

GDR Contribution to Coulomb Excitation. III† ¹⁸O

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Abstract

The discrepancy between the values of $B(E2)$ for the transition between the ground and first excited states in ¹⁸O as obtained from Coulomb excitation measurements and from other measurements is here attributed to the giant dipole resonance (GDR) contribution to Coulomb excitation; an estimate based on shell model calculations predicts this contribution to be much larger than previously assumed.

1. Introduction

Ball *et al.* (1982) have recently made an accurate measurement of the lifetime of the 2^+ first excited state of ¹⁸O using Doppler broadened lineshape analysis (DBLA). They point out that values of $B(E2; 0^+ \rightarrow 2^+)$ obtained in Coulomb excitation measurements on ¹⁸O are about 15% lower than the value corresponding to their lifetime. As a possible explanation for part of this discrepancy, they suggest that the GDR contribution to Coulomb excitation may be larger than the value assumed ($k = 1$). Here k is the ratio of the GDR contribution to the value calculated from the hydrodynamic model, as empirically renormalized by Levinger (1957). A complete explanation of the discrepancy would require $k \gtrsim 3$, but Ball *et al.* do not envisage $k \gtrsim 1.5$.

Large values of k have been measured in other light nuclei: $k = 2.6-3.9$ in ⁶Li (Disdier *et al.* 1971; Häusser *et al.* 1973; Gemmeke *et al.* 1978), $k = 2.3-3.6$ in ⁷Li (Häusser *et al.* 1973) and $k = 5.7 \pm 0.4$ in ¹⁷O (Kuehner *et al.* 1982). Shell model calculations (Barker 1982*a*, 1982*b*, here referred to as Parts I, II) have also given values of k appreciably larger than unity for these nuclei, although smaller than the experimental values, namely $k \approx 1.9$, 2.3 and 2.6 for ⁶Li, ⁷Li and ¹⁷O respectively.

We here present an estimate of k for ¹⁸O, based on shell model wavefunctions for the 0^+ ground state and 2^+ first excited state. Suitable wavefunctions, which use as basis states both two-particle states (relative to a closed ¹⁶O core) and four-particle two-hole (collective) states, have been given by Lawson *et al.* (1976, here referred to as LSF). LSF obtained the expansion coefficients by fitting data which included E2 transition rates and static moments. In the fit, values of E2 matrix elements were needed for both single-particle and collective states, and these were constrained by using experimental data from ¹⁶O, ¹⁷O and ²⁰Ne. The calculation of k involves matrix elements of an operator \mathcal{O}^2 , analogous to the E2 operator but derived from the E1 operator in second order; we proceed as far as possible in the spirit of LSF by using experimental values of the matrix elements of \mathcal{O}^2 where available.

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2. Formulae and Procedures

Notation and formulae are taken from Häusser *et al.* (1973) and from Parts I and II, with the initial state $|i\rangle \equiv |0\rangle$ and the final state $|f\rangle \equiv |2\rangle$. The parameter k is defined by

$$k = X/X_0, \quad (1)$$

where the unit X_0 for ^{18}O is

$$X_0 = 0.001305 \text{ e MeV}^{-1}. \quad (2)$$

The quantity X can be written

$$X = 5^{-\frac{1}{2}}(E_g - E_0)^{-1} \langle 0 \| \theta^2 \| 2 \rangle / \langle 0 \| \mathcal{M}(\text{E}2) \| 2 \rangle, \quad (3)$$

as in Part I. This can be taken as a definition of the energy E_g , irrespective of the distribution of E1 strength from the ground state, but the assumption that this energy is given by

$$E_g - E_0 = \sigma_{-1}/\sigma_{-2}, \quad (4)$$

where σ_n is the n th moment of the photonuclear cross section, is based on the argument that the E1 strength is concentrated in the GDR region, and then equation (4) is exact only if there is a perfect giant dipole state. Also, we have

$$B(\text{E}2; 0^+ \rightarrow 2^+) = |\langle 0 \| \mathcal{M}(\text{E}2) \| 2 \rangle|^2. \quad (5)$$

The wavefunctions of the ground and first excited states are written as in LSF:

$$|0\rangle = a_1(d_{5/2}^2)_0 + a_2(s_{1/2}^2)_0 + a_3 \Psi_0, \quad (6a)$$

$$|2\rangle = b_1(d_{5/2}^2)_2 + b_2(d_{5/2} s_{1/2})_2 + b_3 \Psi_2 + b_4(d_{5/2} d_{3/2})_2 + b_5(d_{3/2} s_{1/2})_2, \quad (6b)$$

where Ψ_0 and Ψ_2 are collective states. LSF give values of the coefficients a_p and b_p for various fits to the experimental data.

We write

$$\langle 0 \| \mathcal{M}(\text{E}2) \| 2 \rangle = \sum_{p=1}^3 \sum_{q=1}^5 a_p b_q M_{pq}, \quad (7a)$$

$$\langle 0 \| \theta^2 \| 2 \rangle = \sum_{p=1}^3 \sum_{q=1}^5 a_p b_q O_{pq}. \quad (7b)$$

Three types of matrix element M_{pq} (and O_{pq}) are required: matrix elements like M_{11} connecting two-particle states, those like M_{13} connecting a two-particle state with a collective state, and M_{33} connecting collective states. We consider separately these three types of matrix element.

Matrix Elements M_{pq} and O_{pq} connecting Two-particle States

Since the centre of mass of the nucleus has been taken as the origin of coordinates in defining the operators $\mathcal{M}(\text{E}\lambda, \mu)$ (see Part I, equation 3), both $\mathcal{M}(\text{E}2, \mu)$ and $\theta^2(\mu)$ contain one- and two-body parts when written in terms of coordinates relative to the centre of the potential well, being of the form

$$\mathcal{M} = \sum_i \mathcal{M}(i) + \sum_{i < j} \mathcal{M}(i, j). \quad (8)$$

Contributions to M_{pq} and O_{pq} from $\mathcal{M}(i)$ occur only when i refers to a (d, s) nucleon, and those from $\mathcal{M}(i, j)$ when i refers to a (d, s) nucleon and j to a p nucleon, or vice versa. Thus, one can write

$$\mathcal{M} = \sum_i \bar{\mathcal{M}}(i), \quad (9)$$

where i refers to a (d, s) nucleon only. Then the matrix elements M_{pq} and O_{pq} can be written simply in terms of matrix elements $\langle \chi_{j'} \| \mathcal{M}(\text{E2}) \| \chi_j \rangle$ and $\langle \chi_{j'} \| \mathcal{O}^2 \| \chi_j \rangle$ taken between (d, s) single-particle states. LSF obtained values of $\langle \chi_{j'} \| \mathcal{M}(\text{E2}) \| \chi_j \rangle$ for j and j' equal to $\frac{1}{2}$ or $\frac{5}{2}$ by fitting the experimental values of $Q_{5/2+}$ and $B(\text{E2}; \frac{5}{2}^+ \rightarrow \frac{1}{2}^+)$ for ^{17}O , and we do likewise. Assuming the same radial wavefunction and effective charge for the $d_{3/2}$ orbit as for the $d_{5/2}$ orbit, LSF expressed the matrix elements for j or j' equal to $\frac{3}{2}$ in terms of those already obtained. We assume the same relations, although these can be derived only for those parts of the ^{17}O wavefunctions given in Part II that do not involve core excitation. This should be sufficiently accurate, since the coefficients b_4 and b_5 of the states containing $d_{3/2}$ nucleons are small. The same approach is used for $\langle \chi_{j'} \| \mathcal{O}^2 \| \chi_j \rangle$. The calculated value of $\langle \chi_{5/2} \| \mathcal{O}^2 \| \chi_{1/2} \rangle$ in Part II led to $k \approx 2.6$ for ^{17}O , whereas Kuehner *et al.* (1982) measured $k \approx 5.7$. We therefore take $\langle \chi_{5/2} \| \mathcal{O}^2 \| \chi_{1/2} \rangle$ to be 2.2 times the calculated value. Also, we calculate $\langle \chi_{5/2} \| \mathcal{O}^2 \| \chi_{5/2} \rangle$ by the method given in Part II, using values of the radial integrals and expansion coefficients given there, and assume that we should likewise enhance this value by a factor of 2.2.

Matrix Elements M_{pq} and O_{pq} connecting a Two-particle and a Collective State

Since the collective states are four-particle two-hole states, only the two-body parts of the operators $\mathcal{M}(\text{E2})$ and \mathcal{O}^2 contribute to these matrix elements. LSF neglected recoil in their E2 operator, which was therefore one-body only, and so did not have matrix elements of this type. There is no obvious way of deriving them from experiment. We therefore calculate them, and for this purpose take the description of the collective states as given by equations (14) and (15) of LSF, which we write as

$$\phi_J = \sum_{jj'} a_J(jj')(l_j l_{j'})_J, \quad (10a)$$

$$\Psi_{JM} = \sum_{J_1 J_2} b_J(J_1 J_2) [\phi_{J_1}(\pi) \times \phi_{J_2}(v)]_{JM} (v p_{1/2})_0^2 (v p_{3/2})_0^4 (\pi p_{3/2})_0^4. \quad (10b)$$

Then we obtain for these matrix elements†

$$M_{pq} \equiv \langle (l_j l_{j'})_J \| \mathcal{M}(\text{E2}) \| \Psi_J \rangle = a_J(jj') b_J(2J')(2J+1)^{\frac{1}{2}} C, \quad (11)$$

where

$$C = -\frac{4.9}{7^{2.9}} (35\pi)^{-\frac{1}{2}} e \langle 1p : r : 1d \rangle \{ \langle 1p : r : 1d \rangle - (10)^{\frac{1}{2}} \langle 1p : r : 2s \rangle \}, \quad (12)$$

and

$$O_{pq} = -\frac{7.5}{7} (30\pi)^{-\frac{1}{2}} e M_{pq}. \quad (13)$$

Values of the radial integrals $\langle 1p : r : 1d \rangle$ and $\langle 1p : r : 2s \rangle$ are taken from Part II.

† The reduced matrix element used here is as defined in Part I, and is $(2J'+1)^{1/2}$ times that used in LSF.

Matrix Elements M_{33} and O_{33} connecting Collective States

LSF assumed that the value of $M_{33} \equiv \langle \Psi_0 \| \mathcal{M}(E2) \| \Psi_2 \rangle$ should be similar to those of the E2 matrix elements between the ground and first excited states of ^{20}Ne , and between the 6.05 and 6.92 MeV states of ^{16}O ; in some fits M_{33} was constrained to lie within the limits imposed by these values, in other fits it was unconstrained. For each fit, LSF gave the value of M_{33} used. Similarly, O_{33} may be taken as the matrix element $\langle \Psi_0(^{20}\text{Ne}) \| \theta^2 \| \Psi_2(^{20}\text{Ne}) \rangle$ between the ground and first excited states of ^{20}Ne . Since this is not known experimentally, we estimate it using for simplicity crude wavefunctions for the ^{20}Ne states. If the radial integrals are renormalized to fit the known $B(E2; 0^+ \rightarrow 2^+)$ value in ^{20}Ne , the resultant value of O_{33} should be insensitive to the choice of ^{20}Ne wavefunctions (as in Part I). We use $T = 0$ states of the lowest configuration and lowest seniority:

$$\Psi_J(^{20}\text{Ne}) = (d_{5/2}^4)_J. \quad (14)$$

Then we find

$$\langle \Psi_0(^{20}\text{Ne}) \| \mathcal{M}(E2) \| \Psi_2(^{20}\text{Ne}) \rangle = -\frac{3}{5}(2\pi)^{-\frac{1}{2}}e\{\langle 1d : r^2 : 1d \rangle + \frac{7}{9}\langle 1p : r : 1d \rangle^2\}, \quad (15a)$$

$$\langle \Psi_0(^{20}\text{Ne}) \| \theta^2 \| \Psi_2(^{20}\text{Ne}) \rangle = -\frac{6}{7}\pi^{-1}(15)^{-\frac{1}{2}}e^2\{\langle 1d : r^2 : 1d \rangle - \frac{7}{5}\langle 1p : r : 1d \rangle^2\}. \quad (15b)$$

Following the procedure in Part I, and using a value of $\langle 1p : r : 1d \rangle$ taken from Part II, we choose the value of $\langle 1d : r^2 : 1d \rangle$ in equation (15a) to fit the experimental value of $B(E2; 0^+ \rightarrow 2^+)$ for ^{20}Ne , and then use these values of the radial integrals in (15b), so obtaining O_{33} . The value of O_{33} does not depend sensitively on the choice of $\langle 1p : r : 1d \rangle$.

3. Numerical Values

Numerical values of the matrix elements M_{pq} and O_{pq} are given in Tables 1a and 1b respectively. In Table 1a, use has been made of the experimental values $Q_{5/2^+} = -2.578 \text{ e fm}^2$ and $\tau_m(\frac{1}{2}^+ \rightarrow \frac{5}{2}^+) = 258.6 \text{ ps}$ for ^{17}O (Ajzenberg-Selove 1982), of the calculated values $\langle 1p : r : 1d \rangle = 2.715 \text{ fm}$ and $\langle 1p : r : 2s \rangle = -1.586 \text{ fm}$ from Part II, and of the value $q_{02} = -19.64 \text{ e fm}^2$ from the constrained II fit of LSF. In Table 1b, we have taken $\langle \chi_{5/2} \| \theta^2 \| \chi_{1/2} \rangle = -0.557 \text{ e}^2 \text{ fm}^2$ for ^{17}O from Part II and renormalized this by a factor of 2.2 to fit the experimental value $k = 5.7$ for ^{17}O (Kuehner *et al.* 1982). Similarly, we take $\langle \chi_{5/2} \| \theta^2 \| \chi_{5/2} \rangle = -0.425 \times 2.2 \text{ e}^2 \text{ fm}^2$. Also, we use $\tau_m(2^+ \rightarrow 0^+) = 1.05 \text{ ps}$ for ^{20}Ne (Ajzenberg-Selove 1978), giving

$$\begin{aligned} B(E2; 0^+ \rightarrow 2^+) &= 334 \text{ e}^2 \text{ fm}^4, & \langle \Psi_0(^{20}\text{Ne}) \| \mathcal{M}(E2) \| \Psi_2(^{20}\text{Ne}) \rangle &= -18.3 \text{ e fm}^2, \\ \langle 1d : r^2 : 1d \rangle &= 41.7 \text{ fm}^2, & \langle \Psi_0(^{20}\text{Ne}) \| \theta^2 \| \Psi_2(^{20}\text{Ne}) \rangle &= -2.211 \text{ e}^2 \text{ fm}^2. \end{aligned}$$

With values of a_p and b_p taken from the constrained II fit of LSF, we then obtain from equations (7)

$$\langle 0 \| \mathcal{M}(E2) \| 2 \rangle = 3.688 - 0.064 + 2.024 = 5.648 \text{ e fm}^2, \quad (16a)$$

$$\langle 0 \| \theta^2 \| 2 \rangle = 1.154 + 0.071 + 0.228 = 1.453 \text{ e}^2 \text{ fm}^2, \quad (16b)$$

where in each case the contributions from the three types of matrix element are shown separately.

From equation (4), with values of σ_{-1} and σ_{-2} for ^{18}O taken from Woodworth *et al.* (1979), we obtain $E_g - E_0 = 20$ MeV. Then equation (3) gives $X = 0.00575$ e MeV $^{-1}$ and (1) gives $k = 4.4$.

Table 1. Values of matrix elements
Units for M_{pq} and O_{pq} are e fm 2 and e 2 fm 2 respectively

p	1	2	q 3	4	5
(a) M_{pq}					
1	-2.721	-2.050	-0.083	-0.962	0
2	0	-3.550	-0.119	0	2.899
3	-0.034	-0.070	-19.64	-0.024	0.058
(b) O_{pq}					
1	-0.763	-0.707	0.091	-0.270	0
2	0	-1.225	0.132	0	1.000
3	0.037	0.078	-2.211	0.026	-0.063

4. Discussion

Woodworth *et al.* (1979) measured the ^{18}O photonuclear cross sections for $E_\gamma \lesssim 42$ MeV only, so that σ_{-1}/σ_{-2} would be increased if appreciable cross sections existed at higher energies; it seems unlikely, however, that this increase would be more than say 20%, suggesting $k \gtrsim 3.7$.

Use of values of a_p and b_p given by the other fits of LSF does not change significantly the values of Section 3. It is seen from equations (16) that the contributions from matrix elements connecting two-particle and collective states are small, so that the use of purely calculated values for them is unlikely to cause appreciable error in the value of k . Since the dominant contribution to $\langle 0 \parallel \mathcal{O}^2 \parallel 2 \rangle$ comes from the matrix elements connecting two-particle states, which were renormalized to fit the measured value of k in ^{17}O , it is clear that the deduced value of k for ^{18}O is very sensitive to the assumed value of k for ^{17}O . In fact if $k(^{17}\text{O})$ is treated as a parameter and other values are unchanged, then we obtain

$$k(^{18}\text{O}) = 0.615 k(^{17}\text{O}) + 0.907. \quad (17)$$

A defect of the values in Section 3 is that the value (16a) implies from equation (5) that $B(\text{E}2; 0^+ \rightarrow 2^+) = 31.9 \text{ e}^2 \text{ fm}^4$. Although this is reasonably close to the LSF constrained II value of $35.3 \text{ e}^2 \text{ fm}^4$, which was obtained from a fit to data that included the experimental value $B(\text{E}2; 0^+ \rightarrow 2^+) = 37.1 \text{ e}^2 \text{ fm}^4$ obtained by Berant *et al.* (1974), it is seen from Table 2 of Ball *et al.* (1982) that the Berant *et al.* value of $B(\text{E}2; 0^+ \rightarrow 2^+)$ is lower than all other experimental values, and much lower than the value of $47.6 \text{ e}^2 \text{ fm}^4$ that Ball *et al.* took to be correct. If we merely replace the value (16a) for $\langle 0 \parallel \mathcal{M}(\text{E}2) \parallel 2 \rangle$ by 6.90 e fm^2 in order to fit $B(\text{E}2; 0^+ \rightarrow 2^+) = 47.6 \text{ e}^2 \text{ fm}^4$,

then k would be reduced to 3.6. It seems likely, however, that any change in the wavefunctions that would increase the value of $\langle 0 \| \mathcal{M}(E2) \| 2 \rangle$ would also increase the value of $\langle 0 \| \mathcal{O}^2 \| 2 \rangle$, thus leading to a value of k intermediate between 3.6 and 4.4.

These calculations suggest that k for ^{18}O should be large compared with unity, and probably about 4. The reason why Ball *et al.* (1982) regarded 1.5 as an approximate upper limit for k seems to be due to confusion regarding the meaning or definition of k . Many authors, including Ball *et al.*, have taken k to be equal to

$$k' = \sigma_{-2} / (3.5 A^{5/3} \mu\text{b MeV}^{-1}). \quad (18)$$

The denominator in (18) is the hydrodynamic model value of σ_{-2} , as empirically renormalized by Levinger (1957) in order to fit experimental values of σ_{-2} for nuclei with $A \gtrsim 20$, so that it is not surprising that $k' \approx 1$ for ^{18}O ; the results of Woodworth *et al.* (1979) give $k' = 1.26$. But $k = k'$ implies that the parameter η_0 has its hydrodynamic value, namely $\frac{4}{3}\pi^{\frac{1}{2}}\langle 0 \| \mathcal{M}(E2) \| 2 \rangle / ZeR_0^2$, with $R_0 = 1.2 A^{1/3} \text{ fm}$ (Häusser *et al.* 1973), and it is not at all obvious that this should be a good approximation for ^{18}O .

Use of $k = 4$ in the analysis of the Coulomb excitation measurements on ^{18}O would increase the derived value of $B(E2; 0^+ \rightarrow 2^+)$. For example, the published value of $39.0 e^2 \text{ fm}^4$ of Fewell *et al.* (1979) (for $k = 1$ and destructive interference, as implied by the LSF wavefunctions) would increase to $47.1 e^2 \text{ fm}^4$ for $k = 4$ (Kuehner *et al.* 1982), in good agreement with the adopted value $47.6 e^2 \text{ fm}^4$ of Ball *et al.* The $k = 1$ value of $45.3 e^2 \text{ fm}^4$ of Flaum *et al.* (1977) is already consistent within experimental errors with the value of Ball *et al.*, but Flaum *et al.* found that their derived value of $B(E2; 0^+ \rightarrow 2^+)$ was very insensitive to the value of k , being only $0.7 e^2 \text{ fm}^4$ less for $k = 0$. Thus, the use of $k = 4$ in the Coulomb excitation analyses would seem to make the derived values of $B(E2; 0^+ \rightarrow 2^+)$ more consistent among themselves, and also with the DBLA value obtained by Ball *et al.*

Changing k in the analysis of Fewell *et al.* (1979) also changes the derived value of Q_{2+} from $-2.3 e \text{ fm}^2$ for $k = 1$ to $1.0 e \text{ fm}^2$ for $k = 4$ (Kuehner *et al.* 1982). The Q_{2+} value of Flaum *et al.* (1977) is more sensitive to k , and would probably also be near zero for $k = 4$. Most model calculations have given $Q_{2+} \approx -5 e \text{ fm}^2$ (see Table 1 of Fewell *et al.*). Vold *et al.* (1977) have pointed out that values of Q_{2+} near zero can be obtained with the LSF model provided the collective states belong to a triaxially deformed band rather than one with axial symmetry. Positive values of Q_{2+} have been predicted in calculations based on energy-weighted sum rules (Koo 1979; Koo and Tassie 1979).

We note that the crude wavefunctions (14) used here for ^{20}Ne give $k(^{20}\text{Ne}) \approx 2.3$ (assuming $E_g - E_0 \approx 20 \text{ MeV}$). Such a moderately large value of k could contribute to the discrepancy between calculated and experimental values of $Q_{2+}(^{20}\text{Ne})$, which was pointed out for example by Spear (1981), since the experimental values were based on the assumption that $k = 1$.

In summary, these calculations and estimates suggest that $k(^{18}\text{O}) \approx 4$, which is sufficiently large to remove the discrepancy between values of $B(E2; 0^+ \rightarrow 2^+)$ for ^{18}O derived from Coulomb excitation and from other measurements.

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References

- Ajzenberg-Selove, F. (1978). *Nucl. Phys. A* **300**, 1.
- Ajzenberg-Selove, F. (1982). *Nucl. Phys. A* **375**, 1.
- Ball, G. C., Alexander, T. K., Davies, W. G., Forster, J. S., and Mitchell, I. V. (1982). *Nucl. Phys. A* **377**, 268.
- Barker, F. C. (1982a). *Aust. J. Phys.* **35**, 291.
- Barker, F. C. (1982b). *Aust. J. Phys.* **35**, 301.
- Berant, Z., Broude, C., Engler, G., and Start, D. F. H. (1974). *Nucl. Phys. A* **225**, 55.
- Disdier, D. L., Ball, G. C., Häusser, O., and Warner, R. E. (1971). *Phys. Rev. Lett.* **27**, 1391.
- Fewell, M. P., Baxter, A. M., Kean, D. C., Spear, R. H., and Zabel, T. H. (1979). *Nucl. Phys. A* **321**, 457.
- Flaum, C., Barrette, J., Le Vine, M. J., and Thorn, C. E. (1977). *Phys. Rev. Lett.* **39**, 446.
- Gemmeke, H., Deluigi, B., Lassen, L., and Scholz, D. (1978). *Z. Phys. A* **286**, 73.
- Häusser, O., McDonald, A. B., Alexander, T. K., Ferguson, A. J., and Warner, R. E. (1973). *Nucl. Phys. A* **212**, 613.
- Koo, W. K. (1979). *Phys. Lett. B* **87**, 307.
- Koo, W. K., and Tassie, L. J. (1979). *Nucl. Phys. A* **315**, 21.
- Kuehner, J. A., Spear, R. H., Vermeer, W. J., Esat, M. T., Baxter, A. M., and Hinds, S. (1982). A measurement of the giant-dipole-resonance contribution to the Coulomb excitation of ^{17}O . *Phys. Lett.* (to be published).
- Lawson, R. D., Serduke, F. J. D., and Fortune, H. T. (1976). *Phys. Rev. C* **14**, 1245.
- Levinger, J. S. (1957). *Phys. Rev.* **107**, 554.
- Spear, R. H. (1981). *Phys. Rep.* **73**, 369.
- Vold, P. B., Cline, D., Russo, P., Sprinkle, J. K., Scharenberg, R. P., and Mitchell, R. J. (1977). *Phys. Rev. Lett.* **39**, 325.
- Woodworth, J. G., *et al.* (1979). *Phys. Rev. C* **19**, 1667.

