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Doppler Shift of Radio Signals Transmitted between Orbiting Satellites

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Abstract

A general expression, applicable at VHF and above, is derived for the Doppler shift of radio signals transmitted between two satellites embedded in the ionosphere. The Doppler shift is made up of several contributions which depend on (a) the rate of change of the free space path between the satellites, (b) the components, perpendicular to the line of sight between the satellites, of both the mean velocity of the satellites and the electron concentration gradients, (c) the moment of the perpendicular electron concentration gradients and the deviations from the mean of the individual satellite perpendicular velocities, (d) the velocity components along the line of sight between the satellites, and the electron concentration values at each satellite, and (e) changes occurring in the ionosphere with time. On applying the expression to the case of a plane stratified ionosphere it is found that the Doppler shift can be used to determine the electron density gradient in the direction of the average velocity vector of the two satellites. In the case of wave-like ionospheric irregularities, Doppler measurements could be used with direct measurement techniques to give the component of the irregularity wave vector in the plane containing the satellites and their average velocity vector. The Space Shuttle could readily be used either to launch or tether satellites in similar orbits in order to make Doppler measurements between the Shuttle and the satellites. Orbits covering the altitude range 250–400 km would be most suitable since this covers the region of the F_2 peak of the ionosphere where relatively intense spatial structures occur.

1. Introduction

Ground-based measurements of the Doppler shift of radio signals transmitted from satellites have been used for many years to study properties of the ionosphere (see e.g. Ross 1960; de Mendonça 1962; Al'pert 1964; Evans and Holt 1973; Tyagi 1974; Gay and Grossi 1975). Current interest in anomalies in the Earth's gravitational field has led to proposals to use the transmission of radio signals between orbiting satellites as a means of detecting gravitational anomalies (Schwarz 1970; Weiffenbach and Grossi 1976). Such transmissions could also be used for ionospheric studies and in fact ionospheric effects must be identified for accurate determination of gravitational anomalies.

Gay and Grossi (1975) have reported ionospheric results obtained using VHF transmissions between the Apollo and Soyuz spacecraft. In the present paper we derive an expression for the Doppler shift of radio signals transmitted between two orbiting vehicles. The expression gives physical insight into the various factors contributing to the Doppler shift and it is applicable at VHF and above.

A basic starting point in any discussion of radio wave propagation is to consider the phase path P defined by

$$P = \int_A^B \mu \cos\alpha \, \mathrm{d}s, \tag{1}$$

where μ is the refractive index, s is the arc length along the ray from A to B and α is the angle between the ray and wave-normal directions. If the phase path is affected by the change in some parameter u, then the first order perturbation of the ray can be expressed as (Bennett 1969)

$$\delta_u P = \int_A^B \frac{\partial \mu}{\partial u} \delta u \cos\alpha \, \mathrm{d}s + \left(\mu \hat{\boldsymbol{p}} \, . \, \delta u \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}u}\right)_A^B, \tag{2}$$

where $\delta_u P = (\mathrm{d}P/\mathrm{d}u)\delta u$ and $\hat{\boldsymbol{p}}$ is a unit vector in the ray direction, and $\boldsymbol{q} = (q^1, q^2, q^3)$ is the position vector of a point on the ray. Equation (2) has been expressed in terms of variations of u rather than derivatives because the variational form is more convenient for the purposes of this paper. An application of (2) which is of particular interest here is the case when u is identified as time t. Then, taking $\delta u = 1$, the Doppler shift is given by

$$\Delta f = -\frac{f}{c}\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{f}{c}\left(\int_A^B \frac{\delta \mu}{\delta t} \cos\alpha \, \mathrm{d}s + (\mu \hat{\boldsymbol{p}} \, . \, \boldsymbol{v})_A^B\right), \tag{3}$$

where f is the frequency, c the speed of light and $\boldsymbol{v}$ the velocity of a point on the ray.

Equation (3) expresses the Doppler shift in a form readily applied to HF and MF propagation in the ionosphere (Dyson 1975, 1978). At VHF and above, another formulation of the Doppler shift is particularly useful. At these frequencies the Earth's magnetic field may be neglected and in fact the ionosphere itself may be regarded as simply producing a perturbation to the free space ray. Thus we get

$$P = P_0 + \delta_m P, \tag{4}$$

where P_0 is the free space phase path and $\delta_m P$ is the first order perturbation produced by the ionospheric medium and which can be calculated using (2). Furthermore, if the ionosphere is itself perturbed by an irregularity, we have

$$P = P_0 + \delta_m P + \delta_i P, \tag{5}$$

where $\delta_i P$ is the perturbation to the phase path due to the irregularity and which may again be determined using (2). The Doppler shift is now given by

$$\Delta f = -\frac{f}{c}\left(\frac{\mathrm{d}P_0}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\delta_m P + \frac{\mathrm{d}}{\mathrm{d}t}\delta_i P\right). \tag{6}$$

The second and third terms of (6) involve second order terms and can be derived from (2) (Dyson and Bennett 1978; Bennett and Dyson 1981). Both these terms are of the same form (Bennett and Dyson 1981) so we will designate them by $(\mathrm{d}/\mathrm{d}t)\delta_r P$ where r may be either m or i, depending on whether the background ionosphere or an irregularity is being considered. The meaning of these terms will be made clearer in discussing applications in Section 3.

2. Doppler Shift for Satellite-to-satellite Transmission at and above VHF

Bennett and Dyson (1981) used equation (6) to derive an expression for the Doppler shift in the case of a signal transmitted between a moving satellite and a stationary observer on the ground. The method given in their Appendix B is easily modified to allow for both the transmitter and receiver to be moving as is required for satellite-to-satellite transmission. The expression obtained may be written as

$$(\mathrm{d}/\mathrm{d}t)\delta_r P = (L\langle\delta\mu_r/\delta t\rangle + V^B_{\perp\mathrm{C}}\langle G_{\perp\mathrm{C}}\rangle + V^A_{\perp\mathrm{C}}\langle G'_{\perp\mathrm{C}}\rangle + V^B_{\perp\mathrm{N}}\langle G_{\perp\mathrm{N}}\rangle + V^A_{\perp\mathrm{N}}\langle G'_{\perp\mathrm{N}}\rangle + V^B_{\parallel}\mu_{rB} - V^A_{\parallel}\mu_{rA})\delta r, \tag{7}$$

where V^A and V^B are the velocities of the two satellites, μ_{rA} and μ_{rB} are the values of μ_r at A and B (the subscripts $\parallel$, $\perp$C and $\perp$N indicate respectively, components parallel to AB, orthogonal to AB but in the plane AB, and orthogonal to both AB and the plane AB), and

$$\langle G_\alpha\rangle\delta r = \frac{1}{L}\int_0^L l\frac{\partial\mu_r}{\partial q^\alpha}\delta r\,\mathrm{d}l, \qquad \alpha = \perp\mathrm{C}, \perp\mathrm{N}, \tag{8}$$

$$\langle G'_\alpha\rangle\delta r = \frac{1}{L}\int_0^L (L-l)\frac{\partial\mu_r}{\partial q^\alpha}\delta r\,\mathrm{d}l, \qquad \alpha = \perp\mathrm{C}, \perp\mathrm{N}. \tag{9}$$

Any practical configuration of satellites is likely to consist of satellites in similar orbits with closely matched velocities. It is therefore convenient to express the satellite velocities in terms of their average $\bar{V}$ given by

$$\bar{V} = \tfrac{1}{2}(V^A + V^B),$$

and we also write

$$V^B = V^A + \Delta V, \qquad \bar{\mu}_r = \tfrac{1}{2}(\mu_{rA} + \mu_{rB}), \qquad \mu_{rB} = \mu_{rA} + \Delta\mu_r.$$

Equation (7) then becomes

$$(\mathrm{d}/\mathrm{d}t)\delta_r P = (L\langle\partial\mu_r/\partial t\rangle + \bar{V}_{\perp\mathrm{C}}\langle H_{\perp\mathrm{C}}\rangle + \Delta V_{\perp\mathrm{C}}\langle H'_{\perp\mathrm{C}}\rangle + \bar{V}_{\perp\mathrm{N}}\langle H_{\perp\mathrm{N}}\rangle + \Delta V_{\perp\mathrm{N}}\langle H'_{\perp\mathrm{N}}\rangle + \bar{V}_{\parallel}\Delta\mu_r + \bar{\mu}_r\Delta V_{\parallel})\delta r, \tag{10}$$

where

$$\langle H_\alpha\rangle\delta r = \int_0^L \frac{\partial\mu_r}{\partial q^\alpha}\delta r\,\mathrm{d}l, \qquad \langle H'_\alpha\rangle\delta r = \int_0^L \frac{l-\frac{1}{2}L}{L}\frac{\partial\mu_r}{\partial q^\alpha}\delta r\,\mathrm{d}l, \quad \alpha = \perp\mathrm{C}, \perp\mathrm{N}. \tag{11a, b}$$

The first term in (10) gives the contribution due to time changes in the ionosphere and these are often negligible for the frequencies under consideration (Bennett and Dyson 1981).

Terms of the form $\langle H_\alpha\rangle\delta r$ are the refractive index gradients in the α direction, averaged along AB and multiplied by L, whereas $\langle H'_\alpha\rangle\delta r$ represents the moment of the appropriate perpendicular gradient about the mid-point of AB and is a measure of the inhomogeneity of the perpendicular gradient. If $\bar{V}_{\perp\mathrm{C}}$ and $\bar{V}_{\perp\mathrm{N}}$ are zero, $\Delta V_{\perp\mathrm{C}}$

and $\Delta V_{\perp N}$ represent rotation of AB about its mid-point. This will produce a Doppler shift only if the moments of the perpendicular gradients about the mid-point are not zero. The term $\bar{V}_{\parallel}\,\Delta\mu_r$ arises because even if a transmitter and receiver have the same velocity along AB there is still a contribution to the Doppler shift if the refractive index is different at the end points. Finally, the term $\bar{\mu}_r\,\Delta V_{\parallel}$ gives the contribution due to any difference in the velocity of the end points.

In practical experiments, the differential Doppler shift of two harmonically related frequencies is usually measured. This has the effect of removing the $\mathrm{d}P_0/\mathrm{d}t$ term in (6) and multiplying the other terms by a factor dependent on the two frequencies. Thus our discussion applies equally well to differential Doppler measurements. Another important advantage of differential Doppler measurements is that errors due to uncertainty in the absolute values of the frequencies are diminished (Gay and Grossi 1975).

3. Applications

Plane Stratified Ionosphere

For satellites in similar orbits and separations of a few hundred kilometres or less, the smooth or average background ionosphere between the satellites may often be approximated by a plane stratified ionosphere. Although the ionosphere may often be horizontally stratified we do not make this restriction. Initially then, we consider the ionosphere to be stratified in some particular plane and to have a constant gradient perpendicular to that plane. Then we may write

$$\bar{V}\,\Delta\mu_m\,\delta m = \bar{V}_{\parallel}\frac{\partial\mu_m}{\partial q^{\parallel}}L\,\delta m\,, \qquad \langle H_\alpha\rangle\delta m = \frac{\partial\mu_m}{\partial q^{\alpha}}L\,\delta m\,.$$

Several of the terms in equation (10) are proportional to the velocity difference of the two satellites and hence this equation is considerably simplified if the two satellites have exactly the same velocity. Although considering the satellite velocities to be identical is a rather special case, it is in fact a situation which would be highly desirable in experiments designed to detect gravitational anomalies. Consequently we initially consider the two satellite velocities to be equal. In general, time changes in the medium are unimportant and so, with these assumptions, equation (10) becomes

$$(\mathrm{d}/\mathrm{d}t)\delta_m P = \{\bar{V}.\nabla(\mu_m\,\delta m)\}L\,. \tag{12}$$

Now $\mu = 1 - KN/2f^2 - \dots$, where K is a constant, N is the electron concentration and f the frequency, and thus

$$\mu_m\,\delta m = -KN/2f^2\,. \tag{13}$$

A plane stratified ionosphere with constant gradient may be represented by

$$N = N_0 + \nabla N.(\boldsymbol{q}-\boldsymbol{q}_0)\,, \tag{14}$$

where N_0 is the electron concentration at $\boldsymbol{q}_0$ and ∇N is the gradient of N. Consequently equation (12) becomes

$$(\mathrm{d}/\mathrm{d}t)\delta_m P = -(\bar{V}.\nabla N)KL/2f^2\,. \tag{15}$$

It is therefore apparent that in this case the Doppler shift due to the ionosphere is dependent on the component of the electron concentration gradient along the velocity vector of the satellites. It follows that the relative positions of the satellites are unimportant, i.e. it does not matter whether the satellites follow each other along the same orbit or travel side by side in adjacent orbits: the Doppler shift is determined by the common velocity vector of the satellite, the electron concentration gradient in the direction of this velocity vector, and the distance between the satellites. In other words, under the conditions assumed, no further information on the ionosphere is obtained by having more than two satellites in similar orbits, and with identical velocities. However, an experiment using three (or more) satellites with known and equal velocities would measure the gradient of N in the direction of the common velocity vector, with redundancy. Any inconsistencies could be ascribed to either $\Delta V_{\parallel}$ terms or the non-constancy of $V_{\parallel} N$, and we now examine these cases.

Firstly, if the satellites do not have exactly the same velocity, the ΔV terms must be included. However, if we consider ∇N to remain constant, the terms involving $\langle H'_\alpha \rangle$ will still be zero and the only additional term to be included is

$$\Delta V_{\parallel}\, \bar{\mu}_m \delta m = -\Delta V_{\parallel}\, \bar{N}(K/2f^2), \tag{16}$$

where $\bar{N}$ is the average of the electron concentrations at the two satellites. If more than two satellites are used different Doppler shifts will generally occur for the different paths between the satellites because $\Delta V_{\parallel}$ and $\bar{N}$ will usually be different for these different paths. The difference in the Doppler shifts will depend on the difference in the product $\Delta V_{\parallel}\, \bar{N}$ between different paths, so that it is not possible to determine both $\Delta V_{\parallel}$ and $\bar{N}$ but one may be determined if the other is known.

In general the additional term due to $\Delta V_{\parallel}$ may be ignored compared with the $V.\nabla N$ term if

$$|\{\bar{V}.\nabla(\mu_m \delta m)\}L| \gg |\Delta V_{\parallel}\, \bar{\mu}_m \delta m|,$$

which, using (13) and (14), reduces to

$$|\Delta V_{\parallel}/\bar{V}| \ll |\Delta N/\bar{N}|, \tag{17}$$

where ΔN is the difference in the electron concentrations at A and B. As an example, if $|\Delta V_{\parallel}/\bar{V}| \sim 1\%$ the term $\Delta V_{\parallel}\, \bar{\mu}_m \delta m$ can be ignored if $|\Delta N/\bar{N}| \sim 10\%$.

If the ionospheric gradient is not constant the $\langle H'_\alpha \rangle$ terms must be considered. However, these terms remain unimportant if

$$|\Delta V_{\perp}/\bar{V}| \ll |\langle H_{\perp}\rangle/\langle H'_{\perp}\rangle|. \tag{18}$$

It follows from equations (11) that the RHS of (18) is >1 provided the gradient does not change sign along AB. In many instances this will be true in the absence of irregularities. Thus, the $\langle H'_{\perp}\rangle$ terms may be safely ignored if

$$|\Delta V_{\perp}/\bar{V}_{\perp}| \ll 1.$$

To examine this in detail, let us suppose the perpendicular gradient in electron density at A has a value a and that this gradient changes linearly with distance l along the line of sight between A and B, so that we may write

$$(\partial \mu_m/\partial q)\delta m = a + bl,$$

where b is a constant. It follows that

$$\langle H_\alpha \rangle = aL + \tfrac{1}{2}bL^2, \qquad \langle H'_\alpha \rangle = \tfrac{1}{12}bL^2,$$

so that

$$\langle H_\alpha \rangle / \langle H'_\alpha \rangle = 6 + 12a/bL.$$

Clearly this ratio is >6 if a and b have the same sign and (18) will be easily satisfied for expected values of $|\Delta V_\perp / \bar{V}|$ even if b is much larger than a. If a and b have different signs

$$|\langle H_\alpha \rangle / \langle H'_\alpha \rangle| \geqslant 1 \quad \text{provided} \quad a/bL \geqslant -\tfrac{5}{12}.$$

This may therefore be taken as the condition under which (18) is valid for the expected values of $|\Delta V_\perp / \bar{V}|$.

One of the problems in interpreting ground based measurements of Doppler shifts of satellite signals is that the Doppler shift depends on both the total electron content and the horizontal electron concentration gradients (see e.g. Bennett and Dyson 1981). Usually the Doppler measurements are interpreted in terms of the total electron content, and horizontal gradients are either ignored or estimated by other means (Evans and Holt 1973). As has been shown here, satellite-to-satellite Doppler measurements give a measure of the background gradients in the ionosphere and could be used to correct for gradient effects near the satellite height in simultaneous satellite-to-ground Doppler measurements. Of course horizontal gradients well below the satellite may still have an important affect on the satellite-to-ground Doppler shift as has been shown elsewhere (Dyson and Bennett 1978; Bennett and Dyson 1981).

Ionospheric Irregularities

Many different types of ionospheric irregularities could be considered, however, any irregularity may be represented by a spectrum of waves. Consequently, as an illustrative example, we will consider a plane wave irregularity in a background ionosphere of constant electron concentration N_b, and note that more complicated irregularity structures could be treated by considering the appropriate spectrum of waves.

We describe the irregularity by the equation

$$\delta N_i = N_b \exp\{j(\boldsymbol{k}.\boldsymbol{q} - \Omega t + \phi)\}\delta i, \tag{19}$$

where $\boldsymbol{q} = (x, y, z)$ is the position vector, ϕ a phase term, $\boldsymbol{k} = (k_x, k_y, k_z)$ the irregularity wave vector, Ω the wave frequency and δi the fractional amplitude of the irregularity. At time t we choose a fixed coordinate system centred at the mid-point between the satellites. We choose the z-axis to be along the straight line AB between the satellites and the x-axis such that $\bar{\boldsymbol{V}}$ is in the xz plane. Note that $\bar{\boldsymbol{V}}$ is also the velocity of the mid-point between the satellites. Furthermore we choose i so that

$$\mu_i \delta i = -K\delta N_i / 2f^2,$$

where K is the same constant as in (13). Now, we have

$$\langle \partial\mu/\partial t \rangle \delta i = \frac{1}{L}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} \frac{\partial \mu}{\partial t}\delta i \,\mathrm{d}z,$$

and therefore

$$\begin{aligned}|L\langle\partial\mu_i/\partial t\rangle\delta i| &= \left|\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{jK\Omega}{2f^2}N_\mathrm{b}\exp\{j(\boldsymbol{k}.\boldsymbol{q}-\Omega t+\phi)\}\delta i\,\mathrm{d}z\right| \\ &= \left|j\frac{K\Omega N_\mathrm{b}L\delta i}{2f^2}\frac{\sin(\frac{1}{2}k_z L)}{\frac{1}{2}k_z L}\exp\{j(\phi-\Omega t)\}\right| \\ &\leqslant (K\Omega N_\mathrm{b}L/2f^2)\delta i\,.\end{aligned}$$

For the rather extreme case of an irregularity with an amplitude of 10 MHz in plasma frequency and a period of 5 min this term gives a contribution to the Doppler shift of less than $3\cdot 5\ \mu$Hz for $L = 100$ km. Consequently this term may usually be ignored.

To consider the velocity terms we first consider the case in which the two satellites have identical uniform velocities so that $\Delta V = 0$. By the choice of axes, we have $\bar{V}_y = 0$, so that

$$\bar{V}_y\langle H_y\rangle\delta i = 0\,,\qquad (\partial\mu_i/\partial x)\delta i = -jk_x M\exp(\boldsymbol{k}.\boldsymbol{q}-\Omega t+\phi)\,,$$

where the constant M equals $KN_\mathrm{b}\,\delta i/2f^2$. It follows that

$$\begin{aligned}(\bar{V}_x\langle H_x\rangle+\bar{V}_{\|}\,\Delta\mu_i)\delta i &= -j2M\{(k_x/k_z)\bar{V}_x+\bar{V}_{\|}\}\sin(\tfrac{1}{2}k_z L)\exp(\phi-\Omega t) \\ &= -j2M(\bar{\boldsymbol{V}}.\boldsymbol{k}/k_z)\sin(\tfrac{1}{2}k_z L)\exp(\phi-\Omega t)\,. \qquad (20)\end{aligned}$$

It is apparent from (20) that the form of the dependence of the amplitude of the Doppler shift variation on k_z leads to a resonance condition such that the Doppler shift is most sensitive to waves for which

$$k_z L = (2n+1)\pi\,,\qquad n = 0,1,2,\ldots, \qquad (21)$$

that is, waves for which L equals an odd number of half-wavelengths. This is to be expected from the nature of the terms on the LHS of (20); $\langle H_x\rangle$ depends on the integral of spatial gradients due to the irregularity and so maximum values will occur when a minimum in the irregularity electron concentration occurs at one satellite and a maximum at the other, i.e. when L equals an odd number of half-wavelengths. Similarly $\Delta\mu_i$, which depends on the difference in electron concentration at the satellites, is also a maximum when an irregularity maximum occurs at one satellite and a minimum at the other.

If $\mu_i\,\delta i$ is also sampled independently at one or both of the end-points, there is a phase difference with respect to the Doppler shift given by

$$\psi = \pm\tfrac{1}{2}k_z L - \tfrac{1}{2}\pi\,. \qquad (22)$$

When the resonance condition given by (21) occurs, we have

$$\psi = \pm n\pi\,,\qquad n = 0,1,2,\ldots.$$

There is also an amplitude difference which can be expressed as the ratio

$$\frac{|(\mathrm{d}/\mathrm{d}t)\delta_i P|}{|\mu_i\,\delta i|} = 2\frac{(k_x^2\,\bar{V}_x^2+k_z^2\,\bar{V}_z^2)^{\frac{1}{2}}}{k_z}\sin(\tfrac{1}{2}k_z L)\,. \qquad (23)$$

It follows that if $\bar{V}$ and L are known, combining measurements of electron concentration at one satellite with Doppler measurements on a VHF link between two satellites would enable several parameters of the wave irregularity to be determined. The electron concentration measurements give δN_i, M and $\mu_i \delta i$, while k_z can be determined from (22) and k_x from (23) provided V_x is not small. In fact k_x is most readily determined when the average velocity of the satellites is perpendicular to the straight line joining them.

If the Doppler variation is zero at successive times t_1 and t_2, then from (20) we get

$$\Delta\phi - \Omega(t_2 - t_1) = 2\pi, \tag{24}$$

where $\Delta\phi$ is the spatial phase difference between the points at which the mid-point of the line joining the two satellites is located at times t_1 and t_2. Now

$$\Delta\phi = \boldsymbol{k}\,.\,\bar{V}(t_2 - t_1), \tag{25}$$

so unless $\Omega \ll \boldsymbol{k}\,.\,\bar{V}$, Ω can be determined from (24) and (25) and k_y is the only undetermined wave parameter.

The assumption of uniform motion can, of course, only be an approximation applicable to portions of an orbit. In general $\bar{V}$ will vary with time as will k_x, k_y and k_z because at each instant they are determined with respect to a co-ordinate system centred at the mid-point between the satellites and oriented so that the z-axis is aligned along the straight line joining the satellites. Nevertheless, (20)–(23) are still valid provided the appropriate values of the parameters are used at each instant of time.

In general $\Delta V \neq 0$ and, from (10), the additional terms to be taken into account are given by

$$(\Delta V_\perp \langle H_\perp' \rangle + \bar{\mu}_i \Delta V_\parallel)\delta i$$

$$= M \frac{\Delta \boldsymbol{V}.\boldsymbol{k}}{k_z} \left\{ \left(1 - \frac{\Delta V_\parallel \, k_z}{\Delta \boldsymbol{V}.\boldsymbol{k}} \right) \frac{\sin(\frac{1}{2}k_z L)}{\frac{1}{2}k_z L} - \cos(\tfrac{1}{2}k_z L) \right\} \exp\{j(\phi - \Omega t)\}. \tag{26}$$

This contribution to the Doppler shift is in quadrature with the contribution due to the terms involving $\bar{V}$ (see equation 20).

As $\frac{1}{2}k_z L$ increases, the maxima and minima in $\sin(\frac{1}{2}k_z L)/\frac{1}{2}k_z L$ decrease in magnitude and (26) has a resonance condition given approximately by

$$\cos(\tfrac{1}{2}k_z L) = 1\,; \qquad \text{i.e.} \quad k_z L = 2n\pi, \quad n = 1, 2, \ldots. \tag{27}$$

In (26) the term enclosed by the large braces expresses the effect of the resonance conditions already discussed, i.e. the terms involving $\Delta \boldsymbol{V}.\boldsymbol{k}$ dominate when the spacing between the satellites equals an integral number of the irregularity wavelength as measured along the line joining the satellites. The multiplicative factor shows that the relative importance of (20) and (26) depends on the ratio of the ΔV and V components along the irregularity wave vector $\boldsymbol{k}$. This is to be expected since a velocity perpendicular to $\boldsymbol{k}$ represents motion along a contour of constant electron density.

4. Conclusions

A general expression has been derived for the Doppler shift for radio signals at VHF and above, transmitted between two satellites. The Doppler shift is made up of several contributions which depend on (a) the rate of change of the free space path between the satellites, (b) the components, perpendicular to the line of sight between the satellites, of both the mean velocity of the satellites and of the integrated electron concentration gradients, (c) the moment of the perpendicular electron concentration gradients and the deviations from the mean of the individual satellite perpendicular velocities, (d) the velocity components along the line of sight between the satellites, and the electron concentration values at each satellite, and (e) changes occurring in the ionosphere with time.

In a plane stratified ionosphere, Doppler measurements between two satellites with a common velocity vector can be used to determine the electron concentration gradient in the direction of the velocity. Additional satellites do not provide any more information about a plane stratified ionosphere unless the satellite velocities differ significantly. For wave-like irregularities it is found that the contribution to the Doppler shift dependent on the average velocity of the satellites is in phase quadrature with respect to the contribution dependent on the velocity difference of the satellites. These contributions also have different resonance conditions. The average velocity contribution is greatest for irregularities which have an odd number of half-wavelengths along the line of sight between the two satellites, whereas for the term dependent on the velocity difference, irregularities having an integral number of wavelengths between the two satellites have the largest contributions. The relative importance of these velocity terms depends also on the relative magnitude of the components of the relative and difference velocity vectors along the irregularity wave vector direction. Combined Doppler and in-situ measurements of electron concentration enable the component of the irregularity wave vector in the plane containing the satellites and the average velocity vector to be determined.

The Space Shuttle provides excellent potential to carry out Doppler measurements between orbiting satellites since satellites can easily be launched from it into similar orbits or tethered above or below the Shuttle itself. The satellites need only contain transmitters and/or transponders. The actual Doppler measurements and processing could be done on board the Shuttle. Orbits covering the altitude range of 250–400 km are within the Shuttle's capabilities and would encompass the F_2 peak of the ionosphere and so be in the region in which the steepest horizontal gradients and the maximum ionospheric response to gravity waves occur. Other irregularities, such as plasma bubbles associated with equatorial spread F could also be studied.

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