GDR Contribution to Coulomb Excitation. II[†] ¹⁷O

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Abstract

The contribution to the Coulomb excitation of the first excited state of ¹⁷O due to virtual excitation of the giant dipole resonance (GDR) is calculated, using shell model wavefunctions for the ground and first excited states. A large value is obtained.

1. Introduction

In calculating the Coulomb excitation probability of the first excited state of a nucleus, the values of at least four quantities are needed. These are B(E2) for the E2 transition from the ground to the first excited state, the quadrupole moments Q and Q^* of the two states, and a parameter k measuring the strength of the contribution due to virtual excitation of the GDR. Values of k have usually been taken from estimates based on a hydrodynamic model, which is not necessarily reliable for light nuclei. Values of k obtained from shell model calculations in Part I (Barker 1982, present issue p. 291) agreed reasonably with experimental values for ⁶Li, ⁷Li and ¹⁰B, but only after radial integrals had been renormalized in order to fit experimental B(E2) values in these nuclei. This renormalization was presumably necessary because higher configurations were neglected in the wavefunctions for the ground and first excited states. It is desirable to compare calculated and measured values of k for a light nucleus for which wavefunctions including higher configurations are available, so that such renormalization is not needed. A suitable case is ¹⁷O.

There have been many studies of the wavefunctions of the ground and first excited states of ¹⁷O, in which higher configurations (core excitations) have been included with the aim of fitting the values of Q and B(E2). In comparison with other light nuclei, ¹⁷O has the advantages of being stable (so that Q has been measured accurately), of having a particle-stable first excited state with spin differing by two from that of the ground state (so that B(E2) can be obtained from a lifetime measurement), and of having the spin of the first excited state equal to $\frac{1}{2}$ (so that $Q^* = 0$). Further, the excitation energy of the first excited state is small (0.87 MeV), making the excitation probability sensitive to the value of k, and there is a large energy gap between the $\frac{1}{2}^+$ first excited state and higher states that might feed this $\frac{1}{2}^+$ state by virtual E2 excitation, so that their contribution can be neglected.

Since ¹⁷O seems to be such a good test case, we here give a shell model calculation of the GDR contribution. A measurement of the Coulomb excitation of the first excited state of ¹⁷O is in progress at Canberra (R. H. Spear, personal communication).

† Part I, Aust. J. Phys., 1982, 35, 291-9.

2. Formulae

Notation and formulae are taken from Häusser *et al.* (1973) and Part I. The parameter k, which measures the strength of the GDR contribution, is defined by the ratio

$$k = X/X_0, \tag{1}$$

where

$$X = S(E1)/\langle i \parallel \mathcal{M}(E2) \parallel f \rangle, \qquad (2)$$

with

$$S(E1) = \sum_{n} W(11I_{i}I_{f}, 2I_{n}) \langle i \parallel \mathcal{M}(E1) \parallel n \rangle \langle n \parallel \mathcal{M}(E1) \parallel f \rangle / (E_{n} - E_{i}).$$
(3)

Here $|i\rangle$ is the ¹⁷O ground state with $I_i = \frac{5}{2}$, and $|f\rangle$ is the first excited state with $I_f = \frac{1}{2}$. The unit X_0 in equation (1) is given by

$$X_0 = 0.00058 \, A/Z \ e \, \mathrm{MeV}^{-1} = 0.00123 \ e \, \mathrm{MeV}^{-1} \,. \tag{4}$$

Contributions to S(E1) come only from states $|n\rangle$ with $I_n = \frac{3}{2}$. We assume that all the E1 strength from the ground state to states of spin $\frac{3}{2}$ is concentrated in one eigenstate with energy E_g . Then (3) reduces to

$$S(E1) = 5^{-\frac{1}{2}} (E_g - E_i)^{-1} \langle i \| \mathcal{O}^2 \| f \rangle.$$
(5)

The denominator in (2) is related to B(E2) by

$$B(E2; i \to f) = (2I_i + 1)^{-1} |\langle i || \mathcal{M}(E2) || f \rangle|^2.$$
(6)

The wavefunctions of the ground and first excited states of 17 O, and of the analogue states in 17 F, have been discussed by many authors, mainly in connection with the problem of fitting the values of Q and B(E2) for the two nuclei. We assume that they have the form (Barker 1964)

$$\Psi(M_T \tfrac{5}{2}M) = \Psi([00\tfrac{5}{2}]M_T \tfrac{5}{2}M) + a_1 \Psi([02\tfrac{5}{2}]M_T \tfrac{5}{2}M) + a'_1 \Psi([12\tfrac{5}{2}]M_T \tfrac{5}{2}M) + a_2 \Psi([02\tfrac{1}{2}]M_T \tfrac{5}{2}M) + a'_2 \Psi([12\tfrac{1}{2}]M_T \tfrac{5}{2}M),$$
(7a)

$$\Psi(M_T \frac{1}{2}M) = \Psi([00\frac{1}{2}]M_T \frac{1}{2}M) + a_3 \Psi([02\frac{5}{2}]M_T \frac{1}{2}M) + a'_3 \Psi([12\frac{5}{2}]M_T \frac{1}{2}M),$$
(7b)

where $M_T = \frac{1}{2}$ for ¹⁷O $(-\frac{1}{2}$ for ¹⁷F) and $[\overline{TJ}j]$ represents an A = 16 core state of isospin \overline{T} and spin \overline{J} coupled to an odd nucleon in the state nl_j (either $1d_{5/2}$ or $2s_{1/2}$). In Barker (1964), the terms in (7) involving a_i, a'_i , which represent distortion of the ¹⁶O core, were treated in first-order perturbation theory, the coefficients a_i, a'_i being of order $0 \cdot 1 - 0 \cdot 2$. These distortion terms are necessary in order to explain the appreciable measured values of Q and B(E2) for ¹⁷O, since the zeroth-order terms vanish when the usual E2 operator (no recoil) is used. For ¹⁷F the zeroth-order terms do not vanish, while the distortion contributions are smaller than in ¹⁷O but are not negligible.[†] Recoil contributions, obtained by taking the origin of

[†] The formulae (25) in Barker (1964), which display these distortion contributions, are incorrect; the quantity $\langle r^2 \rangle_{00}^{\pm}$ should in each case be replaced by $2 \cdot 73 \text{ fm } \langle r^2 \rangle_{00}^{\pm}$. Because $\langle r^2 \rangle_{00}^{\pm} = 2 \cdot 64 \text{ fm}$ from (26), this change is numerically small and later results of that paper are not changed appreciably.

coordinates in the E2 operator as the centre of mass rather than the usual centre of the potential well, are relatively small. With the approximations made in Barker (1964), it was found that simultaneous fits to Q and B(E2) for ¹⁷O could not be obtained with sets of expansion coefficients a_i, a'_i derived from any reasonable twoparticle interaction. A similar result was found in more elaborate calculations, in which a distribution of E2 strength in the ¹⁶O core was included by actual diagonalization of a given Hamiltonian (see e.g. Mavromatis and Singh 1969; Chung and Shin 1980). Other discussions of E2 matrix elements in ¹⁷O and ¹⁷F have been based on the concept of effective charges (Brown *et al.* 1977; Harvey 1978), but this does not seem to be a useful approach in the calculation of S(E1) (see Part I). We therefore evaluate S(E1) using wavefunctions of the form (7), working to first order in the coefficients a_i, a'_i , which are adjusted to fit Q and B(E2).

Since \emptyset^2 has one- and two-body terms, while the $\overline{J} = 2$, ¹⁶O core states (with $\overline{T} = 0, 1$) are obtained by operating on the ¹⁶O ground state with the E2 operators (taken as one-body operators by neglecting recoil), matrix elements between ¹⁷O states of the lowest configuration would involve one-, two- and three-body operators. It is preferable to construct the $\overline{J} = 2$, ¹⁶O core states explicitly, since then it is necessary to calculate matrix elements of only one- and two-body operators between the ¹⁷O states based on these and the ¹⁷O states of the lowest configurations.

Thus we take the ¹⁶O ground state as the single state of the closed shell configuration 1s⁴1p¹², and write the $\overline{J} = 2$, ¹⁶O states relative to this as

$$\Phi_{\overline{T}2m} = \sum_{n=1}^{\infty} c_{1n}(\overline{T}) | (1s^{-1}nd)\overline{T}02, 02m) + \sum_{n=1}^{\infty} c_{2n}(\overline{T}) | (1p^{-1}nf)\overline{T}02, 02m) + \sum_{n=2}^{\infty} c_{3n}(\overline{T}) | (1p^{-1}np)\overline{T}02, 02m),$$
(8)

where values of the quantum numbers TSL, $M_T JM$ are given. The $c_{jn}(\overline{T})$ can easily be expressed in terms of one-body matrix elements such as the radial integral $\langle 1s; r^2; nd \rangle$, where

$$\langle nl: r^q: n'l' \rangle = \int_0^\infty u_{nl}(r) u_{n'l'}(r) r^q r^2 dr, \qquad \int_0^\infty u_{nl}^2(r) r^2 dr = 1.$$
 (9a, b)

The matrix elements $\langle [00J]M_TJ \parallel 0^2 \parallel [\overline{T}2j]M_TJ' \rangle$ can be calculated, the coefficients of $c_{jn}(\overline{T})$ involving radial integrals such as $\langle 1p : r : nd \rangle$. By using relations such as

$$\sum_{n} \langle 1s; r^{2}; nd \rangle \langle 1p; r; nd \rangle = \langle 1s; r^{3}; 1p \rangle, \qquad (10)$$

one obtains

$$\begin{split} S(\text{E1}) &= \frac{96}{1445} \pi^{-1} (E_{\text{g}} - E_{\text{i}})^{-1} e^{2} [\langle 1\text{d} : r^{2} : 2\text{s} \rangle - 2 \langle 1\text{p} : r : 1\text{d} \rangle \langle 1\text{p} : r : 2\text{s} \rangle \\ &+ N_{0}^{-\frac{1}{2}} \{ (\frac{1}{5} \sqrt{14}) \langle 1\text{p} : r : 1\text{d} \rangle N_{3} a_{1} - (\frac{1}{12} \sqrt{42}) \langle 1\text{p} : r : 1\text{d} \rangle N_{3} a_{1}' \\ &+ (\frac{145}{64} N_{0} - \frac{145}{32} \langle 1\text{s} : r : 1\text{p} \rangle N_{1} - \langle 1\text{p} : r : 2\text{s} \rangle N_{3}) a_{2} \\ &+ \sqrt{3} (-\frac{17}{192} N_{0} + \frac{17}{96} \langle 1\text{s} : r : 1\text{p} \rangle N_{1} + \frac{5}{12} \langle 1\text{p} : r : 2\text{s} \rangle N_{3}) a_{2}' \\ &+ \sqrt{3} (\frac{145}{192} N_{0} - \frac{145}{96} \langle 1\text{s} : r : 1\text{p} \rangle N_{1} + \frac{5}{12} \langle 1\text{p} : r : 2\text{s} \rangle N_{3}) a_{2}' \\ &+ \sqrt{3} (\frac{145}{192} N_{0} - \frac{145}{96} \langle 1\text{s} : r : 1\text{p} \rangle N_{1} - \frac{1}{5} \langle 1\text{p} : r : 1\text{d} \rangle N_{2} - \frac{1}{30} \langle 1\text{s} : r^{2} : 1\text{d} \rangle N_{4}) a_{3} \\ &+ (-\frac{17}{192} N_{0} + \frac{17}{96} \langle 1\text{s} : r : 1\text{p} \rangle N_{1} + \frac{1}{4} \langle 1\text{p} : r : 1\text{d} \rangle N_{2} - \frac{113}{960} \langle 1\text{s} : r^{2} : 1\text{d} \rangle N_{4} a_{3}' \}], \quad (11) \end{split}$$

where

$$N_0 = \langle 1s; r^4; 1s \rangle + 3\langle 1p; r^4; 1p \rangle - \frac{6}{5}\langle 1p; r^2; 1p \rangle^2, \qquad (12a)$$

$$N_1 = 2\langle 1s; r^3; 1p \rangle - \langle 1p; r^2; 1p \rangle \langle 1s; r; 1p \rangle, \qquad (12b)$$

$$N_2 = \langle 1p; r^3; 1d \rangle - \langle 1p; r^2; 1p \rangle \langle 1p; r; 1d \rangle, \qquad (12c)$$

$$N_3 = \langle 1p; r^3; 2s \rangle - \langle 1p; r^2; 1p \rangle \langle 1p; r; 2s \rangle, \qquad (12d)$$

$$N_4 = \langle 1s; r^2; 1d \rangle - 2\langle 1s; r; 1p \rangle \langle 1p; r; 1d \rangle.$$
(12e)

In the same notation we have

$$Q = -\frac{32}{223}e[\langle 1d;r^{2};1d\rangle - \frac{7}{5}\langle 1p;r;1d\rangle^{2} + N_{0}^{-\frac{1}{2}}\{\sqrt{14}(-\frac{271}{16}N_{0} - \frac{9}{4}\langle 1s;r;1p\rangle N_{1} + \frac{7}{25}\langle 1p;r;1d\rangle N_{2} + \frac{3}{20}\langle 1s;r^{2};1d\rangle N_{4} - \frac{3}{350}\langle 1p;r;1d\rangle \langle 1p;r^{3};1d\rangle)a_{1} + \sqrt{42}(\frac{85}{16}N_{0} + \frac{17}{12}\langle 1s;r;1p\rangle N_{1} - \frac{7}{60}\langle 1p;r;1d\rangle N_{2} - \frac{11}{40}\langle 1s;r^{2};1d\rangle N_{4} + \frac{1}{280}\langle 1p;r;1d\rangle \langle 1p;r^{3};1d\rangle + \frac{289}{80}\langle 1s;r^{2};1d\rangle^{2})a_{1}' - \frac{1}{5}\langle 1p;r;2s\rangle \langle 7N_{2} + 3\langle 1p;r^{3};1d\rangle \ranglea_{2} + (\frac{1}{4}\sqrt{\frac{1}{3}})\langle 1p;r;2s\rangle \langle 7N_{2} + 3\langle 1p;r^{3};1d\rangle \ranglea_{2}'\}], \qquad (13)$$

and $B(E2)$ is given by equation (6) with
 $\langle i \parallel \mathcal{M}(E2) \parallel f \rangle = \frac{24}{289}(6\pi)^{-\frac{1}{2}}e[\langle 1d;r^{2};2s\rangle - 2\langle 1p;r;1d\rangle \langle 1p;r;2s\rangle$

$$+N_{0}^{-\frac{1}{2}}\{(\frac{1}{5}\sqrt{14})\langle 1p;r;1d\rangle N_{3}a_{1} - (\frac{1}{12}\sqrt{42})\langle 1p;r;1d\rangle N_{3}a_{1}' + (\frac{271}{8}N_{0} + \frac{9}{2}\langle 1s;r;1p\rangle N_{1} - \langle 1p;r;2s\rangle N_{3})a_{2} + \sqrt{3}(-\frac{85}{8}N_{0} - \frac{17}{6}\langle 1s;r;1p\rangle N_{1} + \frac{5}{12}\langle 1p;r;2s\rangle N_{3})a_{2}' + \sqrt{3}(\frac{271}{24}N_{0} + \frac{3}{2}\langle 1s;r;1p\rangle N_{1} - \frac{1}{5}\langle 1p;r;1d\rangle N_{2} - \frac{1}{30}\langle 1s;r^{2};1d\rangle N_{4})a_{3} + (-\frac{85}{8}N_{0} - \frac{17}{6}\langle 1s;r;1p\rangle N_{1} + \frac{1}{4}\langle 1p;r;1d\rangle N_{2} + \frac{11}{60}\langle 1s;r^{2};1d\rangle N_{4} - \frac{289}{120}\langle 1s;r^{2};1d\rangle^{2})a_{3}'\}].$$
(14)

3. Calculated Values

For harmonic oscillator single-particle wavefunctions, the zeroth-order contribution to S(E1) vanishes, for the same reason† as in 1p shell nuclei (see Part I). We evaluate the radial integrals using single-particle wavefunctions belonging to a real Woods-Saxon potential, with central and surface-peaked spin-orbit terms, plus the Coulomb potential of a uniformly charged sphere. Parameter values for such a potential appropriate to ¹⁷O were found by Brown *et al.* (1977) by fitting the measured ¹⁶O r.m.s. charge radius, giving $r_0 = 1.324$ fm and a = 0.65 fm. We adjust the central potential depth to fit binding energies of 4.144 and 3.273 MeV for the 1d and 2s neutron states respectively (Ajzenberg-Selove 1982), the spin-orbit

† This is not the same as the reason for the vanishing of B(E2) in ¹⁷O in zeroth order, since the zeroth-order contribution to S(E1) for ¹⁷F also vanishes.

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strength for the 1d ($\equiv 1d_{5/2}$) state being $V_{ls} = 15 \text{ MeV fm}^2$ as given by Brown *et al.* From the ¹⁶O(e, e'p)¹⁵N reaction, the 1s state is centred at $E_x \approx 41 \text{ MeV}$, while the 1p strength is split between the $\frac{1}{2}^-$ ground state and the $\frac{3}{2}^-$ state at 6.32 MeV (Ajzenberg-Selove 1981). We therefore fit a 1s neutron binding energy of 56.7 MeV and, since we do not distinguish between $1p_{1/2}$ and $1p_{3/2}$ wavefunctions, we fit a 1p neutron binding energy of 19.8 MeV with $V_{ls} = 0$. Then we get

$$\langle 1s:r^{4}:1s \rangle = 23 \cdot 45 \text{ fm}^{4}, \qquad \langle 1s:r:1p \rangle = 1 \cdot 917 \text{ fm}, \\ \langle 1s:r^{3}:1p \rangle = 12 \cdot 87 \text{ fm}^{3}, \qquad \langle 1s:r^{2}:1d \rangle = 4 \cdot 503 \text{ fm}^{2}, \\ \langle 1p:r^{2}:1p \rangle = 7 \cdot 520 \text{ fm}^{2}, \qquad \langle 1p:r^{4}:1p \rangle = 82 \cdot 69 \text{ fm}^{4}, \\ \langle 1p:r:1d \rangle = 2 \cdot 715 \text{ fm}, \qquad \langle 1p:r^{3}:1d \rangle = 32 \cdot 43 \text{ fm}^{3}, \\ \langle 1p:r:2s \rangle = -1 \cdot 586 \text{ fm}, \qquad \langle 1p:r^{3}:2s \rangle = -30 \cdot 45 \text{ fm}^{3}, \\ \langle 1d:r^{2}:1d \rangle = 13 \cdot 31 \text{ fm}^{2}, \qquad \langle 1d:r^{2}:2s \rangle = -14 \cdot 02 \text{ fm}^{2}.$$
(15)

Values of the coefficients a_i, a_i' given in Barker (1964) were derived from a twoparticle interaction, but they do not fit simultaneously the experimental values of both Q and B(E2) for ¹⁷O. Here we choose the a_i, a_i' to fit Q and B(E2). Some further restrictions on the a_i, a_i' are needed, and we assume $a_i' = -a_i \frac{1}{2} \sqrt{3} (\Delta E / \Delta E')$, a relation found to be valid for a wide range of interactions (Barker 1964). Also from that paper we take $\Delta E / \Delta E' = 0.61$, and $a_3 = 2a_2$. The latter assumption is not critical, since Q depends essentially only on the value of a_1 while B(E2) depends only on the combination $\sqrt{3}a_2 + a_3$ (this is clearly seen from the formulae (15) and (16) of Barker (1964), which neglect recoil). Also, S(E1) depends essentially only on a_1 and $\sqrt{3}a_2 + a_3$ (since the coefficients of a_2 and a_3 in (11) are each dominated by their first two terms, and the coefficients of a_2' and a_3' are relatively small). Then we fit $Q = -2.578 e \text{ fm}^2$ and $\tau_m({}^{17}\text{O}, \frac{1}{2}^+) = 258.6 \text{ ps}$ (Ajzenberg-Selove 1982), which gives $B(E2; \frac{5}{2}^+ \rightarrow \frac{1}{2}^+) = 2.101 e^2 \text{ fm}^4$ and $\langle i \parallel \mathcal{M}(E2) \parallel f \rangle = -3.550 e \text{ fm}^2$, with

$$a_1 = -0.134, \qquad \sqrt{3}a_2 + a_3 = -0.495.$$
 (16a, b)

Then equation (11) gives

$$S(E1) = -0.249(E_g - E_i)^{-1} e^2 \text{ fm}^2.$$
(17)

As in the calculations for 1p shell nuclei in Part I, we assume that $E_{\rm g} - E_{\rm i} = \sigma_{-1}/\sigma_{-2}$, where σ_n is the *n*th moment of the photonuclear cross section. Total photonuclear cross sections (Ahrens *et al.* 1975) have not been measured for isotopic ¹⁷O but only for oxygen of natural isotopic composition (99.8% ¹⁶O); these give $\sigma_{-1}/\sigma_{-2} \approx 26$ MeV (for $E_{\gamma,\text{max}} = 140$ MeV). Measurements of photoneutron cross sections (Jury *et al.* 1980), for $E_{\gamma,\text{max}} \approx 40$ MeV, give $\sigma_{-1}/\sigma_{-2} \approx 20$ MeV for ¹⁷O and 24–27 MeV for ¹⁶O. Although the photoneutron and total photonuclear cross sections yield about the same values of σ_{-1}/σ_{-2} for ¹⁶O, in which the proton threshold is 3.5 MeV below the neutron threshold, the same need not be true for ¹⁷O, in which the proton threshold (at 13.8 MeV) is 9.6 MeV above the neutron threshold. We assume the photoproton and photoneutron cross sections to be about equal for $E_{\gamma} \gtrsim 14$ MeV, and so estimate $\sigma_{-1}/\sigma_{-2} \approx 22$ MeV for the total photonucleon cross section. We therefore take $E_{\rm g} - E_{\rm i} = 22$ MeV. Then $S({\rm E1}) = -0.0113 \ e^2 \ {\rm fm}^2$ MeV⁻¹, giving $X = 0.00319 \ e \ {\rm MeV}^{-1}$ and k = 2.59.

4. Discussion

One of the main uncertainties in the calculation of the GDR contribution for 1p shell nuclei in Part I was due to the use of wavefunctions belonging entirely to the lowest shell model configuration, which necessitated the renormalization of the radial integrals in order to fit experimental B(E2) values. This is avoided in the present case by including terms in the ¹⁷O wavefunctions belonging to higher configurations, and adjusting the coefficients of these to fit the experimental Q and B(E2) values (the contributions from the lowest configurations being almost negligible). These higher configuration components then contribute about 54% of S(E1) and therefore of the GDR contribution.

The other major uncertainty for 1p shell nuclei still remains, in the estimation of $E_{\rm g} - E_{\rm i}$. It is an approximation to take $E_{\rm g} - E_{\rm i} = \sigma_{-1}/\sigma_{-2}$, where the σ_n are taken from the photonuclear cross sections, since the value of $E_{\rm g} - E_{\rm i}$ should depend only on the location of the E1 strength to $\frac{3}{2}^{-}$ states of ¹⁷O. Also the values of σ_n that we use are rather uncertain because total photonuclear cross sections are not available for ¹⁷O.

From these calculations, it is expected that k for ¹⁷O should be large, about 2.6. Such a large value is another reason, additional to those given in Section 1, why ¹⁷O is a suitable nucleus for studying the GDR contribution to Coulomb excitation of low-lying excited states.

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