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The Extremely Large Effective Coulomb Matrix Element of $^{66}Ge \rightarrow ^{66}Ga$

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Abstract

A calculation using Nilsson wavefunctions and an effective one-body spheroidal Coulomb potential yields extremely large values for the Coulomb matrix element even for relatively small deformations of ^{66}Ge.

1. Introduction

The Fermi nuclear matrix element is given by

$$M_{\mathrm{F}} = \langle \mathrm{f} \,|\, F_{\pm} \,|\, \mathrm{i} \rangle,$$

where $|\,\mathrm{i}\rangle$ and $|\,\mathrm{f}\rangle$ are respectively the initial and final state wavefunctions and $F_{\pm}$ the appropriate operator. In the CVC theory (Feynman and Gell-Mann 1958) we have

$$F_{\pm} = T_{\pm} = \tau_{\pm} + \pi_{\pm} + \dots .$$

Here $\tau_{\pm} = \int \bar{\psi}_{\mathrm{N}}(x)\, \tau_{\pm}\, \psi_{\mathrm{N}}(x)\, \mathrm{d}^3 x$ is the isospin operator for the bare nucleons, $\pi_{\pm}$ that for π mesons, and other operators can be included if we do not restrict ourselves to just the pions. This naturally leads to the following selection rule for $0^+ \rightarrow 0^+$ β decays:

$$\Delta T = 0 \qquad \text{and} \qquad \Delta J = 0 .$$

Therefore, the Fermi nuclear matrix element for a β transition between states that differ in isospin should be zero, and a nonvanishing M_{F} for such decays should therefore be attributed to the presence of isospin impurities from charge-dependent effects. In this paper, the charge-dependent effect we consider is that due to the Coulomb potential from which we obtain the effective Coulomb matrix elements $\langle V_{\mathrm{CD}} \rangle$.

If the CVC theory is not valid, then there is another reason why M_{F} should not be zero. In the conventional theory we have $F_{\pm} = \tau_{\pm}$ (and not $F_{\pm} = T_{\pm}$), and therefore exchange currents inside the nucleus can induce Fermi transitions with $\Delta T \neq 0$. These contributions to M_{F} are as large as those due to charge-dependent effects (Yap 1968). However, we assume the validity of the CVC theory in this paper.

Recent studies (Raman *et al.* 1975) have shown that the effective Coulomb matrix elements $|\langle V_{\mathrm{CD}} \rangle|$ as deduced from β-decay experiments are generally rather small, almost all of which are less than 20 keV. The three exceptions are $^{64}Ga \rightarrow ^{64}Zn$,

^{57}Ni → ^{57}Co and ^{66}Ge → ^{66}Ga. The first has been studied in our previous paper (Yap and Saw 1981). The second is rather difficult to study theoretically because of the presence of the Gamow–Teller contributions. Here we look at the last: the β^+ decay from the 0^+ state of ^{66}Ge to the 0^+ state of ^{66}Ga, which has the experimental value of $|\langle V_{\rm CD}\rangle| \leqslant 143$ keV.

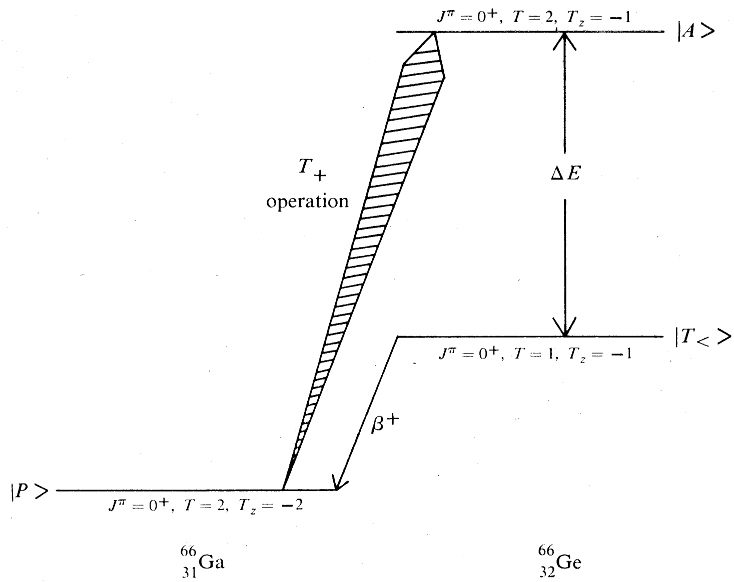

Fig. 1. Partial level diagram for the β^+ decay of ^{66}Ge to ^{66}Ga.

2. Calculation

Using the Nilsson (1955) model we calculate the effective Coulomb matrix elements $\langle V_{\rm CD}\rangle$ for the β^+ decay of the ground state of ^{66}Ge to the ground state of ^{66}Ga. The partial level diagram for this decay is shown in Fig. 1, where $|P\rangle$, $|A\rangle$ and $|T_<\rangle$ are the parent, analogue and antianalogue states respectively. The configuration of the Nilsson orbit for the ground-state wavefunction of ^{66}Ga

$$| J=M=K = 0, T=2, T_z=-2\rangle$$

is shown in Fig. 2*a*, where the values of $\Omega_i^\pi[N_i n_{zi} \Lambda_i], i = 1,2,3$, depend on the deformation parameter β of the parent nucleus in the case of β^+ decay. The assignments for $\Omega_i^\pi[N_i n_{zi} \Lambda_i]$ for various values of the deformation parameter in ^{66}Ge are as follows:

$$i = 1,\ \tfrac{3}{2}^-[312]; \qquad i = 2,\ \tfrac{1}{2}^-[310]; \qquad i = 3,\ \tfrac{3}{2}^-[301].$$

The respective configurations for the wavefunction of the analogue state and the antianalogue state (constructed so that it is orthogonal to the analogue state)

$$| J=M=K=0, T=2, T_z=-1\rangle, \qquad | J=M=K=0, T=1, T_z=-1\rangle$$

are shown in Figs 2*b* and 2*c* respectively.

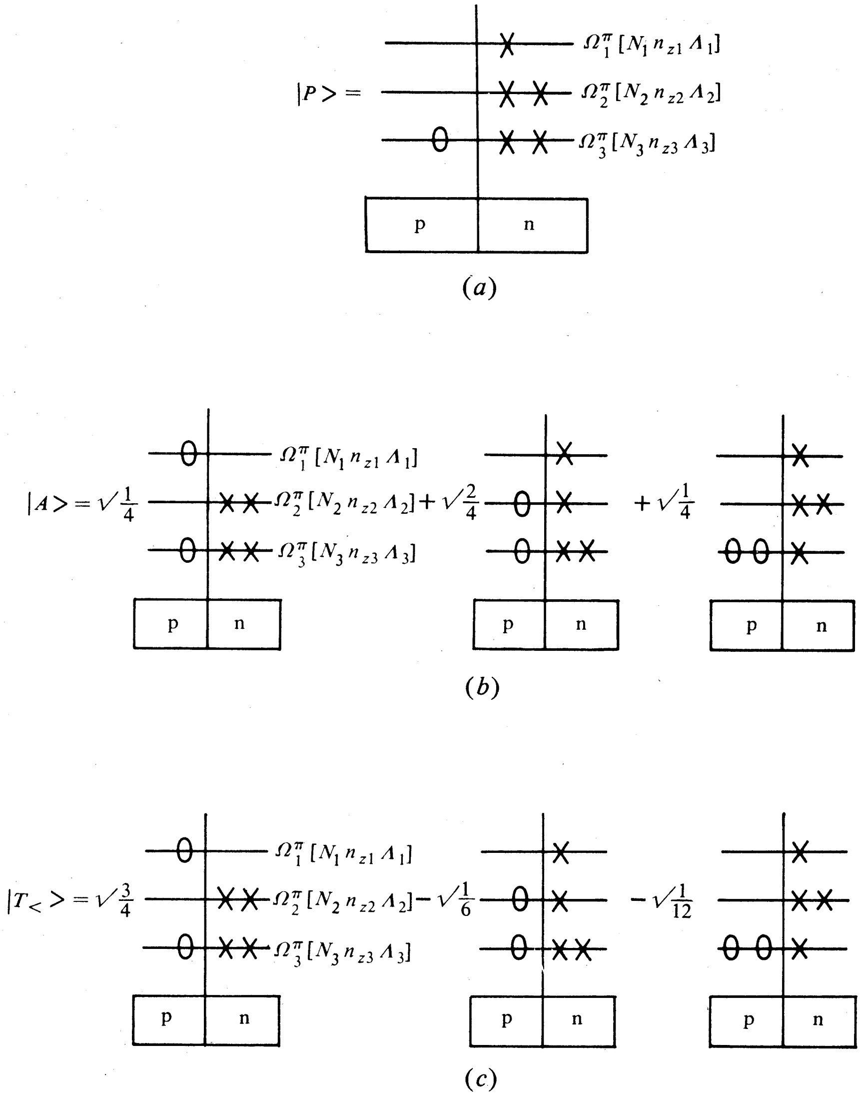

Fig. 2. Configurations for (*a*) the parent ground-state wavefunction in ^{66}Ga, and (*b*) the analogue and (*c*) the antianalogue wavefunctions in ^{66}Ge.

The Fermi matrix element M_{F} due to the Coulomb potential is

$$M_{\mathrm{F}} = \langle \mathrm{f} \mid T_- \mid \mathrm{i} \rangle = \langle P \mid T_- \mid (\mid T_< \rangle + \alpha \mid A \rangle)$$

$$= \alpha \langle P \mid T_- \mid A \rangle = 2\alpha ,$$

with the admixture coefficient α given by

$$\alpha = -\langle T_< \mid V_{\mathrm{CD}} \mid A \rangle / \Delta E ,$$

and V_{CD} given by (Damgard 1966)

$$V_{CD} = \{(Z-1)e^2/R\}\{\tfrac{3}{2}-\tfrac{1}{2}(r/R)^2\}+a(r/R)^2 Y_{20}, \qquad \text{for } r < R; \qquad (1a)$$

$$= \{(Z-1)e^2/r\}+a(R/r)^3 Y_{20}, \qquad \text{for } r > R, \qquad (1b)$$

where R is the radius of the nucleus and a is related to the Bohr deformation parameter β by

$$a = \tfrac{3}{5}\beta(Z-1)e^2/R.$$

Using equations (1) and Figs 2*b* and 2*c*, together with the assignments of orbitals given above, it is straightforward, though rather tedious, to calculate the effective Coulomb matrix elements $\langle T_< \mid V_{CD} \mid A\rangle = \langle V_{CD}\rangle$.

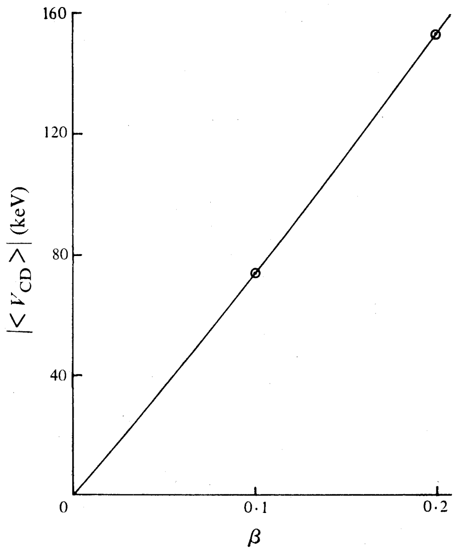

Fig. 3. Variation of $|< V_{CD} >|$ as a function of the deformation parameter β.

3. Results and Discussion

As $|\langle V_{CD}\rangle|$ could depend sensitively on the deformation parameter β, the theoretical values of $|\langle V_{CD}\rangle|$ should be calculated as a function of β, which is expected to be around 0·2 in the present case (Møller and Nix 1981). Fig. 3 shows the variation of $|\langle V_{CD}\rangle|$ with the deformation parameter β. We see that even if the prolate deformation of ^{66}Ga has the very small value of $\beta = 0{\cdot}1$, the effective Coulomb matrix element still has the extremely large value of 73 keV. We feel that it would be of much interest to have a reasonably accurate determination of the *ft* value of this decay. Furthermore, it is of interest to note that all three decays mentioned with large values of the Coulomb matrix element are β^+ transitions of medium-weight nuclei with $N \approx Z$.

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