Kellogg 1980). With this interpretation, the Langmuir waves, when above the threshold for observation, occur consistently tens of minutes later than the radio emission they supposedly generate. Thus the relationship between the radio emission and the Langmuir waves is seemingly acausal. Elsewhere I suggest that the acausality can be avoided by ignoring the large body of circumstantial evidence for the second harmonic and assuming that the radio emission is at the fundamental (Melrose 1982). The foregoing observations also indicate that the observed radio emission may not be directly related to the observed Langmuir waves. This radical suggestion was made by Lin *et al.* (1981), who suggested that alternative 'plasma emission' processes which do not involve Langmuir waves should be explored.

In this paper I discuss four possible mechanisms whereby a stream of fast electrons might emit at the plasma frequency or its second harmonic without involving Langmuir waves. Emission of transverse waves by a charge in constant rectilinear motion is not possible because the resonance (Cerenkov) condition $\omega - \boldsymbol{k}.\boldsymbol{v} = 0$ cannot be satisfied. Thus we are led to consider higher order processes. Of the four processes discussed here, one is a parametric instability which involves ion waves, and the other three explicitly require the presence of ion-sound or other low-frequency turbulence. The role of this turbulence varies from one mechanism to another. There is observational evidence for ion-sound turbulence in the interplanetary medium (Gurnett and Frank 1978; Gurnett *et al.* 1979), and it is thought to be present most of the time (Gurnett *et al.* 1981). Here it is simply assumed that the required low-frequency waves are present.

Only one of the four mechanisms leads to emission at the second harmonic. Lin *et al.* (1973) considered a situation in which all the electrons drift relative to the ions at a speed v_0 much greater than the thermal speeds of either species. Using a one-dimensional model they found a parametric instability which involves ion oscillations at k_i driving a coupled system of a negative-energy wave at $k_0 + k_i$ and a positive-energy wave at k_0 (in the ion frame); for $k_0 v_0 \ll \omega_p$ the instability occurs at $k_i v_0 \approx 2\omega_p$ with both the driven waves having frequency $\approx \omega_p$ in the electron frame. The net effect is to drive oscillations at $\approx 2\omega_p$ in the electron frame, and hence to produce second-harmonic emission. The requirement that *all* the electrons drift relative to the ions at much greater than the thermal speed seems to be an essential requirement. This requirement is not satisfied for type III streams, which involve only a small fraction of the electrons drifting relative to the background ions and electrons. Thus this mechanism seems inapplicable to type III bursts. It is mentioned here simply because it is the only known non-Langmuir-wave mechanism which can produce second-harmonic emission.

A second mechanism is turbulent bremsstrahlung (Tsytovich *et al.* 1975). In this case an electron resonating with an ion-sound wave is accelerated due to the electric field of the wave, and as a result of this accelerated motion, the electron radiates (Kuijpers 1980*a*). In existing discussions the emitted waves are assumed to be small $|\boldsymbol{k}|$ Langmuir waves. Indeed turbulent bremsstrahlung was invoked as a means of converting energy in ion-sound turbulence into energy in Langmuir turbulence (Tsytovich *et al.* 1975), and has become a topic of some controversy (Vlahos and Papadopoulos 1979; Kuijpers 1980*b*; Tsytovich *et al.* 1981; Nambu 1981). It is obvious that the mechanism should produce transverse waves at given $|\boldsymbol{k}|$ at about the same rate as it produces Langmuir waves at that $|\boldsymbol{k}|$. This is confirmed by detailed calculations outlined in Appendix 1 (cf. also Nambu and Shukla 1979; Nambu 1980). The mechanism favours small $|\boldsymbol{k}|$ and hence is a fundamental 'plasma emission' process. From a qualitative discussion of the mechanism based on the analogy with turbulent bremsstrahlung of small $|\boldsymbol{k}|$ Langmuir waves, it is concluded from known results that the mechanism is not particularly favourable for highly suprathermal electrons and hence not particularly favourable for type III bursts.

The remaining two processes will be discussed in detail in this paper. They are scattering of ion-sound waves into transverse waves, and double emission of an ion-sound wave and a transverse wave, by suprathermal electrons. The two processes are related by a crossing symmetry and will be discussed together. These and similar processes have been discussed only incidentally in the literature. Lin *et al.* (1973) considered a 'nonresonant case' in addition to their parametric instability, discussed above. Their 'nonresonant case', which they likened to a free-electron maser, is essentially double emission of transverse waves and ion-sound waves. Kaplan and Tsytovich (1973; p. 85) also discussed amplification of transverse waves due to scattering and double emission. These authors were concerned with transverse waves at $\omega \gg \omega_p$. Here it is pointed out that the processes under consideration become greatly enhanced for $\omega \approx \omega_p$, and hence constitute a fundamental 'plasma emission' process.

The role of the ion-sound turbulence in the scattering and double-emission processes may be regarded as providing a momentum transfer which allows the emission of the transverse waves to occur. The resonance condition is

$$(\omega - \boldsymbol{k}.\boldsymbol{v}) \mp (\omega' - \boldsymbol{k}'.\boldsymbol{v}) = 0, \tag{1}$$

where $\omega, \boldsymbol{k}$ refers to the transverse waves and $\omega', \boldsymbol{k}'$ to the ion-sound waves, and where the upper and lower signs refer to scattering and double emission respectively. We have $\omega > \omega_p \gg \omega'$, and we assume $|\boldsymbol{k}| \ll |\boldsymbol{k}'|$. Then equation (1) becomes

$$\omega \pm \boldsymbol{k}'.\boldsymbol{v} = 0. \tag{2}$$

In a semi-classical description, the emission involves a quantum $\hbar(\omega \mp \omega') \approx \hbar\omega$ of energy carried off by the transverse wave, and a quantum $\hbar(\boldsymbol{k} \mp \boldsymbol{k}') \approx \mp\hbar\boldsymbol{k}'$ of momentum provided by the ion-sound wave.

A semi-classical treatment of the scattering and double-emission process is presented in a general form in Section 2, and applied to ion-sound and transverse waves in Section 3. (A collective-medium treatment is outlined in Appendix 2.) The condition (2) allows an alternative bump-in-the-tail instability which results directly in the growth of transverse waves, as suggested by Lin *et al.* (1973) and by Kaplan and Tsytovich (1973; p. 85). This instability for $\omega \approx \omega_p$ is discussed in Section 4. The effect of the three-wave processes $\mathrm{s}+\mathrm{l} \leftrightarrow \mathrm{t}$ and $\mathrm{s}+\mathrm{t} \leftrightarrow \mathrm{l}$ (s = ion-sound, l = Langmuir, t = transverse) cannot be ignored because the enhancement in the scattering and double-emission processes maximizes when $\omega \mp \omega'$, $\boldsymbol{k} \mp \boldsymbol{k}'$ nearly satisfies the dispersion relation for Langmuir waves; the conditions $\omega \mp \omega' = \omega''$ and $\boldsymbol{k} \mp \boldsymbol{k}' = \boldsymbol{k}''$

with ω'', $\boldsymbol{k}''$ describing a Langmuir wave, are just the conditions for the three-wave processes to occur. These three-wave processes are discussed in Section 5, where it is pointed out that as a result of the so-called nonlinear Landau damping of the Langmuir waves, the ion-sound waves can suppress the familiar bump-in-the-tail instability for the Langmuir waves. Thus the ion-sound waves can play two complementary roles: (a) they can suppress the generation of resonant Langmuir waves, i.e. Langmuir waves satisfying $\omega'' - \boldsymbol{k}''.\boldsymbol{v} = 0$, and (b) they can allow the alternative bump-in-the-tail instability to produce transverse waves directly. The possibility that type III emission may be produced directly through this alternative bump-in-the-tail instability is discussed in Section 6.

2. Single-particle Semi-classical Description

In describing the processes under discussion as 'scattering' and 'double emission', we implicitly assume a single-particle description. An electron (denoted e and e′ in the initial and final states respectively) can scatter a wave in one mode (σ' say) into a wave in another mode (σ say), or it can emit the two waves simultaneously. These processes may be described symbolically by $\mathrm{e}+\sigma' \rightarrow \mathrm{e}'+\sigma$ and $\mathrm{e} \rightarrow \mathrm{e}'+\sigma+\sigma'$ respectively and they are related by a crossing symmetry. In this section a general theory for such scattering and double-emission processes is written down. An alternative collective-medium approach is outlined in Appendix 2.

Waves in a given mode σ are described in terms of their wavevector $\boldsymbol{k}$. They have frequency ω determined by the dispersion relation $\omega = \omega^{\sigma}(\boldsymbol{k})$. Their electric vector is along the unimodular polarization vector $\boldsymbol{e}^{\sigma}(\boldsymbol{k})$ and the ratio of the electric to total energy is denoted $R_{\mathrm{E}}^{\sigma}(\boldsymbol{k})$ (Melrose 1980*a*; p. 47). For the crossing symmetry to have a simple form, it is appropriate to make the choices relating negative- and positive-frequency solutions

$$\omega^{\sigma}(-\boldsymbol{k}) = -\omega^{\sigma}(\boldsymbol{k}), \qquad \boldsymbol{e}^{\sigma}(-\boldsymbol{k}) = \boldsymbol{e}^{*\sigma}(\boldsymbol{k}), \qquad R_{\mathrm{E}}^{\sigma}(-\boldsymbol{k}) = R_{\mathrm{E}}^{\sigma}(\boldsymbol{k}), \tag{3}$$

where the asterisk denotes complex conjugation. Let $w_{+}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')$ be the probability per unit time that a wave quantum in the mode σ' in the range $\mathrm{d}^3k'/(2\pi)^3$ be scattered into a wave quantum in the mode σ in the range $\mathrm{d}^3k/(2\pi)^3$ by a particle with charge q, mass m and momentum $\boldsymbol{p} = \gamma m\boldsymbol{v}$, where $\gamma = (1-v^2/c^2)^{-\frac{1}{2}}$. The corresponding probability for double emission, i.e. with both wave quanta in the final state, is denoted $w_{-}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')$. Crossing symmetry implies

$$w_{-}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}') = w_{+}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},-\boldsymbol{k}'), \tag{4}$$

where equations (3) are used. A general expression for the probability is (Melrose 1980*b*; p. 168)

$$w_{\pm}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}') = \frac{4(2\pi)^3 q^4}{(4\pi\varepsilon_0)^2 m^2}\frac{R_{\mathrm{E}}^{\sigma}(\boldsymbol{k})\,R_{\mathrm{E}}^{\sigma'}(\boldsymbol{k}')}{|\,\omega^{\sigma}(\boldsymbol{k})\,\omega^{\sigma'}(\boldsymbol{k}')\,|}$$

$$\times\,|\,a^{\sigma\sigma'}(\boldsymbol{k},\pm\boldsymbol{k}',\boldsymbol{v})\,|^2\,\delta(\omega^{\sigma}(\boldsymbol{k})\mp\omega^{\sigma'}(\boldsymbol{k}')-(\boldsymbol{k}\mp\boldsymbol{k}').\boldsymbol{v}), \tag{5}$$

with

$$a^{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{v}) = e_i^{*\sigma}(\boldsymbol{k})\,e_j^{\sigma'}(\boldsymbol{k}')\,a_{ij}(\boldsymbol{k},\omega^{\sigma}(\boldsymbol{k});\boldsymbol{k}',\omega^{\sigma'}(\boldsymbol{k}');\boldsymbol{v}). \tag{6}$$

Let us denote $\boldsymbol{k}, \omega$ and $\boldsymbol{k}', \omega'$ collectively by k and k' respectively. One may make the separation

$$a_{ij}(k, k', \boldsymbol{v}) = a_{ij}^{\mathrm{TS}}(k, k', \boldsymbol{v}) + a_{ij}^{\mathrm{NS}}(k, k', \boldsymbol{v}), \tag{7}$$

into a part due to Thomson scattering (TS)

$$a_{ij}^{\mathrm{TS}}(k, k', \boldsymbol{v}) = \frac{1}{\gamma}\left(\delta_{ij} + \frac{k_i' v_j}{\omega' - \boldsymbol{k}' . \boldsymbol{v}} + \frac{k_j v_i}{\omega - \boldsymbol{k} . \boldsymbol{v}} + \frac{(\boldsymbol{k} . \boldsymbol{k}' - \omega\omega'/c^2) v_i v_j}{(\omega - \boldsymbol{k} . \boldsymbol{v})(\omega' - \boldsymbol{k}' . \boldsymbol{v})}\right), \tag{8}$$

and a part due to nonlinear scattering (NS) by the shielding field of the particle

$$a_{ij}^{\mathrm{NS}}(k, k', \boldsymbol{v}) = \frac{\mu_0 m c^2}{q(\omega - \omega')^2} 2\alpha_{ijl}(k, k', k - k') \frac{\lambda_{lm}(k - k')}{\Lambda(k - k')} v_m . \tag{9}$$

In (9), α_{ijl} describes the quadratic response of the plasma in an expansion of the induced current in powers of the vector potential $\boldsymbol{A}(k)$ in the temporal gauge ($\phi(k) = 0$):

$$J_i^{\mathrm{ind}}(k) = \alpha_{ij}(k) A_j(k) + \int \mathrm{d}\lambda^{(2)} \alpha_{ijl}(k, k_1, k_2) A_j(k_1) A_l(k_2)$$
$$+ \int \mathrm{d}\lambda^{(3)} \alpha_{ijlm}(k, k_1, k_2, k_3) A_j(k_1) A_l(k_2) A_m(k_3) + \ldots, \tag{10}$$

where (using $:=$ to define quantities on the left-hand side)

$$\mathrm{d}\lambda^{(n)} := \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \cdots \frac{\mathrm{d}^4 k_n}{(2\pi)^4} (2\pi)^4 \delta^4(k - k_1 - \ldots - k_n) \tag{11}$$

denotes an n-fold convolution integral. The 'photon propagator' $\lambda_{ij}(k)/\Lambda(k)$ in (9) is the inverse of the operator $\Lambda_{ij}(k)$ in the inhomogeneous wave equation

$$\Lambda_{ij}(k) A_j(k) = -(\mu_0 c^2/\omega^2) J_i^{\mathrm{ext}}(k), \tag{12}$$

where $\boldsymbol{J}^{\mathrm{ext}}(k)$ is an extraneous current and with

$$\Lambda_{ij}(k) := (|\boldsymbol{k}|^2 c^2/\omega^2)(\kappa_i \kappa_j - \delta_{ij}) + \varepsilon_{ij}(k), \tag{13}$$

$$\varepsilon_{ij}(k) := \delta_{ij} + (\mu_0 c^2/\omega^2)\alpha_{ij}(k), \qquad \boldsymbol{\kappa} := \boldsymbol{k}/|\boldsymbol{k}| . \tag{14}$$

An application of detailed balancing leads to kinetic equations for the occupation numbers $N^\sigma(\boldsymbol{k})$ and $N^{\sigma'}(\boldsymbol{k}')$ of the wave quanta and for the distribution function $f(\boldsymbol{p})$ of electrons (Melrose 1980a; pp. 171 & 175):

$$\frac{\mathrm{d}N^\sigma(\boldsymbol{k})}{\mathrm{d}t} = \int \mathrm{d}^3 p \int \frac{\mathrm{d}^3 k'}{(2\pi)^3} \sum_{\pm} w_{\pm}^{\sigma\sigma'}(\boldsymbol{p}, \boldsymbol{k}, \boldsymbol{k}')[\{N^{\sigma'}(\boldsymbol{k}') \mp N^\sigma(\boldsymbol{k})\} f(\boldsymbol{p})$$
$$+ N^\sigma(\boldsymbol{k}) N^{\sigma'}(\boldsymbol{k}') \hbar(\boldsymbol{k} \mp \boldsymbol{k}') . \partial f(\boldsymbol{p})/\partial \boldsymbol{p}], \tag{15}$$

$$\frac{\mathrm{d}N^{\sigma'}(\boldsymbol{k}')}{\mathrm{d}t} = \int \mathrm{d}^3 p \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sum_{\pm} w_{\pm}^{\sigma\sigma'}(\boldsymbol{p}, \boldsymbol{k}, \boldsymbol{k}')[\{N^\sigma(\boldsymbol{k}) \mp N^{\sigma'}(\boldsymbol{k}')\} f(\boldsymbol{p})$$
$$\mp N^\sigma(\boldsymbol{k}) N^{\sigma'}(\boldsymbol{k}') \hbar(\boldsymbol{k} \mp \boldsymbol{k}') . \partial f(\boldsymbol{p})/\partial \boldsymbol{p}], \tag{16}$$

$$\frac{\mathrm{d}f(\boldsymbol{p})}{\mathrm{d}t} = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\sum_{\pm}\hbar(\boldsymbol{k}\mp\boldsymbol{k}')\,.\,\frac{\partial}{\partial\boldsymbol{p}}\Big(w_{\pm}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')$$

$$\times\,[\{N^{\sigma'}(\boldsymbol{k}')\mp N^{\sigma}(\boldsymbol{k})\}f(\boldsymbol{p})+N^{\sigma}(\boldsymbol{k})N^{\sigma'}(\boldsymbol{k}')\hbar(\boldsymbol{k}\pm\boldsymbol{k}')\,.\,\partial f(\boldsymbol{p})/\partial\boldsymbol{p}]\Big)\,. \quad (17)$$

In the following discussion we require explicit expressions for the tensors defined by the weak-turbulence expansion (10). A standard calculation using kinetic theory, and retaining only the contribution of one species of particle, leads to

$$\alpha_{ij}(k) = q^2\int \mathrm{d}^3p\,\frac{v_i\,g_{rj}(k,\boldsymbol{v})}{\omega-\boldsymbol{k}\,.\,\boldsymbol{v}}\,\frac{\partial f(\boldsymbol{p})}{\partial p_r}, \quad (18)$$

$$\tilde{\alpha}_{ijl}(k,k_1,k_2) = q^3\int \mathrm{d}^3p\,\frac{v_i\,g_{rj}(k_1,\boldsymbol{v})}{\omega-\boldsymbol{k}\,.\,\boldsymbol{v}}\,\frac{\partial}{\partial p_r}\left(\frac{g_{sl}(k_2,\boldsymbol{v})}{\omega_2-\boldsymbol{k}_2\,.\,\boldsymbol{v}}\,\frac{\partial f(\boldsymbol{p})}{\partial p_s}\right), \quad (19)$$

$$\tilde{\alpha}_{ijlm}(k,k_1,k_2,k_3) = q^4\int \mathrm{d}^3p\,\frac{v_i\,g_{rj}(k_1,\boldsymbol{v})}{\omega-\boldsymbol{k}\,.\,\boldsymbol{v}}\,\frac{\partial}{\partial p_r}$$

$$\times\left\{\frac{g_{sl}(k_2,\boldsymbol{v})}{\omega_2+\omega_3-(\boldsymbol{k}_2+\boldsymbol{k}_3)\,.\,\boldsymbol{v}}\,\frac{\partial}{\partial p_s}\left(\frac{g_{tm}(k_3,\boldsymbol{v})}{\omega_3-\boldsymbol{k}_3\,.\,\boldsymbol{v}}\,\frac{\partial f(\boldsymbol{p})}{\partial p_t}\right)\right\}, \quad (20)$$

with

$$g_{ij}(k,\boldsymbol{v}) := (\omega-\boldsymbol{k}\,.\,\boldsymbol{v})\delta_{ij}+k_i v_j\,, \quad (21)$$

and with α_{ijl} and α_{ijlm} obtained from $\tilde{\alpha}_{ijl}$ and $\tilde{\alpha}_{ijlm}$ by symmetrizing, i.e.

$$\alpha_{ijl}(k,k_1,k_2) = \tfrac{1}{2}\{\tilde{\alpha}_{ijl}(k,k_1,k_2)+\tilde{\alpha}_{ijl}(k,k_2,k_1)\}\,,\quad \text{etc.} \quad (22)$$

3. Processes e+s → e′+t and e → e′+s+t

Let us now assume that the background plasma is isotropic with a thermal distribution of electrons and one species of ion, and find approximate expressions for $w_{\pm}^{\mathrm{st}}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')$ for nonrelativistic electrons.

Relevant plasma parameters are ω_{p}, the electron Debye length λ_{De} and thermal speed $V_{\mathrm{e}} = \omega_{\mathrm{p}}\lambda_{\mathrm{De}}$, and the ion plasma frequency ω_{pi} and ion-sound speed $v_{\mathrm{s}} := \omega_{\mathrm{pi}}\lambda_{\mathrm{De}}$. The properties of the wave modes are:
ion-sound waves (σ = s)

$$\omega^{\mathrm{s}}(\boldsymbol{k}) = \frac{|\boldsymbol{k}|\,v_{\mathrm{s}}}{(1+|\boldsymbol{k}|^2\lambda_{\mathrm{De}}^2)^{\frac{1}{2}}},\qquad e^{\mathrm{s}}(\boldsymbol{k}) = \boldsymbol{\kappa},\qquad R_{\mathrm{E}}^{\mathrm{s}}(\boldsymbol{k}) = \tfrac{1}{2}\left(\frac{\omega^{\mathrm{s}}(\boldsymbol{k})}{\omega_{\mathrm{pi}}}\right)^2; \quad (23)$$

Langmuir waves (σ = l)

$$\omega^{\mathrm{l}}(\boldsymbol{k}) = \omega_{\mathrm{p}}(1+|\boldsymbol{k}|^2\lambda_{\mathrm{De}}^2)^{\frac{1}{2}},\qquad e^{\mathrm{l}}(\boldsymbol{k}) = \boldsymbol{\kappa},\qquad R_{\mathrm{E}}^{\mathrm{l}}(\boldsymbol{k}) = \tfrac{1}{2}\left(\frac{\omega^{\mathrm{l}}(\boldsymbol{k})}{\omega_{\mathrm{p}}}\right)^2; \quad (24)$$

transverse waves (σ = t)

$$\omega^{\mathrm{t}}(\boldsymbol{k}) = (\omega_{\mathrm{p}}^2+|\boldsymbol{k}|^2c^2)^{\frac{1}{2}},\qquad \boldsymbol{e}\,.\,\boldsymbol{\kappa} = 0,\qquad R_{\mathrm{E}}^{\mathrm{t}}(\boldsymbol{k}) = \tfrac{1}{2}\,. \quad (25)$$

The dielectric tensor $\varepsilon_{ij}(k)$ may be separated into longitudinal (l) and transverse (t) parts, and then the photon propagator becomes

$$\frac{\lambda_{ij}(k)}{\Lambda(k)} = \frac{\kappa_i\kappa_j}{\varepsilon^{\mathrm{l}}(k)}+\frac{\delta_{ij}-\kappa_i\kappa_j}{\varepsilon^{\mathrm{t}}(k)-|\boldsymbol{k}|^2c^2/\omega^2}\,. \quad (26)$$

In equation (5), with (6), (7) and (9), the propagator appears with argument $\boldsymbol{k}\mp\boldsymbol{k}',\omega\mp\omega'$ which for $|\boldsymbol{k}'|\gg|\boldsymbol{k}|$ and $\omega\gg\omega'$ corresponds to approximately $\mp\boldsymbol{k}',\omega$. Then we have

$$\varepsilon^{\mathrm{l}}(k\mp k')\approx 1-\frac{\omega_{\mathrm{p}}^2}{\omega^2}\left(1+\frac{3|\boldsymbol{k}'|^2V_{\mathrm{e}}^2}{\omega^2}\right), \tag{27}$$

with $\varepsilon^{\mathrm{t}}(k\mp k')\approx 1-\omega_{\mathrm{p}}^2/\omega^2$. For $\omega\approx\omega_{\mathrm{p}}$ the term involving $\varepsilon^{\mathrm{l}}(k)$ in (26) dominates.

The quadratic response tensor α_{ijl} is dominated by the electronic contribution. If we assume $\omega/|\boldsymbol{k}|,\omega_1/|\boldsymbol{k}_1|\gg V_{\mathrm{e}}\gg\omega_2/|\boldsymbol{k}_2|$, an appropriate approximation is

$$2\alpha_{ijl}(k,k_1,k_2)\approx\frac{e}{\mu_0 m_{\mathrm{e}}c^2}\frac{\omega_2\,\delta_{ij}\,k_{2l}}{|\boldsymbol{k}_2|^2\lambda_{\mathrm{De}}^2}. \tag{28}$$

The factor 2 in (9) and (28) arises from the symmetry property (22), which is now broken. For $v\ll c$, equation (7) with (9)–(11) reduces to

$$a_{ij}(k,k',\boldsymbol{v})\approx\delta_{ij}-\frac{k_i'v_j}{\boldsymbol{k}'\cdot\boldsymbol{v}}+\frac{\omega'}{\omega|\boldsymbol{k}'|^2\lambda_{\mathrm{De}}^2}\frac{\kappa_i'\kappa_j'}{\varepsilon^{\mathrm{l}}(k-k')}, \tag{29}$$

where only the leading term in (26) has been retained. For $|\boldsymbol{k}'|\approx\omega_{\mathrm{p}}/v$ and $\omega'\approx|\boldsymbol{k}'|v_{\mathrm{s}}$, the final term in (29) dominates for $\varepsilon^{\mathrm{l}}(k-k')\ll 43V_{\mathrm{e}}/v$, which is well satisfied for $\omega\approx\omega_{\mathrm{p}}$.

Thus the probability (5) reduces to

$$w_{\pm}^{\mathrm{ts}}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')=\frac{(2\pi)^3e^4}{(4\pi\varepsilon_0)^2m_{\mathrm{e}}^2}\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{pi}}^2V_{\mathrm{e}}^4|\boldsymbol{k}'|^4}\left(\frac{\omega^{\mathrm{s}}(\boldsymbol{k}')}{\omega^{\mathrm{t}}(\boldsymbol{k})}\right)^3$$
$$\times\frac{|\boldsymbol{e}\cdot\boldsymbol{\kappa}'|^2}{|\varepsilon^{\mathrm{l}}(k\mp k')|^2}\delta(\omega^{\mathrm{t}}(\boldsymbol{k})\mp\omega^{\mathrm{s}}(\boldsymbol{k}')-(\boldsymbol{k}\mp\boldsymbol{k}')\cdot\boldsymbol{v}). \tag{30}$$

For $\omega\approx\omega_{\mathrm{p}}$ and $\omega^{\mathrm{s}}(\boldsymbol{k}')\approx|\boldsymbol{k}'|v_{\mathrm{s}}$ further approximation to (30) leads to

$$w_{\pm}^{\mathrm{ts}}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}')\approx\frac{(2\pi)^3e^4}{(4\pi\varepsilon_0)^2m_{\mathrm{e}}^2}\frac{\omega^{\mathrm{s}}(\boldsymbol{k}')\omega_{\mathrm{p}}}{|\boldsymbol{k}'|^2\lambda^2}\frac{|\boldsymbol{e}\cdot\boldsymbol{\kappa}'|^2}{(|\boldsymbol{k}|^2c^2-3|\boldsymbol{k}'|^2V_{\mathrm{e}}^2)^2}\delta(\omega_{\mathrm{p}}\mp\boldsymbol{k}'\cdot\boldsymbol{v}). \tag{31}$$

In the following the sum over the two states of transverse polarization ($|\boldsymbol{e}\cdot\boldsymbol{\kappa}'|^2\rightarrow|\boldsymbol{\kappa}\times\boldsymbol{\kappa}'|^2$) is performed in treating emission, and the average is performed in treating absorption.

Nearly Singular Behaviour

A notable feature of (31) is that it is singular at $|\boldsymbol{k}|^2c^2=3|\boldsymbol{k}'|^2V_{\mathrm{e}}^2$, and this singularity is not integrable. We now argue that this singularity is to be removed by including the imaginary part (Im) of $\varepsilon^{\mathrm{l}}(k)$, and by excluding the region where the real part (Re) satisfies $|\mathrm{Re}\,\varepsilon^{\mathrm{l}}(k)|\lesssim|\mathrm{Im}\,\varepsilon^{\mathrm{l}}(k)|$. The essence of the argument is that for $|\mathrm{Re}\,\varepsilon^{\mathrm{l}}(k)|\lesssim|\mathrm{Im}\,\varepsilon^{\mathrm{l}}(k)|$ the processes in question should be regarded as three-wave processes rather than as scattering and double emission. This point is treated in a more formal way in Appendices 2 and 3; the discussion here is partly heuristic.

From (30), it is evident that the singularity occurs at $\varepsilon^{\mathrm{l}}(k \mp k') = 0$, which corresponds to $\omega \mp \omega'$ being equal to $\omega^{\mathrm{l}}(\boldsymbol{k} \mp \boldsymbol{k}')$. If we write $\boldsymbol{k}'' = \boldsymbol{k} \mp \boldsymbol{k}'$, then the singularity occurs at

$$\boldsymbol{k} \mp \boldsymbol{k}' = \boldsymbol{k}'', \qquad \omega^{\mathrm{t}}(\boldsymbol{k}) \mp \omega^{\mathrm{s}}(\boldsymbol{k}') = \omega^{\mathrm{l}}(\boldsymbol{k}''). \tag{32}$$

The conditions (32) are just those required for the three-wave interactions $\mathrm{s}+\mathrm{l} \leftrightarrow \mathrm{t}$ and $\mathrm{s}+\mathrm{t} \leftrightarrow \mathrm{l}$.

The collective-medium approach developed in Appendix 2 leads to a factor $\varepsilon^{\mathrm{l}}(k \mp k')$ squared in the denominator, as in (30), when one makes the longitudinal approximation. On including the imaginary part of $\varepsilon^{\mathrm{l}}(k \mp k')$, and it is essential to do so in this alternative approach, the relevant factor appears in the form

$$\operatorname{Im}\left(\frac{1}{\varepsilon^{\mathrm{l}}(k \mp k')}\right) = \frac{-\operatorname{Im} \varepsilon^{\mathrm{l}}(k \mp k')}{\{\operatorname{Re} \varepsilon^{\mathrm{l}}(k \mp k')\}^2 + \{\operatorname{Im} \varepsilon^{\mathrm{l}}(k \mp k')\}^2}. \tag{33}$$

Thus the singularity is avoided for $\operatorname{Im} \varepsilon^{\mathrm{l}}(k \mp k') \neq 0$. If we write

$$\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k}) = 2\left(\frac{\operatorname{Im} \varepsilon^{\mathrm{l}}(k)}{(\partial/\partial\omega) \operatorname{Re} \varepsilon^{\mathrm{l}}(k)}\right)_{\omega=\omega^{\mathrm{l}}(\boldsymbol{k})} \tag{34}$$

as the effective absorption coefficient for Langmuir waves in the neighbourhood of the zero in $\operatorname{Re} \varepsilon^{\mathrm{l}}(k \mp k')$, equation (33) may be approximated by

$$\operatorname{Im}\left(\frac{1}{\varepsilon^{\mathrm{l}}(k \mp k')}\right) = \frac{-\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k} \mp \boldsymbol{k}')}{\{\omega \mp \omega' - \omega^{\mathrm{l}}(\boldsymbol{k} \mp \boldsymbol{k}')\}^2 + \{\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k} \mp \boldsymbol{k}')\}^2}. \tag{35}$$

When the frequency mismatch $|\omega \mp \omega' - \omega^{\mathrm{l}}(\boldsymbol{k} \mp \boldsymbol{k}')|$ is well in excess of the absorption coefficient $\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k} \mp \boldsymbol{k}')$, the term involving $\gamma^{\mathrm{l}}_{\mathrm{eff}}$ in the denominator may be neglected, and then the results derived using the theory of Appendix 2 reproduce the result derived using (30). On the other hand, when the frequency mismatch is less than the effective absorption coefficient, the beat at $\omega \mp \omega', \boldsymbol{k} \mp \boldsymbol{k}'$ damps out on a timescale shorter than that over which the frequency mismatch would become evident, and the beat is indistinguishable from a weakly damped Langmuir wave. In this case the processes should be regarded as three-wave interactions. Thus the inclusion of $\operatorname{Im} \varepsilon^{\mathrm{l}}(k) \neq 0$ removes the singularity and implies that the scattering and double-emission processes pass over into three-wave processes when the frequency mismatch becomes less than about the effective absorption coefficient.

The maximum in the probability occurs where the frequency mismatch is equal to $\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k} \mp \boldsymbol{k}') \approx \gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k}')$. The maximum growth rate may be estimated by assuming a Lorentzian profile, as implied by (33), and integrating over it excluding the region where the mismatch is less than $\gamma^{\mathrm{l}}_{\mathrm{eff}}$. For present purposes this is equivalent to making the following approximation in (31):

$$\frac{1}{(|\boldsymbol{k}|^2 c^2 - 3|\boldsymbol{k}'|^2 V_{\mathrm{e}}^2)^2} \approx \frac{1}{|\boldsymbol{k}|^4 c^4}, \qquad \text{for } |\boldsymbol{k}|^2 c^2 \gg 3|\boldsymbol{k}'|^2 V_{\mathrm{e}}^2 \tag{36a}$$

$$\approx \frac{\pi\,\delta(|\boldsymbol{k}|^2 c^2 - 3|\boldsymbol{k}'|^2 V_{\mathrm{e}}^2)}{\omega_{\mathrm{p}}\,\gamma^{\mathrm{l}}_{\mathrm{eff}}(\boldsymbol{k}')}, \qquad \text{for } |\boldsymbol{k}|^2 c^2 \lesssim 3|\boldsymbol{k}'|^2 V_{\mathrm{e}}^2. \tag{36b}$$

The part (36a) applies far from the singularity in (31); using it one may re-derive the result of Kaplan and Tsytovich (1973; p. 85). The part (36b) takes account of the nearly singular behaviour with the singularity cut out as implied by (35). Only the nearly singular part is retained in the discussion below.

4. An Alternative Bump-in-the-tail Instability

The familiar bump-in-the-tail instability causes Langmuir waves to grow due to a stream of suprathermal electrons. The simplest case corresponds to Langmuir waves propagating along the streaming direction and to nonrelativistic electrons with one-dimensional distribution function $F(v)$, defined here such that $\int dv\, F(v)$ equals $n_1 = \int d^3p\, f(\boldsymbol{p})$. The growth rate for such Langmuir waves at $|\boldsymbol{k}''| = \omega_p/v$ is given by minus the absorption coefficient

$$\gamma_0^l(\boldsymbol{k}'') = -(\pi/n_e)\omega_p v^2\, dF(v)/dv\,. \tag{37}$$

In Section 5 it is argued that in the presence of a sufficiently high level of ion-sound waves with $|\boldsymbol{k}'| \approx \omega_p/v$, directed along the streaming direction, this instability can be suppressed. Once suppressed it may be replaced by an alternative one in which the transverse waves grow. A semi-quantitative theory for this alternative instability is developed in the present section.

The alternative instability is due to double emission of ion-sound and transverse waves. (There is a closely analogous instability in which the transverse wave is replaced by a Langmuir wave with small $|\boldsymbol{k}|$.) A simple approximation for the absorption coefficient in this case may be obtained as follows. First insert (31) in (15), assuming $\sigma = \mathrm{t}$, $\sigma' = \mathrm{s}$ and $N^s(\boldsymbol{k}') \gg N^t(\boldsymbol{k})$. Next re-express the N in terms of effective temperatures $T^s(\boldsymbol{k}')$ and $T^t(\boldsymbol{k})$. Now assume that the electron distribution is one-dimensional and strongly concentrated around a particular v. Then carry out the integrals retaining only the nearly singular part (36b) of the probability. Finally, take the average over the states of polarization and the direction of emission of the transverse waves and choose the value of $|\boldsymbol{k}|$ which maximizes the emission, denoting the relevant value of $T^t(\boldsymbol{k})$ by $\bar{T}^t$. The resulting transfer equation is then of the form

$$d\bar{T}^t/dt = T_0^{st} - \gamma^{st}\bar{T}^t, \tag{38}$$

with T_0^{st} describing the effects of 'spontaneous emission' and with γ^{st} the effective absorption coefficient for the two processes $\mathrm{e}+\mathrm{s} \to \mathrm{e}'+\mathrm{t}$ and $\mathrm{e} \to \mathrm{e}'+\mathrm{s}+\mathrm{t}$. These two processes complement each other in the spontaneous emission but contributed with opposite signs to the absorption coefficient (due to the $\mp$ sign in the final term of equation 15). Writing

$$\bar{T}^s = T^s(\boldsymbol{k}') + T^s(-\boldsymbol{k}'), \tag{39}$$

$$A^s = \{T^s(\boldsymbol{k}') - T^s(-\boldsymbol{k}')\}/\bar{T}^s \tag{40}$$

for $\boldsymbol{k}'$ along the streaming direction and $|\boldsymbol{k}'| \approx \omega_p/v$, one finds

$$T_0^{st} \approx \frac{\pi}{9}\frac{r_0 c^2 \omega_p^3}{n_e \gamma_{\mathrm{eff}}^l(\boldsymbol{k}')\, V_e^2}\frac{\bar{T}^s}{T_e} m_e v^2 F(v), \tag{41}$$

$$\gamma^{st} \approx -\frac{\pi}{18}\frac{r_0 c^2 \omega_p^3}{n_e \gamma_{\mathrm{eff}}^l(\boldsymbol{k}')\, V_e^2}\frac{\bar{T}^s}{T_e} A^s v \frac{dF(v)}{dv}, \tag{42}$$

where r_0 is the classical radius of the electron.

Thus a bump-in-the-tail instability develops under similar conditions to that for Langmuir waves, provided that the ion-sound waves are anisotropic favouring the forward streaming direction ($A^{s} > 0$). The double emission tends to cause the waves to grow and the scattering tends to cause them to damp. As a consequence forward-propagating ion-sound waves grow and backward-propagating ion-sound waves damp, causing A^{s} itself to grow; this growth may be described quantitatively using (16). The growth of A^{s} implies that the instability is faster than exponential; it is of the form $d\overline{T}^{t}/dt \propto (\overline{T}^{t})^{2}$.

Quasilinear Relaxation

Suppose that the double-emission instability develops. What effect will this have on the evolution of the stream? In this case when the familiar bump-in-the-tail instability for Langmuir waves develops, the stream evolves through quasilinear relaxation (see e.g. Grognard 1975, 1980; Takakura and Shibahashi 1976; Magelssen and Smith 1977).

The evolution of the stream of electrons due to the alternative double-emission instability may be described using (17). Comparison of (17) and the corresponding equation for the Langmuir-wave case suggests that the evolution should be closely analogous in the two cases. The fact that the alternative instability is faster than exponential is unlikely to be important because quasilinear relaxation occurs almost instantaneously when compared with timescales over which the stream itself evolves (Grognard 1980). Thus the dynamics of the stream should be similar whichever of the two instabilities develops.

5. Three-wave Processes

The three-wave processes $s+l \leftrightarrow t$ and $s+t \leftrightarrow l$ occur when the conditions (32), namely $\boldsymbol{k}'\pm\boldsymbol{k}'' = \pm\boldsymbol{k}$ and $\omega'\pm\omega'' = \pm\omega$, are satisfied. Analogous processes with the t wave replaced by an l wave with small $|\boldsymbol{k}|$ are also possible. In the following discussion, to avoid confusion Langmuir waves generated directly by the stream are referred to as 'resonant' l waves and other l waves are referred to as 'nonresonant' or specifically as 'small $|\boldsymbol{k}|$' l waves; 'small $|\boldsymbol{k}|$ wave' implies either a t wave or a small $|\boldsymbol{k}|$ l wave.

The three-wave process can lead to suppression of the bump-in-the-tail instability for resonant l waves. The nonlinear (NL) absorption coefficient for the resonant l waves is estimated in Appendix 3:

$$\gamma_{\mathrm{NL}}^{l}(\boldsymbol{k}'') \approx \frac{2r_0\, c^2\omega_{\mathrm{p}}^2}{9V_{\mathrm{e}}^2\, v}\frac{\overline{T}^{s}}{T_{\mathrm{e}}}, \tag{43}$$

with $|\boldsymbol{k}''| \approx \omega_{\mathrm{p}}/v$. The result (43) is in qualitative agreement with one quoted by Smith *et al.* (1979) based on the work of Dawson and Oberman (1963), and also Dawson (1968). They expressed γ_{NL}^{l} in terms of the density fluctuations δn associated with the ion-sound waves. One has $T^{s}(\boldsymbol{k}')/T_{\mathrm{e}} = |\delta n(\boldsymbol{k}')|^2/n$, and with

$$|\delta n|^2 := \int \frac{d^3k'}{(2\pi)^3}\,|\delta n(\boldsymbol{k}')|^2,$$

one finds $\gamma_{\mathrm{NL}}^{l}/\omega_{\mathrm{p}} \approx |\delta n/n|^2(k''\lambda_{\mathrm{De}})^{-2}$ provided that the fluctuations δn are dominated by waves of the relevant $k' \approx \omega_{\mathrm{p}}/v$.

The bump-in-the-tail instability for resonant l waves is suppressed for $\gamma_{\rm NL}^{\rm l} \gtrsim |\gamma_0^{\rm l}|$. Inspection of (37), (42) and (43) shows that we have

$$\gamma^{\rm st} \approx \gamma_{\rm NL}^{\rm l}\, \gamma_0^{\rm l}\, A^{\rm s}/\gamma_{\rm eff}^{\rm l}\,. \tag{44}$$

With $\gamma_{\rm eff}^{\rm l} \approx |\gamma_0^{\rm l}|$, it follows that when the generation of the resonant l waves is suppressed the alternative bump-in-the-tail instability takes over with a growth rate $\gamma^{\rm st} \approx \gamma_0^{\rm l}$.

Threshold Level of Ion-Sound Waves

Let us determine the threshold level of ion-sound waves (with $|\boldsymbol{k}'| \approx \omega_{\rm p}/v$) required to suppress the resonant l waves and produce fundamental 'plasma emission' directly. If we write $\{v^2\,{\rm d}F(v)/{\rm d}v\}_{\rm max} = n_1(v/\Delta v)^2$ in (37), the condition $\gamma_{\rm NL}^{\rm l} > |\gamma_0^{\rm l}|$ leads to the threshold condition

$$\frac{\overline{T}^{\rm s}}{T_{\rm e}} \gtrsim \frac{9\pi}{2}\frac{n_1}{n_{\rm e}}\frac{V_{\rm e}^2\, v}{r_0\, c^2 \omega_{\rm p}}\left(\frac{v}{\Delta v}\right)^2. \tag{45}$$

An alternative way of expressing the condition (45) is in terms of the ratio of the energy density $W^{\rm s}$ in the ion-sound waves to the thermal energy density $n_{\rm e}\,T_{\rm e}$ in the electrons:

$$\frac{W^s}{n_{\rm e}\,T_{\rm e}} \gtrsim \frac{n_1}{n_{\rm e}}\left(\frac{V_{\rm e}}{v}\right)^2\left(\frac{v}{\Delta v}\right)^2. \tag{46}$$

It should be emphasized that (45) and (46) include only the ion-sound waves in a range $\Delta k' \approx k'$ at $k' \approx \omega_{\rm p}/v$.

6. Discussion and Conclusions

The main result of the current investigation is that when ion-sound waves are present above a threshold level, the bump-in-the-tail instability (due to a stream of fast electrons) changes its character. Below the threshold the instability produces resonant Langmuir waves (with $|\boldsymbol{k}''| \approx \omega_{\rm p}/v$). Above the threshold it may proceed through double emission of an ion-sound wave (with $|\boldsymbol{k}'| \approx \omega_{\rm p}/v$) and a small $|\boldsymbol{k}|$ transverse wave or Langmuir wave. Thus, in principle it is possible for a stream of electrons, with ${\rm d}F(v)/{\rm d}v > 0$ over some range, to propagate without copious production of resonant Langmuir waves and to generate fundamental 'plasma emission' directly.

For type III streams in the interplanetary medium the threshold condition on the ion-sound waves, in the form (46) say, requires $W^{\rm s}/n_{\rm e}\,T_{\rm e} \gtrsim 10^{-9}$, where we set $n_1/n_{\rm e} \approx 10^{-7}$, $(V_{\rm e}/v)^2 \approx 10^{-3}$, $(v/\Delta v)^2 \approx 10$. This energy density refers to ion-sound waves with $|\boldsymbol{k}'| \approx \omega_{\rm p}/v$, i.e. $|\boldsymbol{k}'|\,\lambda_{\rm De} \approx 0{\cdot}03$. Observations of ion-sound waves in the interplanetary medium (Gurnett *et al.* 1979) show a peak value corresponding to $W^{\rm s}/n_{\rm e}\,T_{\rm e} \approx 10^{-8}$. There is little direct information on the $|\boldsymbol{k}'|$ spectrum of the ion-sound turbulence. However, values $|\boldsymbol{k}'|\,\lambda_{\rm De} \approx 0{\cdot}03$ are likely to be present. It is reasonable to conclude that the threshold level is exceeded some of the time for type III bursts in the interplanetary medium.

Note that emphasis is placed here on ion-sound waves with $|\boldsymbol{k}'| \approx \omega_{\rm p}/v$. These waves combine with the resonant Langmuir waves (below the threshold for suppression of the Langmuir instability) to produce small $|\boldsymbol{k}|$ transverse waves or Langmuir waves. Suppression of the Langmuir instability can also occur due to ion-sound waves with $|\boldsymbol{k}'| \gg \omega_p/v$. These combine with the resonant Langmuir waves to produce large $|\boldsymbol{k}|$ Langmuir waves ($|\boldsymbol{k}| \approx |\boldsymbol{k}'|$). The effects of the ion-sound waves with $|\boldsymbol{k}'| \approx \omega_p/v$ dominate provided that the energy density W^s in the ion-sound waves satisfies $\mathrm{d}(W^s/|\boldsymbol{k}'|^2)/\mathrm{d}|\boldsymbol{k}'| < 0$ for $|\boldsymbol{k}'| \gtrsim \omega_p/v$, which is assumed to be the case here.

Generation of small $|\boldsymbol{k}|$ transverse waves or Langmuir waves can also occur through turbulent bremsstrahlung. For a stream of fast electrons the growth rate for this process is much smaller than that for the double-emission process (Appendix 1). Thus turbulent bremsstrahlung may be neglected here.

The possible application of the double-emission process to Type III bursts appears favourable in several ways, but also encounters a number of difficulties. Favourable aspects of the process are that it does indeed allow 'plasma emission' to occur without Langmuir waves necessarily being involved. It is particularly interesting that the conditions for it to occur seem likely to be satisfied at least some of the time. A further favourable aspect is that the threshold condition (46) on the ion-sound waves is most easily satisfied at the front of the stream where n_1 is small and v is large. This allows the possibility that initially the radio emission is produced by the double-emission process without any Langmuir waves being generated. Later, as n_1 increases and v decreases, the instability could revert to the generation of resonant Langmuir waves. In this way, one could account for the appearance of Langmuir waves only after the appearance of radio emission.

Now let us turn to the difficulties encountered with the application of double emission to type III bursts. First, one cannot account for second-harmonic emission without invoking Langmuir waves. Second-harmonic emission certainly occurs for some bursts in the solar corona, and it is much simpler to attribute fundamental plus harmonic emission in the corona to the conventional forms of plasma emission, with the generation of the fundamental enhanced by low-frequency turbulence (see e.g. Melrose 1980*c*). There is no need to invoke double emission in the corona. Second, the hypothesis that double emission occurs in the interplanetary medium requires that the emission be at the fundamental, rather than the second harmonic as is now widely accepted. There are strong arguments for favouring fundamental emission (Melrose 1982) but these do not require that the fundamental be due to double emission. Third, the basic reason for invoking double emission is to account for plasma emission without Langmuir waves, but double emission itself should produce Langmuir waves. This occurs due to the process $\mathrm{e} \rightarrow \mathrm{e}' + \mathrm{s} + \mathrm{l}$, with the small $|\boldsymbol{k}|$ ($\ll \omega_p/v$) Langmuir waves growing at the same rate as do transverse waves. Thus although the production of resonant Langmuir waves can be suppressed by the ion-sound waves, it is difficult to see how one can argue that double emission will produce transverse waves without also producing observable nonresonant Langmuir waves.

In view of these difficulties it seems unlikely that 'plasma emission' can occur in the absence of detectable Langmuir waves. However, it seems possible that in the presence of ion-sound turbulence a type III stream may generate transverse waves and nonresonant (small $|\boldsymbol{k}|$) Langmuir waves, but no resonant Langmuir waves. If this alternative bump-in-the-tail instability is important then

(1) the fundamental 'plasma emission' is generated directly by the stream;

(2) the nonresonant Langmuir waves also generated by the stream should be distributed roughly isotropically, and should appear at the same time as the fundamental;

(3) the small $|\boldsymbol{k}|$ Langmuir waves generated directly by the stream can more easily undergo Langmuir collapse than would an equivalent energy density in resonant Langmuir waves.

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Postscript

After this paper was written, an alternative treatment of the plasma processes by Goldman and DuBois (1981) has cast further doubt on the viability of the postulated growth of transverse waves due to the processes $\mathrm{e} \rightarrow \mathrm{e}'+\mathrm{s}+\mathrm{t}$ and $\mathrm{e}+\mathrm{s} \rightarrow \mathrm{e}'+\mathrm{t}$ in type III bursts in the interplanetary medium. Goldman and DuBois included the damping of the ion-sound waves, which is neglected in the present paper, and found that the net absorption coefficient for the transverse waves is the sum of the (negative) value (42) and the (positive) Landau damping rate of the ion-sound waves. The latter is the much larger in magnitude, implying that the net effect is one of damping. Thus the growth of transverse waves due to the instability discussed in Section 4 above evidently occurs only if the low-frequency waves have an intrinsic damping rate much less than that of ion-sound waves, and less than the value (42) in magnitude. Goldman (personal communication) also argued that for a spectrum of ion-sound waves the k values ($k \approx \omega_{\mathrm{p}}/v$) which contribute to growth can be more than offset by k values leading to damping, i.e. to $\gamma^{\mathrm{st}} > 0$.

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Appendix 1. Turbulent Bremsstrahlung

Turbulent bremsstrahlung, and also the scattering and double-emission processes, may be treated as nonlinear damping processes. The absorption coefficient is given in general by (cf. Melrose 1980*a*; p. 47)

$$\gamma^{\sigma}(\boldsymbol{k}) = -\{2\mathrm{i}\,\mu_0\, c^2/\omega^{\sigma}(\boldsymbol{k})\}\, R_{\mathrm{E}}^{\sigma}(\boldsymbol{k})\, e_i^{*\sigma}(\boldsymbol{k})\, e_j^{\sigma}(\boldsymbol{k})\, \alpha_{ij}^{(\mathrm{A})}(\boldsymbol{k}, \omega^{\sigma}(\boldsymbol{k}))\,, \tag{A1}$$

where $\alpha_{ij}^{(\mathrm{A})}$ is the anti-hermitian part of the linear response tensor (cf. equation 10). Nonlinear damping is treated by including the nonlinear response tensor $\alpha_{ij}^{\mathrm{NL}}$ in (A1). One contribution, $\alpha_{ij}^{\mathrm{NL}}$, arises from the cubic response in (10) with $k_3 = -k_1$, $k_2 = k$. If we write

$$A^{\sigma'}(k') = \boldsymbol{e}^{\sigma'}(\boldsymbol{k}')\exp\{\mathrm{i}\,\psi^{\sigma'}(\boldsymbol{k}')\}\left(\frac{\hbar V\, R_{\mathrm{E}}^{\sigma'}(\boldsymbol{k}')\, N^{\sigma'}(\boldsymbol{k}')}{\varepsilon_0\,|\,\omega^{\sigma'}(\boldsymbol{k}')\,|}\right)^{\frac{1}{2}} 2\pi\,\delta(\omega' - \omega^{\sigma'}(\boldsymbol{k}))\,, \tag{A2}$$

where V is the normalization volume, an average over phases $\psi^{\sigma'}(\boldsymbol{k}')$ gives

$$\langle A_i^{\sigma'}(k')\, A_j^{\sigma'}(k'')\rangle = \{\mu_0 \hbar c^2/\omega^{\sigma'}(k')\}\, R_{\mathrm{E}}^{\sigma'}(\boldsymbol{k}')\, N^{\sigma'}(\boldsymbol{k}')\, e_i^{\sigma'}(\boldsymbol{k}')\, e_j^{*\sigma'}(\boldsymbol{k}')$$
$$\times\, 2\pi\,\delta(\omega' - \omega^{\sigma'}(\boldsymbol{k}'))(2\pi)^4\delta^4(k' + k'')\,, \tag{A3}$$

and we have

$$\alpha_{ij}^{\mathrm{NL1}}(k) = \frac{\hbar}{\varepsilon_0}\int \frac{\mathrm{d}^3 k'}{(2\pi)^3}\, 3\alpha_{irjs}(k, k', k, -k')\, e_r^{\sigma'}(\boldsymbol{k})\, e_s^{*\sigma'}(\boldsymbol{k})\frac{R_{\mathrm{E}}^{\sigma'}(\boldsymbol{k}')}{|\,\omega^{\sigma'}(\boldsymbol{k}')\,|}\, N^{\sigma'}(\boldsymbol{k}')\,. \tag{A4}$$

A second contribution, $\alpha_{ij}^{\mathrm{NL2}}(k)$, arises from the quadratic response of $A^{\sigma'}(k')$ to the beat between $A^{\sigma}(k)$ and $A^{\sigma'}(k)$. After averaging over phases using (A2) and (A3) one finds

$$\alpha_{ij}^{\mathrm{NL2}}(k) = \frac{\hbar}{\varepsilon_0}\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\,2\alpha_{ira}(k,k',k-k')\left(\frac{-\mu_0 c^2}{(\omega-\omega')^2}\right)\frac{\lambda_{ab}(k-k')}{\Lambda(k-k')}$$

$$\times\, 2\alpha_{bjs}(k-k',k,-k')\,e_r^{\sigma'}(\boldsymbol{k})\,e_s^{*\sigma'}(\boldsymbol{k}')\,\frac{R_{\mathrm{E}}^{\sigma'}(\boldsymbol{k}')}{|\omega^{\sigma'}(\boldsymbol{k}')|}\,N^{\sigma'}(\boldsymbol{k}'). \qquad \text{(A5)}$$

The factors 3 in (A4) and 2 in (A5) arise from the symmetrization indicated in (22), and are to be omitted when appropriate unsymmetrized forms of (20) and (19) respectively are used.

To evaluate the anti-hermitian part of $\alpha_{ij}^{\mathrm{NL1}}(k)$ we start from (20) and partially integrate, using the identity (cf. equations 8 and 21)

$$a_{ij}^{\mathrm{TS}}(k,k_1,\boldsymbol{v}) = \frac{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}{\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v}}\,m\frac{\partial}{\partial p_r}\left(\frac{v_i\,g_{rj}(k_1,\boldsymbol{v})}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\right), \qquad \text{(A6)}$$

to find

$$\tilde{\alpha}_{ijlm}(k,k_1,k_2,k_3) = -\frac{q^4}{m^2}\int\mathrm{d}^3p\,\frac{(\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v})\,a_{ij}^{\mathrm{TS}}(k,k_1,\boldsymbol{v})}{(\omega-\boldsymbol{k}\cdot\boldsymbol{v})\{\omega_2+\omega_3-(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{v}\}}$$

$$\times\left(\frac{\omega_2-\boldsymbol{k}_2\cdot\boldsymbol{v}}{\omega_3-\boldsymbol{k}_3\cdot\boldsymbol{v}}\,a_{lm}^{\mathrm{TS}}(k_2,k_3,\boldsymbol{v})\,\boldsymbol{k}_3\cdot\frac{\partial f(\boldsymbol{p})}{\partial\boldsymbol{p}}\right.$$

$$\left.+\,m\,g_{sl}(k_2,\boldsymbol{v})\,g_{tm}(k_3,\boldsymbol{v})\frac{\partial^2 f(\boldsymbol{p})}{\partial p_s\,\partial p_t}\right). \qquad \text{(A7)}$$

The denominators are to be evaluated in the usual way by giving ω an infinitesimal imaginary part and using

$$\frac{1}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}+\mathrm{i}0} = \mathrm{P}\frac{1}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}} - \mathrm{i}\pi\,\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}), \qquad \text{(A8)}$$

where P denotes the Cauchy principal value. The anti-hermitian part arises from the semi-residue term in (A8). In treating turbulent bremsstrahlung the relevant contribution is from the denominator $\omega_3-\boldsymbol{k}_3\cdot\boldsymbol{v}$ in (A7) and thence in (A4); there is no relevant contribution from $\alpha_{ij}^{\mathrm{NL1}}$. On retaining only the relevant resonant part and setting $k_3 = -k_1$, one finds the contribution to turbulent bremsstrahlung (TB) from (A7):

$$\tilde{\alpha}_{ijlm}^{\mathrm{TB}}(k,k_1,k,-k_1) = \frac{\mathrm{i}\pi q^4}{m_{\mathrm{e}}^2}\int\frac{\mathrm{d}^3p}{\gamma^2}\,v_m\,\delta(\omega_1-\boldsymbol{k}_1\cdot\boldsymbol{v})\,\boldsymbol{k}_1\cdot\frac{\partial f(\boldsymbol{p})}{\partial\boldsymbol{p}}$$

$$\times\left[\frac{1}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\left(-\delta_{ij}k_{1l}+k_{1i}\delta_{jl}\right)+\frac{1}{(\omega-\boldsymbol{k}\cdot\boldsymbol{v})^2}\right.$$

$$\times\left\{\left(\frac{\omega\omega_1}{c^2}-\boldsymbol{k}\cdot\boldsymbol{k}_1\right)(\delta_{ij}v_l-\delta_{il}v_j-v_1\delta_{jl})+k_{1i}v_jk_l+\right.$$

$$+2k_{1i}v_jk_l - v_ik_jk_{1l}\Big\} + \frac{1}{(\omega - \boldsymbol{k}\,.\,\boldsymbol{v})^3}$$

$$\times\left\{-2\left(\frac{\omega^2}{c^2} - |\boldsymbol{k}|^2\right)k_{1i}v_jv_l - \left(\frac{\omega\omega_1}{c^2} - \boldsymbol{k}\,.\,\boldsymbol{k}_1\right)(k_iv_jv_l - 3v_iv_jk_l)\right\}$$

$$+\frac{1}{(\omega - \boldsymbol{k}\,.\,\boldsymbol{v})^4}\left\{3\left(\frac{\omega^2}{c^2} - |\boldsymbol{k}|^2\right)\left(\frac{\omega\omega_1}{c^2} - \boldsymbol{k}\,.\,\boldsymbol{k}_1\right)v_iv_jv_l\right\}\Bigg]. \qquad \text{(A9)}$$

Only terms even in k_1 are retained in (A9) (cf. Tsytovich *et al.* 1975). Now inserting (A4) in (A1) and identifying the relevant part of $3\alpha_{irjs}$ as $\tilde{\alpha}^{\mathrm{TB}}_{irjs}$ one finds the absorption coefficient for turbulent bremsstrahlung:

$$\gamma^{\mathrm{TB}\sigma}(k) = -2\mathrm{i}\frac{\hbar}{\varepsilon_0^2}\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\frac{R^{\sigma}_{\mathrm{E}}(\boldsymbol{k})\,R^{\sigma'}_{\mathrm{E}}(\boldsymbol{k}')}{|\,\omega^{\sigma}(\boldsymbol{k})\,\omega^{\sigma'}(\boldsymbol{k}')\,|}N^{\sigma'}(\boldsymbol{k}')$$

$$\times\, e_i^{*\sigma}(\boldsymbol{k})\,e_j^{\sigma}(\boldsymbol{k})\,e_r^{\sigma'}(\boldsymbol{k}')\,e_s^{*\sigma'}(\boldsymbol{k}')\,\tilde{\alpha}^{\mathrm{TB}}_{irjs}(k,k',k,-k'). \qquad \text{(A10)}$$

with k and k' now denoting $\boldsymbol{k}, \omega^{\sigma}(\boldsymbol{k})$ and $\boldsymbol{k}', \omega^{\sigma'}(\boldsymbol{k}')$ respectively.

Let us first use (A10) with (A9) to reproduce the existing results (Tsytovich *et al.* 1975; Kuijpers 1980*a*) for the turbulent bremsstrahlung of Langmuir waves off ion-sound waves. In this case the waves are longitudinal ($\boldsymbol{e}^{\sigma}(\boldsymbol{k}) = \boldsymbol{\kappa}$, $\boldsymbol{e}^{\sigma'}(\boldsymbol{k}') = \boldsymbol{\kappa}'$) and in (A10) we have

$$\kappa_i\kappa_j\kappa'_r\kappa'_s\tilde{\alpha}^{\mathrm{TB}}_{irjs}(k,k',k,-k') = -\mathrm{i}\pi\frac{q^4}{m^2}\frac{\omega^2\omega'^2}{|\boldsymbol{k}|^2|\boldsymbol{k}'|^2}$$

$$\times\int\frac{\mathrm{d}^3p}{\gamma^2}\,\delta(\omega' - \boldsymbol{k}'\,.\,\boldsymbol{v})\,\boldsymbol{k}'\,.\,\frac{\partial f(\boldsymbol{p})}{\partial\boldsymbol{p}}\,\frac{1}{(\omega - \boldsymbol{k}\,.\,\boldsymbol{v})^4}\Bigg\{3|\boldsymbol{k}|^2\boldsymbol{k}\,.\,\boldsymbol{k}'$$

$$-\frac{\omega\omega'}{c^2}|\boldsymbol{k}|^2 - \frac{2\boldsymbol{k}\,.\,\boldsymbol{v}}{c^2}(\omega\boldsymbol{k}\,.\,\boldsymbol{k}' + \omega'|\boldsymbol{k}|^2) + \left(\frac{\boldsymbol{k}\,.\,\boldsymbol{v}}{c}\right)^2\left(\frac{3\omega\omega'}{c^2} - \boldsymbol{k}\,.\,\boldsymbol{k}'\right)\Bigg\}. \qquad \text{(A11)}$$

The existing results are reproduced in the nonrelativistic limit, which corresponds to $\gamma = 1$ and $c = \infty$ in the integrand in (A11). The nonrelativistic and longitudinal approximations were made at the outset by Tsytovich *et al.* (1975) and Kuijpers (1980*a*). The relativistic corrections, derived here for the first time, turn out not to be important under the conditions envisaged in proposed applications of turbulent bremsstrahlung.

For transverse waves we average over the two states of polarization. Then in place of (A11) we have

$$\tfrac{1}{2}(\delta_{ij} - \kappa_i\kappa_j)\kappa'_r\kappa'_s\tilde{\alpha}^{\mathrm{TB}}_{irjs}(k,k',k,-k')$$

$$= \frac{\mathrm{i}\pi q^4}{m^2}\frac{\omega^2\omega'^2}{|\boldsymbol{k}|^2|\boldsymbol{k}'|^2}\int\frac{\mathrm{d}^3p}{\gamma^2}\,\delta(\omega' - \boldsymbol{k}'\,.\,\boldsymbol{v})\,\boldsymbol{k}'\,.\,\frac{\partial f(\boldsymbol{p})}{\partial\boldsymbol{p}}\,\frac{1}{(\omega - \boldsymbol{k}\,.\,\boldsymbol{v})^4}$$

$$\times\,(|\boldsymbol{k}|^2\boldsymbol{k}\,.\,\boldsymbol{k}' + \text{relativistic corrections}), \qquad \text{(A12)}$$

where the 'relativistic corrections' are terms proportional to c^{-2}. As for Langmuir waves, the relativistic corrections are not important. If we neglect them, the absorption coefficient from transverse waves due to turbulent bremsstrahlung is just one third that for Langmuir waves. The absorption coefficient is

$$\gamma_{\mathrm{TB}}^{\mathrm{t}}(\boldsymbol{k}) = \frac{(2\pi)^3 e^4}{(4\pi\varepsilon_0)^2 m_{\mathrm{e}}^2 \omega_{\mathrm{pi}}^2 \{\omega^{\mathrm{t}}(\boldsymbol{k})\}^3} \int \frac{\mathrm{d}^3 k'}{(2\pi)^3} \int \mathrm{d}^3 p \, \frac{\boldsymbol{k} \cdot \boldsymbol{k}'}{|\boldsymbol{k}'|^2} \times T^{\mathrm{s}}(\boldsymbol{k}') \{\omega^2(\boldsymbol{k}')\}^2 \, \delta(\omega^{\mathrm{s}}(\boldsymbol{k}') - \boldsymbol{k}' \cdot \boldsymbol{v}) \, \boldsymbol{k}' \cdot \partial f(\boldsymbol{p})/\partial \boldsymbol{p}, \tag{A13}$$

where the nonrelativistic approximation has been made.

An order of magnitude comparison between the growth rates for turbulent bremsstrahlung and for double emission leads to

$$\frac{\gamma_{\mathrm{TB}}^{\mathrm{t}}}{\gamma^{\mathrm{st}}} \approx \left(\frac{|\boldsymbol{k}|c}{\omega_{\mathrm{p}}}\right)\left(\frac{\gamma_{\mathrm{eff}}^{\mathrm{l}}}{\omega_{\mathrm{p}}}\right)\frac{V_{\mathrm{e}}^6}{cv^5}\,\frac{(|\boldsymbol{k}'|^4 T^{\mathrm{s}} A^{\mathrm{s}})_{\mathrm{TB}}}{(\omega_{\mathrm{p}}/v)^4 \bar{T}^{\mathrm{s}} A^{\mathrm{s}}}, \tag{A14}$$

where $(k'^4 T^{\mathrm{s}} A^{\mathrm{s}})_{\mathrm{TB}}$ denotes the maximum value of this quantity. (All ion-sound waves, and not just those with $|\boldsymbol{k}'| \approx \omega_{\mathrm{p}}/v$, contribute to turbulent bremsstrahlung.) Although the last factor in (A14) may be large, all the other factors are small, and one has $|\gamma_{\mathrm{TB}}^{\mathrm{t}}| \ll |\gamma^{\mathrm{st}}|$ for a bump-in-the-tail distribution of electrons.

Appendix 2. Collective-medium Treatment of Scattering and Double Emission

The method used in Appendix 1 may be used to derive the effective absorption coefficient due to the scattering and double-emission processes. The relevant effective absorption coefficient from the single-particle approach follows directly from (15):

$$\gamma^{\sigma}(\boldsymbol{k}) = \int \mathrm{d}^3 p \int \frac{\mathrm{d}^3 k'}{(2\pi)^3} \sum_{\pm} w_{\pm}^{\sigma\sigma'}(\boldsymbol{p},\boldsymbol{k},\boldsymbol{k}') N^{\sigma'}(\boldsymbol{k}') \hbar(\boldsymbol{k} \mp \boldsymbol{k}') \cdot \partial f(\boldsymbol{p})/\partial \boldsymbol{p}. \tag{A15}$$

We now re-derive (A15) starting from (A1) with (A4) and (A5).

The relevant anti-hermitian part of $\alpha_{ij}^{\mathrm{NL1}}$ will be called the Thomson scattering (TS) part, for reasons which will become evident. This part arises from the resonance in the denominator $\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3) \cdot \boldsymbol{v}$ in (A7). We have

$$\tilde{\alpha}_{ijlm}^{\mathrm{TS}}(k, k', k, -k') = \frac{\mathrm{i}\pi q^4}{m^2} \int \mathrm{d}^3 p \, a_{ij}^{\mathrm{TS}}(k, k', \boldsymbol{v}) \times a_{lm}^{\mathrm{TS}}(k, k', \boldsymbol{v}) \, \delta(\omega - \omega' - (\boldsymbol{k} - \boldsymbol{k}') \cdot \boldsymbol{v})(\boldsymbol{k} - \boldsymbol{k}') \cdot \partial f(\boldsymbol{p})/\partial \boldsymbol{p}. \tag{A16}$$

If we insert (A16) in (A4) and then in (A1), there is a contribution from (A16) as written with $\omega' > 0$ and another contribution with $\omega' < 0$. The negative-frequency part may be rewritten by replacing k' by $-k'$ with the new $\omega' > 0$. Then the former contribution describes scattering and the latter describes double emission. Direct comparison then shows that the result (A15) is reproduced, but with only the Thomson scattering term in (5) with (6) and (7).

Now consider the contribution from $\alpha_{ij}^{\mathrm{NL2}}$, given by (A5). There are three resonant denominators which contribute in this case. One arises from the photon propagator

λ_{ab}/Λ. Including the anti-hermitian part of the linear response tensor (18), i.e. the part

$$\alpha_{ij}^{(\mathrm{A})}(k) = -\mathrm{i}\pi q^2 \int \mathrm{d}^3p\, v_i v_j\, \delta(\omega - \boldsymbol{k}.\boldsymbol{v})\, \boldsymbol{k}.\partial f(\boldsymbol{p})/\partial \boldsymbol{p}, \tag{A17}$$

one finds, to first order in dissipative terms,

$$\lambda_{ij}^{(\mathrm{A})}(k) = \frac{\mu_0 c^2}{\omega^2}\varepsilon_{irm}\varepsilon_{jsn}\alpha_{sr}^{(\mathrm{A})}(k)\,\Lambda_{nm}(k), \tag{A18}$$

$$\mathrm{Im}\left(\frac{1}{\Lambda(k)}\right) = -\frac{\mu_0 c^2}{\omega^2}\frac{\lambda_{rs}(k)\,\alpha_{sr}^{(\mathrm{A})}(k)}{\{\Lambda(k)\}^2}, \tag{A19}$$

where 'Im' denotes the imaginary part. Together (A18) and (A19) imply

$$\left(\frac{\lambda_{ij}(k)}{\Lambda(k)}\right)^{(\mathrm{A})} = -\frac{\mu_0 c^2}{\omega^2}\frac{\lambda_{is}(k)\,\lambda_{rj}(k)}{\Lambda^2(k)}\alpha_{sr}^{(\mathrm{A})}(k). \tag{A20}$$

In deriving (A18)–(A20), the facts that Λ is the determinant of Λ_{ij} and that λ_{ij} is the cofactor of Λ_{ji} have been used, together with a matrix identity

$$\lambda_{ij}\lambda_{rs} = \lambda_{is}\lambda_{rj} + \Lambda\varepsilon_{irm}\varepsilon_{jsn}\Lambda_{nm}. \tag{A21}$$

On retaining this contribution from $\alpha_{ij}^{\mathrm{NL1}}$ in (A1), one finds that (A15) is reproduced, but now with only the nonlinear-scattering term in (5) with (6) and (7). To complete the re-derivation of (A15), we need the cross terms between Thomson scattering and nonlinear scattering. These arise from resonant denominators in α_{ira} and α_{bjs} in (A5). On partially integrating, we find (19) gives

$$\tilde{\alpha}_{ijl}(k, k_1, k_2) = -\frac{q^2}{m}\int \mathrm{d}^3p\,\frac{\omega_1 - \boldsymbol{k}_1.\boldsymbol{v}}{(\omega - \boldsymbol{k}.\boldsymbol{v})(\omega_2 - \boldsymbol{k}_2.\boldsymbol{v})}$$
$$\times\, a_{ij}^{\mathrm{TS}}(k, k_1, \boldsymbol{v})\, g_{sl}(k_2, \boldsymbol{v})\,\partial f(\boldsymbol{p})/\partial p_s, \tag{A22}$$

where (A6) has been used. The relevant resonances arise from the zeros of $\omega_2 - \boldsymbol{k}_2.\boldsymbol{v}$ and $\omega - \boldsymbol{k}.\boldsymbol{v}$ in the cases of α_{ira} and α_{bjs} respectively in (A5).

In summary, (A1) reproduces (A15) with (i) the Thomson-scattering term in (5), with (6)–(9), reproduced by the resonance in the denominator $\omega_2 + \omega_3 - (\boldsymbol{k}_2 + \boldsymbol{k}_3).\boldsymbol{v}$ in (A4) with (A7); (ii) the nonlinear-scattering term reproduced by the resonance in the photon propagator in (A5); (iii) the cross terms between Thomson and non-linear scattering reproduced by resonances in the quadratic response tensor (A22) in (A5) (cf. Tsytovich 1977; pp. 79–81). The terms with $\omega' > 0$ in (A4) and (A5) are interpreted as describing the scattering process, and those with $\omega' < 0$ as describing the double-emission process, in accord with the crossing symmetry (4).

Two remarks on (A20) are appropriate. First, an expansion in the ratio of the anti-hermitian part to the hermitian part is performed, and in this context $\Lambda(k)$ in the denominator in (A20) is real. Alternatively, one could include $\alpha_{ij}^{(\mathrm{A})}(k)$ in $\Lambda_{ij}(k)$ and then $\Lambda(k)$ itself is complex. When we make the longitudinal approximation in the latter case, the factor $\Lambda^2(k)$ is replaced by $\{\varepsilon^{\mathrm{l}}(k)\}^2$, with $\varepsilon^{\mathrm{l}}(k)$ including both the real and imaginary parts, as in (33). Second, the prescription used in deriving (A20) is consistent with and a generalization of the prescriptions used by Tsytovich (1977; pp. 79–81) and Davidson (1972; Appendix E). However, it appears to be different from a prescription used by Akhiezer *et al.* (1975; p. 88) and also by Rönnmark (1977). The apparent inconsistency in the literature requires further investigation.

Appendix 3. Three-wave Processes

The evolution of resonant Langmuir waves (k'') due to three-wave processes involving an ion-sound wave s and another wave, which is either a transverse wave or a small $|\boldsymbol{k}|$ Langmuir wave, may be described by the kinetic equation (see e.g. Melrose 1980*a*; p. 173)

$$\frac{\mathrm{d}N^{\mathrm{l}}(\boldsymbol{k}'')}{\mathrm{d}t} = -\int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\sum_{\pm} u_{\pm}^{\mathrm{l}\sigma\mathrm{s}}(\boldsymbol{k}'',\boldsymbol{k},\boldsymbol{k}')$$
$$\times\{N^{\mathrm{l}}(\boldsymbol{k}'')N^{\mathrm{s}}(\boldsymbol{k}')\mp N^{\mathrm{l}}(\boldsymbol{k}'')N^{\sigma}(\boldsymbol{k})-N^{\mathrm{s}}(\boldsymbol{k}')N^{\sigma}(\boldsymbol{k})\}, \tag{A23}$$

with (cf. Melrose 1980*c*)

$$u_{\pm}^{\mathrm{l}\sigma\mathrm{s}}(\boldsymbol{k}'',\boldsymbol{k},\boldsymbol{k}') = \frac{(2\pi)^5}{2(4\pi\varepsilon_0)}\frac{\hbar e^2}{m_{\mathrm{e}}^2}\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{pi}}^2}\frac{\{\omega^{\mathrm{s}}(\boldsymbol{k}')\}^3}{k'^2V_{\mathrm{e}}^4}R_{\mathrm{E}}^{\sigma}(\boldsymbol{k})\,|\,\boldsymbol{e}^{*\sigma}(\boldsymbol{k})\,.\,\boldsymbol{\kappa}''\,|^2$$
$$\times\,\delta^3(\boldsymbol{k}-\boldsymbol{k}''\mp\boldsymbol{k}')\,\delta(\omega^{\sigma}(\boldsymbol{k})-\omega^{\mathrm{l}}(\boldsymbol{k}'')\mp\omega^{\mathrm{s}}(\boldsymbol{k}')). \tag{A24}$$

We retain only the leading term $N^{\mathrm{l}}(\boldsymbol{k}'')N^{\mathrm{s}}(\boldsymbol{k}')$ in (A23) in evaluating the nonlinear absorption coefficient for the Langmuir waves. In addition we assume $|\,\omega^{\mathrm{l}}(\boldsymbol{k}'')-\omega^{\sigma}(\boldsymbol{k})\,|\gg\omega^{\mathrm{s}}(\boldsymbol{k}')$. On adding the effects of the processes $\mathrm{l+s\to t}$, $\mathrm{l\to t+s}$, $\mathrm{l+s\to l'}$ and $\mathrm{l\to l'+s}$, one finds

$$\gamma_{\mathrm{NL}}^{\mathrm{l}}(k'') = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3k'}{(2\pi)^3}\sum_{\pm}\sum_{\sigma=\mathrm{t,l'}} u_{\mp}^{\mathrm{l}\sigma\mathrm{s}}(\boldsymbol{k}'',\boldsymbol{k},\boldsymbol{k}')\,N^{\mathrm{s}}(\boldsymbol{k}')$$
$$\approx\frac{2}{9}\frac{r_0\,c^2|\,\boldsymbol{k}''\,|\,\omega_{\mathrm{p}}}{V_{\mathrm{e}}^2}\frac{\overline{T}^{\mathrm{s}}}{T_{\mathrm{e}}}, \tag{A25}$$

with $\overline{T}^{\mathrm{s}}$ given by (35). For resonant Langmuir waves, $|\,\boldsymbol{k}''\,|\approx\omega_{\mathrm{p}}/v$ in (A25) leads to (40).

The kinetic equation for waves in mode σ due to the three-wave processes is

$$\frac{\mathrm{d}N^{\sigma}(\boldsymbol{k})}{\mathrm{d}t} = \int\frac{\mathrm{d}^3k'}{(2\pi)^3}\int\frac{\mathrm{d}^3k''}{(2\pi)^3}\sum_{\pm} u_{\pm}^{\mathrm{l}\sigma\mathrm{s}}(\boldsymbol{k}'',\boldsymbol{k},\boldsymbol{k}')$$
$$\times\{N^{\mathrm{l}}(\boldsymbol{k}'')N^{\mathrm{s}}(\boldsymbol{k}')\mp N^{\mathrm{l}}(\boldsymbol{k}'')N^{\sigma}(\boldsymbol{k})-N^{\mathrm{s}}(\boldsymbol{k}')N^{\sigma}(\boldsymbol{k})\}. \tag{A26}$$

For $N^{\mathrm{s}}(\boldsymbol{k}')\ll N^{\mathrm{l}}(\boldsymbol{k}'')$, equation (A26) implies a nonlinear damping with effective absorption coefficient

$$\gamma_{\mathrm{NL}}^{\sigma}(\boldsymbol{k}) = \int\frac{\mathrm{d}^3k'}{(2\pi)^3}\int\frac{\mathrm{d}^3k''}{(2\pi)^3}\sum_{\pm} u_{\pm}^{\mathrm{l}\sigma\mathrm{s}}(\boldsymbol{k}'',\boldsymbol{k},\boldsymbol{k}')\,N^{\mathrm{s}}(\boldsymbol{k}'). \tag{A27}$$

Formula (A27) may be re-derived from (A1) by retaining only the anti-hermitian part which arises from the resonance in (A5) at the dispersion relation for Langmuir waves. Quite generally, for $k-k' = k''$ and $\omega'' \approx \omega^{\sigma''}(\boldsymbol{k}'')$ for any mode σ'', we have

$$\Lambda(k'') \approx (\omega'' - \omega^{\sigma''}(\boldsymbol{k}'') + \mathrm{i}\,0)\{\partial\Lambda(k'')/\partial\omega''\}_{\omega''=\omega^{\sigma''}(k'')} . \tag{A28}$$

The relevant anti-hermitian part of (A5) then arises from

$$\left(\frac{1}{(\omega-\omega')^2}\frac{\lambda_{ab}(k-k')}{\Lambda(k-k')}\right)^{\mathrm{A}} = -\mathrm{i}\pi\int\frac{\mathrm{d}^3k}{(2\pi)^3}\,(2\pi)^3\,\delta^3(\boldsymbol{k}-\boldsymbol{k}'-\boldsymbol{k}'')$$

$$\times\frac{R_{\mathrm{E}}^{\sigma''}(\boldsymbol{k}'')}{|\,\omega^{\sigma''}(\boldsymbol{k}'')\,|}\,e_a^{\sigma''}(\boldsymbol{k}'')\,e_b^{*\sigma''}(\boldsymbol{k}'')\,\delta(\omega-\omega'-\omega^{\sigma''}(\boldsymbol{k}'')) , \tag{A29}$$

which includes the negative-frequency solution $\omega'' = \omega^{\sigma''}(-\boldsymbol{k}'')$ implicitly. With $\sigma'' = 1$, equation (A29) in (A5) and thence in (A1) reproduces (A27).

Note that the resonant part (A27) is derived by ignoring the anti-hermitian part of $\alpha_{ij}(k)$ and by giving ω an infinitesimal imaginary part in accord with the causal condition. This prescription is essentially that used by Akhiezer *et al.* (1975; p. 88). However, they used the prescription in place of (A20) in treating scattering and double emission (referred to as nonlinear Landau damping). As remarked at the end of Appendix 2, an apparent inconsistency exists in the literature on this point. What has been shown here is that the prescriptions (A20) and (A27) used as they are here, reproduce the results of an independent method of calculation.

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