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Drift and Diffusion of Electrons in an Atomic Gas between Closely Spaced Electrodes. II* Effect of Inelastic Collisions

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Abstract

Previously derived source functions and generalized electron absorption coefficients are employed in a new method to obtain the electron flux in an atomic gas between closely spaced electrodes taking into account inelastic collisions. Energy losses due to elastic collisions are assumed to be relatively unimportant compared with inelastic collisions. The initial energy of the electrons at the source and the applied electric field are assumed to be insufficiently large to produce more than one inelastic collision for any electron, but otherwise there is no restriction on the number of inelastic thresholds. Given these restrictions it is shown that the results obtained are consistent with the usual two-term approximation to the Boltzmann equation. It is further demonstrated that the electrodes have little effect on the electron flux when the generalized electron absorption coefficients are dominated by electron–atom collisions. Conditions under which inelastic collisions have a significant effect on the electron flux are derived. Finally, the connection between the present steady state analysis and quantities obtainable by a pulsed electron source experiment are discussed.

1. Introduction

Previously reported work which we refer to as Part I (McMahon 1983, present issue p. 27) is extended to include the effect of inelastic collisions on the electron flux between closely spaced electrodes. As emphasized in Part I, this theory is relevant to recent attempts to treat the boundary layer problem for electron swarms (Lowke *et al.* 1977; Braglia and Lowke 1979; Robson 1981, 1982; K. Ness and R. E. Robson, unpublished data; Chantry 1982; Braglia 1982). The approach used by the present author in Part I differs from the others by omitting the effect of energy losses by elastic collisions. Although this pays the price of restricting the analysis to heavy atomic gases with electrodes and sources well within the boundary layer thickness of each other, it does enable an analytical solution of the flux problem unrestricted by any particularly convenient forms of the energy dependence of the momentum transfer cross section. One payoff of this approach is a better physical understanding of how the absorbing boundary influences the flux of electrons. A second advantage is the discovery of methods by which electrode sensitive terms can be eliminated so as to obtain potentially useful relations between experimentally measurable quantities and the momentum transfer cross section. The third profitable outcome here is to show how inelastic collisions can be introduced into the theory and, although

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this no longer enables an analytical solution for the flux, the results can be described in straightforward physical terms that would be virtually impossible in a purely numerical approach. What the results show is that the inclusion of inelastic collisions in the theory is closely related to the boundary layer problem in that the generalized absorption coefficients and source strengths introduced in Part I can be used to describe the loss and gain of electrons in a given energy range by inelastic electron–atom collisions. It is believed that the insights provided by the present work will prove to be of value for more general developments of the boundary layer theory.

2. Inelastic Collision Effects on the Electron Flux and Number Density

To construct the electron flux with inelastic collisions we employ the somewhat unconventional approach of using physical arguments and results already obtained in Part I to guess the required result. This is then verified as the correct solution of the scalar and vector equations in the two-term approximation to the Boltzmann equation [Huxley and Crompton (1974), equations (6.36) and (6.37)]. For initial simplicity, we consider just one inelastic threshold. This case is readily extended provided that the initial energy of the electrons at the source and the electric field strength enable only one actual inelastic collision for any electron. Thus the theory applies for energies up to twice the lowest threshold energy. As in Part I we develop the basic results for infinite plane parallel electrodes A and B of separation L. As before, let there be a parallel source S of strength S electrons per second per unit area situated at a distance d from B.

To help guess the structure of the solution, we begin by regarding the swarm as undergoing elastic collisions everywhere between S and A except in the range $z, z+\mathrm{d}z$ measured from S (see Fig. 1 p. 31). Let the source strength for electrons of energy $\varepsilon'(z) = \varepsilon(z)-\varepsilon_i$ be $\mathrm{d}S_i$, which is just the rate of inelastic collisions in a slab of thickness $\mathrm{d}z$. We find then that

$$\mathrm{d}S_i = \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,\mathrm{d}z\,, \tag{1a}$$

where

$$\rho(z) = 4N q_i(\varepsilon-\varepsilon_i)\,. \tag{1b}$$

Here $q_i(\varepsilon-\varepsilon_i)$ is the inelastic cross section for the threshold energy ε_i and N is the number density of gas atoms. That $\mathrm{d}S_i$ is defined to be proportional to the local random flux introduces the assumption, as is shown later, that the major effect of inelastic collisions is through the distribution of electron energies f_0 rather than through their contribution to the momentum transfer cross section. Given the source $\mathrm{d}S_i$, the methods of Part I can be easily used to calculate the fluxes of electrons that have lost energy ε_i. The new problem is to find how to describe the effect of a distribution of sinks $\mathrm{d}S_i$ on the flux of those electrons that have not yet lost energy through inelastic collisions. This problem is equivalent to the boundary value problem of Part I but complicated by effective 'boundaries' distributed throughout the gas. We introduce the notation $\Gamma_{Ae}(z)$ and $\Gamma_{Ai}(z)$ to denote the flux from S to A at z of electrons that have not undergone and have undergone inelastic collisions respectively. By the continuity equation we have in general

$$\Gamma_{Ae}(z)+\Gamma_{Ai}(z) = \Gamma_A\,, \tag{2}$$

where Γ_{A} is independent of z. By considering the production of electrons in an infinitesimal slab one also finds by the continuity relation that

$$\mathrm{d}\Gamma_{\mathrm{Ai}}(z)/\mathrm{d}z = \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,. \tag{3}$$

With inelastic collisions only at $z, z+\mathrm{d}z$, the absorption coefficient $\alpha_{\mathrm{A}}(\bar{z})$ for $\bar{z} \geqslant z+\mathrm{d}z$ is still that given by equation (36a) of Part I. For $\bar{z} \leqslant z$ we introduce the notation $\alpha_{\mathrm{A}}(\bar{z};z)$ for the absorption coefficient at z, given inelastic collisions at $z, z+\mathrm{d}z$. By definition we have

$$\Gamma_{\mathrm{Ae}}(\bar{z}) = \tfrac{1}{4}\alpha_{\mathrm{A}}(\bar{z};z)\,n(\bar{z})\,c(\bar{z}), \qquad \bar{z} \leqslant z\,; \tag{4a}$$

$$= \tfrac{1}{4}\alpha_{\mathrm{A}}(\bar{z})\,n(z)\,c(z), \qquad \bar{z} \geqslant z+\mathrm{d}z\,. \tag{4b}$$

By the continuity relation the $\Gamma_{\mathrm{Ae}}(\bar{z})$ are actually both independent of $\bar{z}$. For convenience we can use equation (4a) at $\bar{z} = z$ to denote $\Gamma_{\mathrm{Ae}} \equiv \Gamma_{\mathrm{Ae}}(z,z)$ and $\Gamma_{\mathrm{Ae}}(\bar{z})$ at $\bar{z} \geqslant z+\mathrm{d}z$ we can denote as Γ'_{Ae}. We also write

$$\alpha_{\mathrm{A}}(z;z) = \alpha_{\mathrm{A}}(z)+\mathrm{d}\alpha_{\mathrm{A}}(z)\,, \tag{5}$$

and the problem is to relate $\mathrm{d}\alpha_{\mathrm{A}}(z)$ to $\rho(z)\,\mathrm{d}z$. Note that Γ_{Ae} only differs from Γ'_{Ae} by including a contribution which makes up those electrons that undergo inelastic collisions. We can write therefore

$$\Gamma_{\mathrm{Ae}} = \Gamma'_{\mathrm{Ae}}+\mathrm{d}\Gamma_{\mathrm{Ai}}+\mathrm{d}\Gamma_{\mathrm{Bi}}\,, \tag{6}$$

but by definition we have

$$\mathrm{d}\Gamma_{\mathrm{Ai}}+\mathrm{d}\Gamma_{\mathrm{Bi}} = \mathrm{d}S_{\mathrm{i}}\,. \tag{7}$$

By combining equations (4)–(7) and (1a) we find

$$\mathrm{d}\alpha_{\mathrm{A}}(z) = \rho(z)\,\mathrm{d}z\,. \tag{8}$$

Using equations (5) and (8) we can find $\alpha_{\mathrm{A}}(\bar{z};z)$ by applying the coordinate transformation equation (36a) of Part I, giving

$$\frac{1}{\alpha_{\mathrm{A}}(\bar{z};z)} = \frac{\varepsilon(\bar{z})}{\{\alpha_{\mathrm{A}}(z)+\rho(z)\,\mathrm{d}z\}\varepsilon(z)} + \frac{3\varepsilon(\bar{z})}{4eE}\int_{\varepsilon(^{-})}^{\varepsilon(z)} \frac{\mathrm{d}\varepsilon'}{\varepsilon' l(\varepsilon')}\,. \tag{9}$$

To be able to guess the required expressions for $\Gamma_{\mathrm{Ae}}(\bar{z})$, $\Gamma_{\mathrm{Ai}}(\bar{z})$ and $n(\bar{z})$ it is necessary to add the effect of one more region of inelastic collisions at say $z', z'+\mathrm{d}z'$, where we choose $z' > z+\mathrm{d}z$. All of these quantities in the three regions bounded by z and z' are affected in different ways by inelastic collisions. For $\bar{z} \leqslant z$, we need to define $\alpha_{\mathrm{A}}(\bar{z};z;z')$. This is easily obtained by the logical extension of equations (5) and (9) and we have

$$\frac{1}{\alpha_{\mathrm{A}}(\bar{z};z;z')} = \frac{\varepsilon(\bar{z})}{\{\alpha_{\mathrm{A}}(z;z')+\rho(z)\,\mathrm{d}z\}\varepsilon(z)} + \frac{3\varepsilon(\bar{z})}{4eE}\int_{\varepsilon(\bar{z})}^{\varepsilon(z)} \frac{\mathrm{d}\varepsilon'}{\varepsilon' l(\varepsilon')}\,. \tag{10}$$

For $z+\mathrm{d}z \leqslant \bar{z} \leqslant z'$ we can use the absorption coefficient $\alpha_{\mathrm{A}}(\bar{z};z')$ and for $\bar{z} \geqslant z'+\mathrm{d}z'$ the coefficient $\alpha_{\mathrm{A}}(\bar{z})$ applies. It is then possible to construct $\Gamma_{\mathrm{Ae}}(\bar{z})$, $\Gamma_{\mathrm{Ai}}(\bar{z})$ and $n(\bar{z})$

in the three regions of $\bar{z}$. This can be done by a somewhat brute force calculation; however, a much more elegant approach makes use of the coordinate transformation properties described below.

To begin we can extend equations (9) and (10) to absorption coefficients for the back electron flux in the B direction. Thus, in addition to equation (36b) of Part I, we define for $\alpha_B(\bar{z})$

$$\frac{1}{\alpha_B(\bar{z};z)} = \frac{\varepsilon(\bar{z})}{\{\alpha_B(z)+\rho(z)\,dz\}\varepsilon(z)} + \frac{3\varepsilon(\bar{z})}{4eE}\int_{\varepsilon(z)}^{\varepsilon(\bar{z})} \frac{d\varepsilon'}{\varepsilon' l(\varepsilon')}, \tag{11}$$

$$\frac{1}{\alpha_B(\bar{z};z';z)} = \frac{\varepsilon(\bar{z})}{\{\alpha_B(z';z)+\rho(z')\,dz'\}\varepsilon(z')} + \frac{3\varepsilon(\bar{z})}{4eE}\int_{\varepsilon(z')}^{\varepsilon(\bar{z})} \frac{d\varepsilon'}{\varepsilon' l(\varepsilon')}, \tag{12}$$

where $\alpha_B(\bar{z};z)$ is the absorption coefficient at $z+dz \leqslant \bar{z} \leqslant z'$, influenced by inelastic collisions at $z, z+dz$, and $\alpha_B(\bar{z};z';z)$ is the absorption coefficient at $\bar{z} \geqslant z'+dz' > z+dz$ influenced by both regions of inelastic collisions. Using equations (36) of Part I and equations (9)–(12) it is found that the following quantities are invariant under changes to $\bar{z}$:

$$\varepsilon(\bar{z})\,\alpha_A(\bar{z};z;z')\,\alpha_B(\bar{z})/\{\alpha_A(\bar{z};z;z')+\alpha_B(\bar{z})\}, \qquad \bar{z} \leqslant z < z'; \tag{13a}$$

$$\varepsilon(\bar{z})\,\alpha_A(\bar{z};z')\,\alpha_B(\bar{z};z)/\{\alpha_A(\bar{z};z')+\alpha_B(\bar{z};z)\}, \qquad z+dz \leqslant \bar{z} \leqslant z'; \tag{13b}$$

$$\varepsilon(\bar{z})\,\alpha_A(\bar{z})\,\alpha_B(\bar{z};z';z)/\{\alpha_A(\bar{z})+\alpha_B(\bar{z};z';z)\}, \qquad z < z'+dz' \leqslant \bar{z}. \tag{13c}$$

Other quantities of great utility are generalized source functions that take into account inelastic collisions. The obvious extensions of equation (37a) in Part I are

$$\frac{\Gamma_{Ae}(\bar{z})}{S_A(\bar{z})} = \frac{\alpha_A(\bar{z};z;z')}{\alpha_A(\bar{z};z;z')+\alpha_B(\bar{z})}, \qquad \bar{z} \leqslant z < z'; \tag{14a}$$

$$\frac{\Gamma_{Ae}(\bar{z})}{S_A(\bar{z};z)} = \frac{\alpha_A(\bar{z};z')}{\alpha_A(\bar{z};z')+\alpha_B(\bar{z};z)}, \qquad z+dz \leqslant \bar{z} \leqslant z'; \tag{14b}$$

$$\frac{\Gamma_{Ae}(\bar{z})}{S_A(\bar{z};z';z)} = \frac{\alpha_A(\bar{z})}{\alpha_A(\bar{z})+\alpha_B(\bar{z};z';z)}, \qquad z < z'+dz' \leqslant \bar{z}. \tag{14c}$$

Using equations (13a), (13b), (14a) and (14b) we note that the following quantities are invariant under changes of $\bar{z}$:

$$S_A(\bar{z})/\varepsilon(\bar{z})\,\alpha_B(\bar{z}), \qquad \bar{z} \leqslant z < z'; \tag{15a}$$

$$S_A(\bar{z};z)/\varepsilon(\bar{z})\,\alpha_B(\bar{z};z), \qquad z+dz \leqslant \bar{z} \leqslant z'; \tag{15b}$$

$$S_A(\bar{z};z';z)/\varepsilon(\bar{z})\,\alpha_B(\bar{z};z';z), \qquad z < z'+dz' \leqslant \bar{z}. \tag{15c}$$

The application of these source functions relies on the fact that they are continuous across the regions $z, z+dz$ and $z', z'+dz'$ where inelastic collisions occur. This is easily shown using the continuity relation. To calculate $\Gamma_{Ae}(\bar{z})$ for $z < z'+dz' \leqslant \bar{z}$ using equation (14c), one uses the invariant relation (15c) to introduce $S_A(z';z';z)$.

Then the continuity relation for S_A at z' can be used to get $S_A(z';z)$ and again the invariance relation (15b) can be employed to reduce this in terms of $S_A(z;z)$. After using the continuity of S_A at z and the transformation (15a) one finally obtains the expression for $\Gamma_{Ae}(\bar{z})$ in terms of S. The same method can be applied to all ranges of $\bar{z}$. To get $\Gamma_{Ai}(\bar{z})$ one has to know $n(z)$ and $n(z')$ so that the source strengths can be calculated using equation (1a). These are obtained from the general approach for finding $n(\bar{z})$. To get $n(\bar{z})$, for $z < z'+dz' \leqslant \bar{z}$ say, one can use the definition

$$\tfrac{1}{4}n(\bar{z})\,c(\bar{z}) = \{1/\alpha_A(\bar{z})\}\Gamma_{Ae}(\bar{z})\,, \tag{16}$$

and relate this to S through the calculation of $\Gamma_{Ae}(\bar{z})$ via equations (14). The result is most conveniently parametrized in terms of $R(\bar{z}) = n(\bar{z})/n_0(\bar{z})$, where $n_0(\bar{z})$ is the number density that would have existed if there had not been any inelastic collisions.

By examining the formulae for $n(\bar{z})$, $\Gamma_{Ae}(\bar{z})$ and $\Gamma_{Ai}(\bar{z})$ in the three ranges of $\bar{z}$ and retaining only first powers of $\rho(z)\,dz$ and $\rho(z')\,dz'$, the following are guessed as first approximations:

$$R(z) \approx 1 - \frac{\varepsilon(z)\,\alpha_A^2(z)}{\alpha_A(z)+\alpha_B(z)}\int_z^{L-d}\frac{\rho(z')\,dz'}{\varepsilon(z')\,\alpha_A^2(z')} - \int_0^z\frac{\rho(z')\,dz'}{\alpha_A(z')+\alpha_B(z')}, \tag{17a}$$

$$\Gamma_{Ae}(z) \approx \Gamma_{A0}\left(1 + \int_z^{L-d}\frac{\alpha_B(z')}{\alpha_A(z')}\frac{\rho(z')\,dz'}{\alpha_A(z')+\alpha_B(z')} - \int_0^z\frac{\rho(z')\,dz'}{\alpha_A(z')+\alpha_B(z')}\right), \tag{17b}$$

where Γ_{A0} is the flux that would exist for no inelastic collisions.

The exact form of $\Gamma_{Ai}(z)$ can be guessed immediately as

$$\Gamma_{Ai}(z) = \int_0^z\frac{\alpha'_A(z')}{\alpha'_A(z')+\alpha'_B(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,dz' - \int_z^{L-d}\frac{\alpha'_B(z')}{\alpha'_A(z')+\alpha'_B(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,dz'\,. \tag{18}$$

It is easily checked that equation (18) satisfies (3). The exact number density $n'(z)$ of electrons with energy $\varepsilon'(z) = \varepsilon(z)-\varepsilon_i$ can be deduced from the definition of $\alpha'_A(z)$ and $\alpha'_B(z)$. Due to the elementary source $dS_i(z')$ alone, the number density $n'(z)$ is given by

$$\begin{aligned}\tfrac{1}{4}n'(z)\,c'(z) &= \frac{1}{\alpha'_A(z)}\frac{\alpha'_A(z')}{\alpha'_A(z')+\alpha'_B(z')}\,dS_i(z'), &&\text{for } z \geqslant z'\,;\\ &= \frac{1}{\alpha'_B(z)}\frac{\alpha'_B(z')}{\alpha'_A(z')+\alpha'_B(z')}\,dS_i(z'), &&\text{for } z \leqslant z'\,.\end{aligned}$$

The logical generalization is

$$\tfrac{1}{4}n'(z)\,c'(z) = \int_0^z\frac{\alpha'_A(z')}{\alpha'_A(z)}\tfrac{1}{4}n(z')\,c(z')\frac{\rho(z')}{\alpha'_A(z')+\alpha'_B(z')}\,dz' + \int_z^{L-d}\frac{\alpha'_B(z')}{\alpha'_B(z)}\tfrac{1}{4}n(z')\,c(z')\frac{\rho(z')}{\alpha'_A(z')+\alpha'_B(z')}\,dz'\,. \tag{19}$$

By calculating the z derivative of $n'(z)$ and introducing $\Gamma_{\mathrm{Ai}}(z)$ via equation (18) one finds the result

$$\Gamma_{\mathrm{Ai}}(z) = \frac{eE\,l(\varepsilon'(z))}{3m\,c'(z)}n'(z) - \tfrac{1}{3}c'(z)\,l(\varepsilon'(z))\frac{\mathrm{d}n'(z)}{\mathrm{d}z}, \tag{20}$$

which closely resembles equation (9) of Part I except that the flux is now z dependent.

By the continuity equation (2) and equation (3) we require that the exact expression for $\Gamma_{\mathrm{Ae}}(z)$ satisfies

$$\mathrm{d}\Gamma_{\mathrm{Ae}}(z)/\mathrm{d}z = -\tfrac{1}{4}n(z)\,c(z)\,\rho(z). \tag{21}$$

In contrast the RHS of equation (17b) is easily seen to lead to $n_0(z)$ instead of $n(z)$ in equation (21). Thus equation (17b) is very close to the exact solution and it is obvious that we merely need to replace $\rho(z')$ by $R(z')\,\rho(z')$ in equations (17). One can indeed verify that this procedure is correct by asserting that $\Gamma_{\mathrm{Ae}}(z)$ satisfies the equation

$$\Gamma_{\mathrm{Ae}}(z) = \frac{eE\,l(z)}{3m\,c(z)}n(z) - \tfrac{1}{3}l(z)\,c(z)\frac{\mathrm{d}n(z)}{\mathrm{d}z}, \tag{22}$$

which like equation (20) has exactly the form of equation (9) in Part I. To fully justify the replacement $\rho(z) \rightarrow R(z)\,\rho(z)$ in equation (17a) and verify equation (22) from a more basic standpoint one has to return to the guessing procedure described above, but developed with somewhat greater accuracy. The details are somewhat tedious and the reader is referred to a full treatment of the problem given elsewhere (McMahon 1982).

So far our discussion has only considered inelastic collisions in the domain $0 \leqslant z \leqslant L-d$. The effect of inelastic collisions in the region $-d \leqslant z \leqslant 0$ on Γ_{Ae}, Γ_{Ai} and the electron number densities can be obtained by a symmetry argument after calculating $\Gamma_{\mathrm{Be}}(z)$, $\Gamma_{\mathrm{Bi}}(z)$ and the number densities in the region $z \leqslant 0$ for the case of inelastic collisions for $z \geqslant 0$ only. The details are given elsewhere (McMahon 1982). The final results are

$$\begin{aligned}\tfrac{1}{4}n(z)\,c(z) = \tfrac{1}{4}n_0(z)\,c(z) &- \int_z^{L-d} \frac{1}{\alpha_{\mathrm{B}}(z)}\frac{\alpha_{\mathrm{B}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z' \\ &- \int_{-d}^{z} \frac{1}{\alpha_{\mathrm{A}}(z)}\frac{\alpha_{\mathrm{A}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z',\end{aligned} \tag{23}$$

$$\begin{aligned}\Gamma_{\mathrm{Ae}}(z) = \tfrac{1}{4}\alpha_{\mathrm{A}}(z)\,n_0(z)\,c(z) &+ \int_z^{L-d} \frac{\alpha_{\mathrm{B}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z' \\ &- \int_{-d}^{z} \frac{\alpha_{\mathrm{A}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z'.\end{aligned} \tag{24}$$

Equations (18) and (19) are only slightly modified if the lower limit of 0 in the first integral is replaced by $-d$. Equations (20) and (22) are still satisfied.

It only remains to show that our expressions for $\Gamma_{\mathrm{Ae}}(z)$, $\Gamma_{\mathrm{Bi}}(z)$ and $n(z)$, $n'(z)$ are consistent with the two-term approximation of the Boltzmann equation taking into account inelastic collisions. This demonstration is given in the Appendix and proves that indeed isotropic inelastic collisions can be treated mathematically as electron 'sources' and absorbing 'boundaries' distributed throughout the gas.

3. Discussion of Inelastic Collision Effects on the Electron Flux

Adding $\Gamma_{Ae}(z)$ and $\Gamma_{Ai}(z)$ we have for $z \geqslant 0$

$$\Gamma_A = \Gamma_{A0} + \int_z^{L-d} \tfrac{1}{4}n(z')\,c(z')\,\rho(z')\{P_{B0}(z') - P'_{B0}(z')\}\,dz' - \int_{-d}^{z} \tfrac{1}{4}n(z')\,c(z')\,\rho(z')\{P_{A0}(z') - P'_{A0}(z')\}\,dz'\,, \tag{25a}$$

where

$$P_{A0}(z) = \alpha_A(z)/\{\alpha_A(z)+\alpha_B(z)\}\,, \qquad P_{B0}(z) = 1 - P_{A0}(z)\,, \tag{25b}$$

and similarly for $P'_{A0}(z)$ and $P'_{B0}(z)$. Here $P_{A0}(z)$ is the probability that an electron starting at z is absorbed by electrode A (rather than B) given that there are no inelastic collisions. In equation (25a), $z \geqslant 0$ is arbitrary because Γ_A is not explicitly z-dependent by the continuity relation.

If it happens that $\alpha_A(z)/\alpha_B(z) = \alpha'_A(z)/\alpha'_B(z)$ then $\Gamma_A = \Gamma_{A0}$. In other words, inelastic collisions only affect Γ_A by changing the probability of an electron which suffers an inelastic collision at any z being absorbed by electrode A. It can be seen from equations (36) of Part I that if $q_m(\varepsilon) \propto \varepsilon$ and if the most important z values for inelastic collisions are such that $\alpha_A(z)$, $\alpha_B(z)$, $\alpha'_A(z)$ and $\alpha'_B(z)$ are dominated by electron–atom collisions, then $P_{A0} \approx P'_{A0}$ and $P_{B0} \approx P'_{B0}$ so that inelastic collisions have little influence on Γ_A in this case. At the surface of A, P_{B0} and P'_{B0} are much smaller than P_{A0} and P'_{A0} for reasonable reflection probabilities r_A and r'_A. Then $P_{A0} \approx P'_{A0}$ for $z \approx L-d$, so that the effect of the electrode surface is not as significant as one might expect (see the discussion in Section 4 of Part I). Another interesting case is $E = 0$ where we have

$$\frac{1}{\alpha_A(z)} = \frac{1}{\alpha_A} + \frac{3(L-d-z)}{4l(\varepsilon)}\,, \qquad \frac{1}{\alpha_B(z)} = \frac{1}{\alpha_B} + \frac{3(d+z)}{4l(\varepsilon)}\,, \tag{26a, b}$$

and similarly for $\alpha'_A(z)$ and $\alpha'_B(z)$. Because electron–atom collisions dominate P_{A0}, P_{B0}, P'_{A0} and P'_{B0}, we see that again $P_{A0} \approx P'_{A0}$ and $P_{B0} \approx P'_{B0}$ quite independently of the energy dependence of $l(\varepsilon)$. It appears that significant effects on Γ_A due to inelastic collisions are only possible if we have $q_m(\varepsilon)$ not simply proportional to ε and a nonzero electric field.

As a check on this conclusion we should examine what happens if there is a very large perturbation on $n(z)$ due to inelastic collisions. A model that can be solved analytically is that of a resonant inelastic cross section

$$q_i(\varepsilon - \varepsilon_i) = \Delta\varepsilon_i\,\bar{q}_i(0)\,\delta(\varepsilon - \varepsilon_i)\,,$$

where $\Delta\varepsilon_i$ is a width factor and $\bar{q}_i(0)$ gives the scale of the cross section. Let inelastic collisions only occur for $0 \leqslant z_i \leqslant L-d$, where $\varepsilon(z_i) = \varepsilon_i$. By equation (23), $R(z) = R(z_i)$ for $z \geqslant z_i$ and

$$R(z_i) = \left(1 + \frac{4N\,\Delta\varepsilon_i\,\bar{q}_i(0)}{eE\{\alpha_A(z_i)+\alpha_B(z_i)\}}\right)^{-1}, \tag{27}$$

while Γ_A is given by

$$\Gamma_A = \Gamma_{A0}[R(z_i)+\{1-R(z_i)\}P_{A0}(z_i)/P'_{A0}(z_i)]. \tag{28}$$

A large perturbation on $n(z)$ exists if for instance $\Delta\varepsilon_i \gg eEl_i$, where $l_i = \{N\,\bar{q}_i(0)\}^{-1}$. For $z_i = L-d$ we see that $P_{A0} \approx P'_{A0}$ as explained previously, and by equation (28) we have $\Gamma_A \approx \Gamma_{A0}$. For z_i at least several mean free paths from A then $P_{A0} \approx P'_{A0}$ only for E sufficiently small or $q_m(\varepsilon) \propto \varepsilon$. In all the latter cases, $\Gamma_A \approx \Gamma_{A0}$ even though very large effects of inelastic collisions exist for $n(z)$.

It is useful to obtain an estimate of E which will produce an appreciable inelastic collision effect on Γ_A. If $q_m(\varepsilon')/\varepsilon'$ is expanded out in powers of $\varepsilon'-\varepsilon(z)$ we find for example that in equations (36) of Part I

$$\frac{1}{\alpha_A(z)} \approx \frac{\varepsilon(z)}{\alpha_A\,\varepsilon_A}+\frac{3N(L-d-z)}{4}\left\{q_m(\varepsilon(z))+\tfrac{1}{2}\varepsilon(z)\{\varepsilon_A-\varepsilon(z)\}\left(\frac{\partial q_m(\varepsilon')/\varepsilon'}{\partial\varepsilon'}\right)_{\varepsilon(z)}\right.$$
$$\left.+O[\{\varepsilon_A-\varepsilon(z)\}^2]\right\}, \tag{29a}$$

and that $E \neq 0$ gives for $\alpha_B(z)$

$$\frac{1}{\alpha_B(z)} \approx \frac{\varepsilon(z)}{\alpha_B\,\varepsilon_B}+\frac{3N(d+z)}{4}\left\{q_m(\varepsilon(z))-\tfrac{1}{2}\varepsilon(z)\{\varepsilon(z)-\varepsilon_B\}\left(\frac{\partial q_m(\varepsilon')/\varepsilon'}{\partial\varepsilon'}\right)_{\varepsilon(z)}\right.$$
$$\left.+O[\{\varepsilon(z)-\varepsilon_B\}^2]\right\}. \tag{29b}$$

Hence we find, where electron–atom collisions dominate,

$$\frac{P_{A0}(z)}{P'_{A0}(z)} \approx 1+\tfrac{1}{2}eE(L-d-z)\left\{\varepsilon'(z)\left(\frac{\partial q_m(\varepsilon')/\varepsilon'}{\partial\varepsilon'}\right)_{\varepsilon'(z)}-\varepsilon(z)\left(\frac{\partial q_m(\varepsilon')/\varepsilon'}{\partial\varepsilon'}\right)_{\varepsilon(z)}\right\}$$
$$+O[\{\varepsilon_A-\varepsilon(z)\}^2]+O[\{\varepsilon(z)-\varepsilon_B\}^2]. \tag{30}$$

When the second term on the RHS of equation (30) becomes comparable with ± 1, appreciable inelastic collision effects should exist for Γ_A. Equation (30) explicitly shows that these effects require deviations from a simple linear energy dependence of $q_m(\varepsilon)$. It should be noted that our discussion so far has assumed $\varepsilon'_A \geq 0$ and $\varepsilon'_B \geq 0$ so that electrons which have undergone inelastic collisions are still energetic enough to reach both electrodes. If this is not the case we then have $P'_{A0}(z) = 1$ and because in general $P_{A0}(z) \neq 1$, there is always an effect of inelastic collisions on Γ_A.

To repeat the exercise in Section 8 of Part I of finding relationships between experimentally observable quantities and the momentum transfer cross section is more complicated when inelastic collisions have significant effects on the electron flux. Inelastic collision effects could in principle be distinguished in observations of Γ_A/Γ_B through their influence on the density dependence. When electron–atom collisions dominate we have

$$P_{A0}(z) = \alpha_B^{-1}(z)/\{\alpha_A^{-1}(z)+\alpha_B^{-1}(z)\}$$
$$\approx \left(\int_{\varepsilon_B}^{\varepsilon(z)}\frac{d\varepsilon'}{\varepsilon' l(\varepsilon')}\middle/\int_{\varepsilon_B}^{\varepsilon_A}\frac{d\varepsilon'}{\varepsilon' l(\varepsilon')}\right)$$

$$\times\left[1+\left(\frac{3\varepsilon_{\mathrm{B}}\alpha_{\mathrm{B}}}{4eE}\int_{\varepsilon_{\mathrm{B}}}^{\varepsilon(z)}\frac{\mathrm{d}\varepsilon'}{\varepsilon' l(\varepsilon')}\right)^{-1}\right.$$

$$\left.-\left\{\frac{3}{4}\left(\frac{1}{\varepsilon_{\mathrm{A}}\alpha_{\mathrm{A}}}+\frac{1}{\varepsilon_{\mathrm{B}}\alpha_{\mathrm{B}}}\right)^{-1}\frac{1}{eE}\int_{\varepsilon_{\mathrm{B}}}^{\varepsilon_{\mathrm{A}}}\frac{\mathrm{d}\varepsilon'}{\varepsilon' l(\varepsilon')}\right\}^{-1}\right], \tag{31}$$

which shows that the dominant part of $P_{\mathrm{A}0}(z)$ is density independent and the first correction (assuming α_{A} and α_{B} are density independent) is proportional to N^{-1}. In equation (25a), $\rho(z')$ is proportional to N. The gas density dependence of $n(z)$ is gauged a little easier if we work with the ratio $R(z) = n(z)/n_0(z)$. By the first iteration of equation (17a) for $R(z)$ we see that the most important density dependent correction to $R(z) \approx 1$ is proportional to N^2, partly because $\rho(z') \propto N$ and partly because $\{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')\}^{-1} \propto N$. The resonant inelastic cross section model shows this density dependence (see equation 27). This model shows further that for strong inelastic collision effects $R(z)$ should vary as N^{-2}. For inelastic collision effects on $n(z)$ which are not too strong, equation (25a) shows that $\Gamma_{\mathrm{A}}/\Gamma_{\mathrm{A}0} \approx 1+O(N^2)$. The density dependence of $P_{\mathrm{A}0}(z)$ only contributes to a correction of $O(N^3)$. When we combine these results with the analysis of $\Gamma_{\mathrm{A}0}$ given in Section 8 of Part I we find that equation (40) there is replaced by

$$\frac{\Gamma_{\mathrm{A}}}{\Gamma_{\mathrm{B}}} \approx \left(\int_{\varepsilon_{\mathrm{B}}}^{\varepsilon_{\mathrm{S}}} q_{\mathrm{m}}(\varepsilon')/\varepsilon'\,\mathrm{d}\varepsilon' \Big/ \int_{\varepsilon_{\mathrm{S}}}^{\varepsilon_{\mathrm{A}}} q_{\mathrm{m}}(\varepsilon')/\varepsilon'\,\mathrm{d}\varepsilon'\right)$$

$$\times\left(\frac{1+N^{-1}A_{\mathrm{A}}+NB_{\mathrm{A}}+N^2C_{\mathrm{A}}}{1+N^{-1}A_{\mathrm{B}}+NB_{\mathrm{B}}+N^2C_{\mathrm{B}}}\right), \tag{32}$$

provided that $|R(z)-1| \ll 1$. For a strong inelastic collision effect on the distribution function [i.e. $R(z) \ll 1$] and assuming $R(z) \propto N^{-2}$, we see that a significant deviation from the zeroth density power of equation (32) would exist.

The theory is simplified if the electrons that have undergone inelastic collisions can be separated from the others. A method of 'scanning' the energy distribution is described in Section 9 of Part I. Suppose a fine mesh grid is present at say $z = z_0$. Then the flux of electrons that have lost energy is $\Gamma_{\mathrm{Ai}}(z_0)$ and the rest of the electrons make up the flux $\Gamma_{\mathrm{Ae}}(z_0)$. The grid–anode system acts as a single effective electrode which has some influence on the latter two fluxes. However, we omit the surface effects and assume as usual that electron–atom collisions dominate. We can regard $\Gamma_{\mathrm{Ai}}(z_0)$ and $\Gamma_{\mathrm{Ae}}(z_0)$ as contributing to a source function for electrons near the grid and further let us assume that the grid–anode separation is small enough to neglect further inelastic collisions between them. Then equation (46b) of Part I in the present case is equivalent to

$$F_{\mathrm{A}}(z_0,\varepsilon) = \{\alpha_{\mathrm{A}}(z_0,\varepsilon)+\alpha_{\mathrm{G}}(z_0,\varepsilon)\}^{-1}$$

$$\times\{\Gamma_{\mathrm{Ai}}(z_0)\,\delta(\varepsilon-\varepsilon'(z_0))+\Gamma_{\mathrm{Ae}}(z_0)\,\delta(\varepsilon-\varepsilon(z_0))\}, \tag{33}$$

where α_{G} is introduced here to represent the fact that virtually all back-scattered electrons go to the grid rather than the cathode or electrode B. At $z = z_0$, α_{G} should be much larger than α_{A}. If α_{G} is not too strongly energy dependent then information about each separate Γ_{Ai} and Γ_{Ae} can be obtained via equation (46a) of Part I.

4. Relation of Steady State to Pulsed Electron Source Transport Theory

Rather than use continuous electron currents, an idealized experiment may instead use a pulsed electron source S. It is necessary to solve the full time dependent scalar and vector equations to get the time dependent pulse shapes of electron arrivals at the electrodes. Nevertheless, results obtained by our time independent analysis are still applicable. For instance,

$$P_{\mathrm{Ae}} = \Gamma_{\mathrm{Ae}}(L-d)/(\Gamma_{\mathrm{A}}+\Gamma_{\mathrm{B}}) \tag{34}$$

is the probability that an electron reaches and is absorbed by electrode A without loss of energy in the gas. Similarly one can define P_{Be}, P_{Ai} and P_{Bi}. If these probabilities can be obtained by distinguishing the two groups of electrons by their different contributions to the current pulse shape, then information on $q_{\mathrm{m}}(\varepsilon)$ and $q_{\mathrm{i}}(\varepsilon-\varepsilon_{\mathrm{i}})$ is obtainable using the theoretical results already given. The average time for the electrons in each energy group to reach the electrodes can be calculated from the time independent theory. The time element $\mathrm{d}t$ is related to the number of collisions $\mathrm{d}X$ by

$$\mathrm{d}t = \{l(\varepsilon)/c\}\,\mathrm{d}X.$$

Then using equation (30) of Part I specialized to infinite plane parallel electrodes we find that the average S to A transit time for electrons when there are no inelastic collisions is

$$t_0(\mathrm{S},\mathrm{A}) = \tfrac{8}{3}(\tfrac{1}{2}m)^{\frac{1}{2}}\frac{1}{eE\alpha_{\mathrm{A}}\,\varepsilon_{\mathrm{A}}}(\varepsilon_{\mathrm{A}}^{3/2}-\varepsilon_{\mathrm{S}}^{3/2}) + 3(\tfrac{1}{2}m)^{\frac{1}{2}}\frac{1}{(eE)^2}\int_{\varepsilon_{\mathrm{S}}}^{\varepsilon}\varepsilon^{\frac{1}{2}}\,\mathrm{d}\varepsilon\int_{\varepsilon}^{\varepsilon_{\mathrm{A}}}\frac{\mathrm{d}\varepsilon'}{\varepsilon'\,l(\varepsilon')}. \tag{35}$$

This is not equivalent to the average transit time of electrons that do not suffer inelastic collisions even though inelastic collisions are energetically possible. At first sight this may seem a surprising statement. Before discussing it in detail we examine a closely related phenomenon.

Let us define the probability of electrons in the zero energy loss group starting from S being absorbed by A as

$$P_{\mathrm{A}}(0) = \Gamma_{\mathrm{Ae}}(L-d)/\{\Gamma_{\mathrm{Ae}}(L-d)+\Gamma_{\mathrm{Be}}(-d)\}.$$

This probability can be obtained using equation (24) and its obvious extension to $\Gamma_{\mathrm{Be}}(z)$:

$$P_{\mathrm{A}}(0) = \left(\Gamma_{\mathrm{A0}} - \int_{-d}^{L-d}\frac{\alpha_{\mathrm{A}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z'\right)$$

$$\times\left(S - \int_{-d}^{L-d}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\,\mathrm{d}z'\right)^{-1}, \tag{36}$$

and $P_{\mathrm{A}}(0)$ is apparently not equal to $P_{\mathrm{A0}}(0)$ which is just Γ_{A0}/S. In order for these two probabilities to be equal it is easily shown that the condition

$$\int_{-d}^{L-d}\tfrac{1}{4}n(z')\,c(z')\,\rho(z')\left(\frac{\alpha_{\mathrm{A}}(z')}{\alpha_{\mathrm{A}}(z')+\alpha_{\mathrm{B}}(z')} - \frac{\alpha_{\mathrm{A}}(0)}{\alpha_{\mathrm{A}}(0)+\alpha_{\mathrm{B}}(0)}\right)\mathrm{d}z' = 0$$

needs to be satisfied. There is no reason why this should hold and, for example, it is easily shown to be false in the model of a resonant inelastic cross section. Similar to the statement that the average transit times of electrons that do not undergo inelastic collisions depend on whether inelastic collisions are energetically possible, we may find equally surprising the statement that the probability of transfer from S to A of the same electron group is also dependent on whether inelastic collisions are energetically possible. This initial surprise stems from the fact that any electron which itself does not undergo an inelastic collision does not 'know' anything about inelastic processes in the sense that it does not interact significantly with electrons that have lost energy. The solution to this paradox is that the averaging procedure is influenced by whether or not inelastic collisions occur. For instance when inelastic collisions are possible, Γ_{Ae} is more heavily weighted to those electron trajectories which have the lowest number of electron–atom collisions. Electrons with the longest paths to A are more likely to be selected out of the zero energy loss group.

We can build up the expression for $t(\mathrm{S, A})$ for the zero energy loss electron group when inelastic collisions can occur using the techniques of Section 2, assuming $\rho(z) \neq 0$ only for $z \geqslant 0$. We begin by noting that with no inelastic collisions

$$\mathrm{d}X(\varepsilon) = \frac{4}{eE\, l(\varepsilon)\, \alpha_{\mathrm{A}}(\varepsilon)}\, \mathrm{d}\varepsilon\,,$$

which is equivalent to equation (30) of Part I when $\mathrm{d}a = \mathrm{d}a_{\mathrm{A}}$ for parallel plane electrodes. Let inelastic collisions only occur in the range $z, z+\mathrm{d}z$. The average time interval corresponding to the distance $\mathrm{d}X(\varepsilon;\bar{z};z)$ where $\bar{z} \leqslant z$ is just

$$\mathrm{d}t(\varepsilon;\bar{z};z) = 4(\tfrac{1}{2}m)^{\frac{1}{2}} \frac{\varepsilon^{-\frac{1}{2}}}{eE\, \alpha_{\mathrm{A}}(\bar{z};z)}\, \mathrm{d}\varepsilon\,, \tag{37a}$$

where $\varepsilon(\bar{z}) \equiv \varepsilon$. For $\bar{z} \geqslant z+\mathrm{d}z$ the time interval for electrons that do not suffer inelastic collisions becomes

$$\mathrm{d}t(\varepsilon;\bar{z}) = 4(\tfrac{1}{2}m)^{\frac{1}{2}} \frac{\varepsilon^{-\frac{1}{2}}}{eE\, \alpha_{\mathrm{A}}(\bar{z})}\, \mathrm{d}\varepsilon\,, \tag{37b}$$

whereas for those electrons that do lose energy equation (37b) is to be modified by replacing $\alpha_{\mathrm{A}}(\bar{z})$ by $\alpha'_{\mathrm{A}}(\bar{z})$. The net average times of electron transit are then obtained by integrating between ε_{S} and $\varepsilon(z)$ in the case of equation (37a) and integrating equation (37b) between $\varepsilon(z)$ and ε_{A}. By affecting the diffusion modified drift velocity of the electrons via $\alpha_{\mathrm{A}}(\bar{z};z)$, the average transit time of electrons in the zero energy loss group are influenced by inelastic collisions.

One can go further and introduce two regions of inelastic collisions $z, z+\mathrm{d}z$ and $z', z'+\mathrm{d}z'$. Because of the connection of $\Gamma_{\mathrm{Ae}}(\bar{z})$ with $\alpha_{\mathrm{A}}(\bar{z};z;z')$, $\alpha_{\mathrm{A}}(\bar{z};z')$ and $\alpha_{\mathrm{A}}(\bar{z})$ in the regions $\bar{z} \leqslant z < z'$, $z+\mathrm{d}z \leqslant \bar{z} < z'$ and $\bar{z} \geqslant z'+\mathrm{d}z'$ respectively, it is evident that $\mathrm{d}t(\bar{z})$ is always related to $n(\bar{z})$ and $\Gamma_{\mathrm{Ae}}(\bar{z})$ by

$$\mathrm{d}t(\bar{z}) = \{n(\bar{z})/\Gamma_{\mathrm{Ae}}(\bar{z})\}\mathrm{d}\bar{z}\,,$$

which leads us to

$$t(\mathrm{S, A}) = \int_0^{L-d} n(z)/\Gamma_{\mathrm{Ae}}(z)\, \mathrm{d}z\,. \tag{38}$$

Equation (35) is a special case where $\Gamma_{\mathrm{Ae}}(z) = \Gamma_{\mathrm{A0}}$. The same approach can be applied for electrons that suffer inelastic collisions. The average transit time of electrons that have an inelastic collision at z is

$$t'(\mathrm{S},\mathrm{A};z) = \int_0^z n(z')/\Gamma_{\mathrm{Ae}}(z')\,\mathrm{d}z' + \int_z^{L-d} 4/c'(z')\,\alpha'_{\mathrm{A}}(z')\,\mathrm{d}z', \tag{39a}$$

where we have already substituted for the quantity

$$\mathrm{d}n'(z';z)/\mathrm{d}\Gamma_{\mathrm{Ai}}(z) = 4/c'(z')\,\alpha'_{\mathrm{A}}(z'). \tag{39b}$$

Here $\mathrm{d}n'(z';z)$ represents the contribution to the number density of electrons which have inelastic collisions at $z, z+\mathrm{d}z$, and $\mathrm{d}\Gamma_{\mathrm{Ai}}(z)$ is their contribution to the flux for $z' \geqslant z$. Because these electrons are assumed not to undergo further inelastic collisions this latter flux is not z' dependent. The final expression for $t'(\mathrm{S},\mathrm{A})$ is obtained by weighting over the conditional probability that the inelastic collision occurs in the range $z, z+\mathrm{d}z$ given that an inelastic collision definitely occurs between $z = 0$ and $L-d$. Of the electrons that are absorbed by electrode A, the probability that they have suffered an inelastic collision is $\Gamma_{\mathrm{Ai}}(L-d)/\Gamma_{\mathrm{A}}$. The contribution of $z, z+\mathrm{d}z$ to this probability is

$$\Gamma_{\mathrm{A}}^{-1}\{\mathrm{d}\Gamma_{\mathrm{Ai}}(z)/\mathrm{d}z\}\,\mathrm{d}z = \tfrac{1}{4}\Gamma_{\mathrm{A}}^{-1}\,n(z)\,c(z)\,\rho(z).$$

The required conditional probability is $\Gamma_{\mathrm{A}}/\Gamma_{\mathrm{Ai}}(L-d)$ times this expression, and thus we find

$$t'(\mathrm{S},\mathrm{A}) = \frac{1}{\Gamma_{\mathrm{Ai}}(L-d)}\int_0^{L-d} \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,t'(\mathrm{S},\mathrm{A};z)\,\mathrm{d}z. \tag{40}$$

Equations (38), (39a) and (40) assume that inelastic collisions only occur for $z \geqslant 0$. Equation (38) is not affected by inelastic collisions for $z \leqslant 0$ other than through $n(z)$ and $\Gamma_{\mathrm{Ae}}(z)$ being affected by these collisions. Equations (39a) and (40) need to introduce $\rho(z)$ for $z \leqslant 0$ explicitly. For $z \leqslant 0$ we have

$$t'(\mathrm{S},\mathrm{A};z) = \int_z^0 n(z')/\Gamma_{\mathrm{Be}}(z')\,\mathrm{d}z' + \int_z^{L-d} 4/c'(z')\,\alpha'_{\mathrm{A}}(z')\,\mathrm{d}z'. \tag{41}$$

The expression for the conditional probability to weight equation (41) is not changed. Then combining the positive and negative z contributions we see that in general

$$\begin{aligned} t'(\mathrm{S},\mathrm{A}) = \frac{1}{\Gamma_{\mathrm{Ai}}(L-d)}\biggl(&\int_{-d}^0 \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,\mathrm{d}z \int_z^0 n(z')/\Gamma_{\mathrm{Be}}(z')\,\mathrm{d}z' \\ &+ \int_0^{L-d} \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,\mathrm{d}z \int_0^z n(z')/\Gamma_{\mathrm{Ae}}(z')\,\mathrm{d}z' \\ &+ \int_{-d}^{L-d} \tfrac{1}{4}n(z)\,c(z)\,\rho(z)\,\mathrm{d}z \int_z^{L-d} 4/c'(z')\,\alpha'_{\mathrm{A}}(z')\,\mathrm{d}z'\biggr). \end{aligned} \tag{42}$$

From equations (23) and (24) it can be shown that $n(z)/\Gamma_{\mathrm{Ae}}(z)$ is never larger than $n_0(z)/\Gamma_{\mathrm{A0}}$. It follows from equation (38) that $t(\mathrm{S},\mathrm{A}) \leqslant t_0(\mathrm{S},\mathrm{A})$. That inelastic collisions speed up the average transit time of the zero energy loss electron group is physically sensible because the least likely electrons to undergo inelastic collisions are those that drift and diffuse from S to A the most quickly. The times $t(\mathrm{S,A})$ and $t'(\mathrm{S,A})$ have been derived under steady state conditions and how they relate to actual time dependent pulse profiles requires further development to time dependent transport theory.

5. Summary

The generalized electron source strength functions $S_{\mathrm{A}}(z)$ and $S_{\mathrm{B}}(z)$ and the generalized absorption coefficients $\alpha_{\mathrm{A}}(z)$ and $\alpha_{\mathrm{B}}(z)$ have been used to construct expressions for the electron number density and electron flux under simplifying restrictions appropriate for an atomic gas between a pair of closely spaced electrodes. The new method of taking into account inelastic collisions has been shown to be equivalent to solving the two-term approximation to the Boltzmann equation. Without appealing to special models of elastic and inelastic cross sections it has been possible to obtain a relatively general understanding of how inelastic collisions influence the electron flux. The connection between the results of our steady state analysis and parameters that may be measured in a pulsed electron source experiment has been pointed out. Although requiring further development in order to introduce less restrictive assumptions on the energy loss processes, it is believed that the methods and concepts introduced here have provided useful insights into the transport problem near the gas–electrode boundary.

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Appendix

So far the electron energies $\varepsilon(z)$ and $\varepsilon'(z)$ are parametrically dependent on z. To compare with the usual scalar and vector equations [see Huxley and Crompton (1974), equations (6.36) and (6.37)] it is necessary to make z and ε independent variables. Because the swarm is made up of two different energy groups the obvious generalization of equation (7) in Part I is

$$g_0(z,\varepsilon) = h(z,\varepsilon)\,\delta(\varepsilon-\varepsilon(z)) + h'(z,\varepsilon)\,\delta(\varepsilon-\varepsilon'(z)), \qquad \text{(A1a)}$$

with

$$n(z) = 2\pi(2/m)^{3/2}\varepsilon(z)^{\frac{1}{2}}h(z,\varepsilon(z)), \qquad n'(z) = 2\pi(2/m)^{3/2}\varepsilon'(z)^{\frac{1}{2}}h'(z,\varepsilon'(z)). \tag{A1b, c}$$

Similarly equation (5) Part I is replaced by

$$\varepsilon g_1(z,\varepsilon) = \gamma_{Ae}(z,\varepsilon)\,\delta(\varepsilon-\varepsilon(z)) + \gamma_{Ai}(z,\varepsilon)\,\delta(\varepsilon-\varepsilon'(z)), \tag{A2a}$$

with

$$\Gamma_{Ae}(z) = (8\pi/3m^2)\gamma_{Ae}(z,\varepsilon(z)), \qquad \Gamma_{Ai}(z) = (8\pi/3m^2)\gamma_{Ai}(z,\varepsilon'(z)). \tag{A2b, c}$$

Using these expressions for g_0 and εg_1 the time independent scalar equation [see equation (3a) in Part I] becomes

$$\begin{aligned}\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial z} + eE\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial\varepsilon} &= \left(\frac{\partial\gamma_{Ae}(z,\varepsilon)}{\partial z} + eE\frac{\partial\gamma_{Ae}(-z,\varepsilon)}{\partial\varepsilon}\right)\delta(\varepsilon-\varepsilon(z))\\ &\quad + \left(\frac{\partial\gamma_{Ai}(z,\varepsilon)}{\partial z} + eE\frac{\partial\gamma_{Ai}(z,\varepsilon)}{\partial\varepsilon}\right)\delta(\varepsilon-\varepsilon'(z)),\end{aligned} \tag{A3a}$$

and the vector equation [see (3b) in Part I] becomes

$$\begin{aligned}&\frac{\partial g_0(z,\varepsilon)}{\partial z} + eE\frac{\partial g_0(z,\varepsilon)}{\partial\varepsilon} + \frac{g_1(z,\varepsilon)}{l(\varepsilon)}\\ &\quad = \left(\frac{\partial h(z,\varepsilon)}{\partial z} + eE\frac{\partial h(z,\varepsilon)}{\partial\varepsilon} + \frac{\gamma_{Ae}(z,\varepsilon)}{\varepsilon\, l(\varepsilon)}\right)\delta(\varepsilon-\varepsilon(z))\\ &\quad + \left(\frac{\partial h'(z,\varepsilon)}{\partial z} + eE\frac{\partial h'(z,\varepsilon)}{\partial\varepsilon} + \frac{\gamma_{Ai}(z,\varepsilon)}{\varepsilon\, l(\varepsilon)}\right)\delta(\varepsilon-\varepsilon'(z)).\end{aligned} \tag{A3b}$$

Equations (A3) are derived using the identity

$$\frac{\partial\delta(\varepsilon-\varepsilon(z))}{\partial z} + eE\frac{\partial\delta(\varepsilon-\varepsilon(z))}{\partial\varepsilon} = 0.$$

To advance further we need to postulate explicit expressions for $\gamma_{Ae}(z,\varepsilon)$ and $\gamma_{Ai}(z,\varepsilon)$. There is a degree of non-uniqueness when decoupling z from $\varepsilon(z)$ so that a trial and error approach is required, using as guides equations (A1b), (A1c), (A2b) and (A2c), the definition

$$h(z,\varepsilon(z)) = \int h(z,\varepsilon)\,\delta(\varepsilon-\varepsilon(z))\,\mathrm{d}\varepsilon$$

etc. and equations (18), (23) and (24). It is postulated that

$$\begin{aligned}\gamma_{Ae}(z,\varepsilon) = \tfrac{4}{5}\varepsilon\,\alpha_A(\varepsilon)\,h(z,\varepsilon) + 3\varepsilon\,\alpha_A(\varepsilon)\,N\int\frac{q_i(\varepsilon'-\varepsilon_i)}{\alpha_A(\varepsilon')}\,\mathrm{d}\varepsilon'\\ \times\int_z^{L-d} h(z',\varepsilon')\,\delta(\varepsilon'-\varepsilon(z'))\,\mathrm{d}z',\end{aligned} \tag{A4a}$$

$$\gamma_{\mathrm{Ai}}(z,\varepsilon) = 3N\int \frac{\varepsilon'\alpha_{\mathrm{A}}(\varepsilon'-\varepsilon_{\mathrm{i}})\,q_{\mathrm{i}}(\varepsilon'-\varepsilon_{\mathrm{i}})}{\alpha_{\mathrm{A}}(\varepsilon'-\varepsilon_{\mathrm{i}})+\alpha_{\mathrm{B}}(\varepsilon'-\varepsilon_{\mathrm{i}})}\,\mathrm{d}\varepsilon' \int_0^z h(z',\varepsilon')\,\delta(\varepsilon'-\varepsilon(z'))\,\mathrm{d}z'$$

$$-3N\int \frac{\varepsilon'\alpha_{\mathrm{B}}(\varepsilon'-\varepsilon_{\mathrm{i}})\,q_{\mathrm{i}}(\varepsilon'-\varepsilon_{\mathrm{i}})}{\alpha_{\mathrm{A}}(\varepsilon'-\varepsilon_{\mathrm{i}})+\alpha_{\mathrm{B}}(\varepsilon'-\varepsilon_{\mathrm{i}})}\,\mathrm{d}\varepsilon' \int_z^{L-d} h(z',\varepsilon')\,\delta(\varepsilon'-\varepsilon(z'))\,\mathrm{d}z'. \quad \text{(A4b)}$$

Note that $\gamma_{\mathrm{Ai}}(z,\varepsilon)$ is actually independent of ε. The RHS of equation (A3a) can now be calculated. The result is considerably simplified by collecting terms of the same form as those within the large parentheses in equation (A3b). Both these terms in parentheses are zero as is easily proved from equations (20) and (22). The scalar equation reduces to

$$\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial z} + eE\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial\varepsilon} = 3N\int\left(\varepsilon'\delta(\varepsilon-\varepsilon'(z)) - \frac{\varepsilon\,\alpha_{\mathrm{A}}(\varepsilon)}{\alpha_{\mathrm{A}}(\varepsilon')}\delta(\varepsilon-\varepsilon(z))\right)$$

$$\times\, q_{\mathrm{i}}(\varepsilon'-\varepsilon_{\mathrm{i}})\,h(z,\varepsilon')\,\delta(\varepsilon'-\varepsilon(z))\,\mathrm{d}\varepsilon'. \quad \text{(A5)}$$

It is easily seen that $\alpha_{\mathrm{A}}(\varepsilon)$ and $\alpha_{\mathrm{A}}(\varepsilon')$ actually disappear from the RHS because the only nonzero contribution is for $\varepsilon = \varepsilon'$. As required, our scalar equation can be formulated independent of the boundary condition. Equation (A5) can be rewritten as

$$\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial z} + eE\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial\varepsilon} = 3N\int\{\delta(\varepsilon-\varepsilon'(z))-\delta(\varepsilon-\varepsilon(z))\}$$

$$\times\, q_{\mathrm{i}}(\varepsilon'-\varepsilon_{\mathrm{i}})\,\varepsilon' h(z,\varepsilon')\,\delta(\varepsilon'-\varepsilon(z))\,\mathrm{d}\varepsilon'. \quad \text{(A6)}$$

We can write

$$h(z,\varepsilon')\,\delta(\varepsilon-\varepsilon(z))\,\delta(\varepsilon'-\varepsilon(z)) = \{h(z,\varepsilon)\,\delta(\varepsilon-\varepsilon(z))\}\,\delta(\varepsilon'-\varepsilon),$$

and hence

$$h(z,\varepsilon')\,\delta(\varepsilon-\varepsilon(z))\,\delta(\varepsilon'-\varepsilon(z)) = g_0(z,\varepsilon)\,\delta(\varepsilon'-\varepsilon). \quad \text{(A7a)}$$

Defining $\varepsilon_1 = \varepsilon+\varepsilon_{\mathrm{i}}$ and noting that $\delta(\varepsilon-\varepsilon'(z)) = \delta(\varepsilon_1-\varepsilon(z))$, we find

$$h(z,\varepsilon')\,\delta(\varepsilon-\varepsilon'(z))\,\delta(\varepsilon'-\varepsilon(z)) = g_0(z,\varepsilon_1)\,\delta(\varepsilon'-\varepsilon_1). \quad \text{(A7b)}$$

These results reduce equation (A6) to

$$\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial z} + eE\frac{\partial\varepsilon g_1(z,\varepsilon)}{\partial\varepsilon} + 3N\varepsilon\, q_{\mathrm{i}}(\varepsilon-\varepsilon_{\mathrm{i}})\,g_0(z,\varepsilon) - 3N\varepsilon_1\, q_{\mathrm{i}}(\varepsilon_1-\varepsilon_{\mathrm{i}})\,g_0(z;\varepsilon_1) = 0. \quad \text{(A8)}$$

Equation (6.36) of Huxley and Crompton (1974) can be written as

$$\frac{\partial g_0(r,c,t)}{\partial t} + \tfrac{1}{3}c\,\nabla_r\cdot\boldsymbol{g}_1(r,c,t) + \frac{1}{4\pi c^2}\frac{\partial}{\partial c}\left(n\,\sigma_{\mathrm{E}}(r,c,t) - n\,\sigma_{\mathrm{coll}}(r,c,t)\right) = 0, \quad \text{(A9a)}$$

where

$$n\,\sigma_{\mathrm{E}}(r,c,t) = \tfrac{4}{3}\pi c^2(e\boldsymbol{E}/m)\cdot\boldsymbol{g}_1(r,c,t), \quad \text{(A9b)}$$

$$\sigma_{\mathrm{coll}}(r,c,t) = \sigma_{\mathrm{ce}}(r,c,t)+\sigma_{\mathrm{ci}}(r,c,t). \quad \text{(A9c)}$$

Here $\sigma_{\rm ce}$ and $\sigma_{\rm ci}$ are elastic and inelastic collision cross sections defined for electron fluxes across $c, c+{\rm d}c$ in velocity space. We only need the steady state case neglecting elastic collisions; $\sigma_{\rm coll}$ is then approximated by

$$n\,\sigma_{\rm ci}(z,c) = 4\pi N \int_c^{c_1} g_0(z,c')\,c'^3\,q_{\rm i}(c'-c_{\rm i})\,{\rm d}c'\,, \tag{A10}$$

where $c_{\rm i}$ is the electron speed at the threshold energy and $c_1 = (c^2+c_{\rm i}^2)^{\frac{1}{2}}$. Transforming equations (A9a) and (A10) to the variable ε we find that

$$\frac{\partial \varepsilon g_1(z,\varepsilon)}{\partial z} + \frac{\partial}{\partial \varepsilon}\left(eE\varepsilon\, g_1(z,\varepsilon) - 3N\int_\varepsilon^{\varepsilon_1} \varepsilon' g_0(z,\varepsilon')\,q_{\rm i}(\varepsilon'-\varepsilon_{\rm i})\,{\rm d}\varepsilon'\right) = 0\,, \tag{A11}$$

which is equivalent to equation (A8). Our technique of introducing inelastic collision effects on the electron flux and number density through generalized electron source strengths and 'electrode' absorption coefficients $\alpha_{\rm A}(z)$ and $\alpha_{\rm B}(z)$ is equivalent to solving the scalar equation.

The appropriate steady state vector equation is (Huxley and Crompton, equation 6.37)

$$\begin{aligned}&\frac{\partial g_0(z,\varepsilon)}{\partial z} + eE\frac{\partial g_0(z,\varepsilon)}{\partial \varepsilon} + \frac{g_1(z,\varepsilon)}{l(\varepsilon)}\\ &+N\,q_{\rm i}(\varepsilon-\varepsilon_{\rm i})\,g_1(z,\varepsilon) - (N\varepsilon_1/\varepsilon)q_{1\rm i}(\varepsilon_1-\varepsilon_{\rm i})\,g_1(z,\varepsilon_1) = 0\,,\end{aligned} \tag{A12}$$

where $q_{1\rm i}(\varepsilon-\varepsilon_{\rm i})$ is only nonzero through anisotropy of the inelastic collision cross section (see Huxley and Crompton, equation 6.6). Equation (A3b) omits the two terms in the second line of equation (A12), but otherwise agrees. This discrepancy corresponds to the contribution of inelastic collisions to the momentum transfer; however, for atomic gases where the inelastic cross section is generally much smaller than the elastic cross section this is not a serious omission. It is worth noting that isotropic inelastic collisions can be treated exactly by our method by simply redefining $l(\varepsilon)$ everywhere to incorporate the first term in the second line of (A12) in the mean free path. Our method of solving the electron flux problem taking into account inelastic collisions is equivalent to the assumption that the major effect is through the distribution of energies f_0. Because of the very large energy losses $\varepsilon_{\rm i}$ this is an excellent approximation for atomic gases.

Very little of the theory is changed if more than one inelastic threshold is introduced, provided that any given electron is restricted to no more than one inelastic collision. The g_0 and εg_1 given by equations (A1a) and (A2a) only require the introduction of more terms to take into account more energy classes. Each energy class satisfies equations of the form of (20) and (22). Equations (23) and (24) only need to replace $\rho(z)$ by the sum $\sum_i \rho_i(z)$ and equation (18) is essentially unchanged except that $\Gamma_{{\rm A}i}(z)$ is associated with its own $\rho_i(z)$ and absorption parameters $\alpha_{\rm A}^{(i)}$ and $\alpha_{\rm B}^{(i)}$.

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